216715 NEWCOM++
DR.2.1
Review of the State-of-the-Art

Contractual Date of Delivery to the CEC: T0+3
Actual Date of Delivery to the CEC: T0+5
Editor(s): Raymond Knopp
Participating institutions: CNRS, Technion, UPC, CTTC, CNIT, Chalmers, RWTH, UCL/UGent,FTW/TUWien
Contributors: Jin-Hui Chen, Yonina Eldar, Petros Elia, Thomas Erikson, Lennert Jacobs, Joakim Jalden, Florian Kaltenberger, Raymond Knopp, Gerald Matz, Marc Moeneclaey, Antonio Pascual, Ana Perez, Umer Salim, Shlomo Shamai, Dirk Slock, Dan Zhang, Nizar Zorba
Internal Reviewer(s): S. Benedetto, CNIT
Workpackage number: WPR.2
Nature: R
Total Effort Spent: 7.5 mm
Dissemination Level: Public
Version: 1.0

Abstract:

This deliverable presents an overview of the activities pertaining to feedback and imperfect channel estimation and the interplay between the two in the form that will be considered within WPR.2 of the NEWCOM++ project. Their relation stems from the the fact that feedback of channel state information is becoming an integral part of modern wireless networks (UMTS-HSDPA, UMTS-LTE, 802.16m), both from the point-of-view of resource scheduling and advanced multi-antenna signal processing and therefore the ability to resolve the channel at the transmitter is ultimately related to what can be estimated at the receiver. Moreover, the analysis of such systems should be considered in the two-way setting.
We give an overview of the state-of-the-art in relation to the three tasks of WPR.2 namely, Imperfect channel estimation, Point-to-point and Point-to-multipoint Two-Way Channels, and Precoding Techniques for the MIMO Broadcast Channel. At the end of the discussion of each task, we include the initial planning for joint research research activities during the first year of NEWCOM++.

Keyword list: Channel Estimation, Unknown Channels, Feedback, MIMO Broadcast channels

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1 INTRODUCTION

WPR.2 aims to explore two crucial aspects in broadband wireless communications, namely the resolution of the channel state at the receiver and the use of feedback. These are related primarily since feedback of channel state information is becoming an integral part of modern wireless networks, both from the point-of-view of resource scheduling and advanced multi-antenna signal processing and therefore the ability to resolve the channel at the transmitter is ultimately related to what can be estimated at the receiver.

Channel estimation is the process by which the receiver resolves the channel state. It is well-known that as systems become more and more rich in terms of bandwidth and spatial processing (transmit antennas), which is what we are witnessing in the evolution of 3G networks and will surely continue in the future, the number of degrees of freedom to be estimated increases. This increase is approximately linear in both bandwidth and spatial dimensions for practical channel bandwidths. Moreover, due to the mobility of user terminals and objects in their vicinity the channel does not remain constant for long periods of time (a few milliseconds suffice for the channel to have changed completely). This is particularly true in cellular networks where support for high-mobility is an ever-increasing feature. As a result of both these facts, resolution of all degrees of freedom characterizing the channel state is a difficult task.

Although it is well understood that channels are time-varying, a proper analysis of the resulting effects on system performance even for one-way communications is still incomplete. Significant headway has nonetheless been made in very recent years, both from the point-of-view of information-theoretic limits and error-rate characterization. More analytical work is required for broadband MIMO channels to put to rest the analytical treatment of channel state resolution. Furthermore, more refined practical techniques inspired from this analysis combining channel estimation and decoding are still active areas in the wireless communications research community.

Another key feature of most modern radio systems is the fact that they allow for two-way communication. This allows for a sharing of the medium in both directions for both communication and low-layer signalling of channel quality indicators and decoding capacity indicators. In addition to studying the effect of imperfect channel knowledge at the receiving end we strive to study methods to encode it for the return channel as a function of the allocated bandwidth for feedback. Proper exploitation of incomplete feedback for both point-to-point two-way channels and multiuser/broadcast channels is still a very open area for research.

The third theme of WPR.2 covers what is most likely the key practical problem in wireless communications today, namely precoding for the multi-antenna broadcast channel. This is a model for the downlink in a cellular network, where multiple-antennas (at least 4) will surely be used in evolving standards such as UMTS-LTE and 802.16m combined with feedback channels for channel state information and decoding capacity indicators. Based on existing theoretical research, which focuses on multi-antenna wireless broadcast channels, it is well understood now, that precoding is fundamental in the strive to design modern wireless communications systems that will approach theoretical bounds. This subject attracts recently massive attention of researchers, and has also emerged in future wireless standards. While serious theoretical progress has been reported as of late, addressing a variety of state of art concepts, such as dirty-paper coding. Yet the understanding of efficient robust approaches which cope with practical conditions is in its preliminary stages. One of the central aspects that calls for a deeper understanding is the impact of the accuracy of channel state information (CSI) on various communications strategies. Under ideal assumptions, recent theoretical developments do identify the optimal approaches. Once we deviate from ideal assumptions, it is unclear what constitutes the optimal strategy. Even within the restricted class of linear precoding techniques, such as zero-forcing, there is no clear understanding how to operate under these practical assumptions, and exhibiting robust features. The study focuses on precisely these aspects, which we believe have both theoretical and practical implications. The goal will focus on gaining analytical understanding of the optimal linear approaches under different models which reflect partial knowledge of the CSI parameters

We consider further non linear robust precoding strategies relying on information theoretic insights and aimed at identifying robust techniques within the class of dirty-paper approaches, as well as more practical general vector perturbation methods. Finally, the precoding techniques are intimately linked with proper scheduling and required feedback information about CSI, which essentially touches also upon network aspects. Scheduling, and CSI feedback demands cannot be interpreted as a stand-alone entities, but rather as an inherent part of a unifying approach of robust precoding. We plan on conducting research to the end of understanding the basic role of the quality of CSI needed at transmitting ends, as to provide the full promise of precoding, and primarily the superior multiplexing gain. This effort will include also an attempt to identify ultimate theoretical bounds on minimal demands of the feedback link that maintain optimal performance, in terms of multiplexing gain (degrees of freedom) as well as the gap to capacity. Comparison to actual robust pre-processing schemes is also planned, and is to be compared to the ultimate bounds.

1.1 Organization of the document

The remainder deliverable is organized in four sections, reflecting the three tasks of WPR.2 and an overview of the interaction between WPR.2 and the other WPR in NEWCOM++ (Section 2). Section 3 deals with the area of imperfect channel estimation and strives to present the state-of-the-art methods for channel estimation as well as system analytical tools from information and communication theory pertaining to the effects of imperfect resolution of the channel state. We first provide an overview of wireless channel estimation focusing on broadband OFDM-MIMO systems (in the spirit of UMTS-LTE and WIMAX 802.16m). This should be seen as setting the stage for a common language between the various research groups who will collaborate on the subject. We then provide an overview of recent advances in information theory related to the effects of time-varying MIMO channels and their capacity analysis. We then provide an overview on recent results their error-rate analysis. Finally we present the current avenues for research on these topics in the context of the first year of NEWCOM++.

Section 4 covers the area related to two-way communications, both point-to-point and point-to-multipoint. We begin with a short review of classical feedback communications followed by a survey of recent advances in MIMO Hybrid-Automatic-Repeat-Request systems which constitute a modern practical application of feedback from a link perspective which is now commonplace in evolving wireless standards. We then provide an overview of recent work on representation of wireless channels for feedback channels. A review of recent results on proposals for the use of analog feedback is then provided. In order to show a parallel with the work of WPR.6 which deals with co-operative wireless networks, we review some recent work highlighting the benefits of feedback in such scenarios. We conclude the section with the current directions for research on these topics in the context of the first year of NEWCOM++.

In Section 5 we devote our attention to the important problem of precoding strategies for the MIMO broadcast channel. We begin with a summary of the fundamental limits of MIMO broadcast channels followed by an overview of precoding techniques, with a special emphasis on non-linear vector perturbation. We then provide a review of recent results on opportunistic beamforming with limited-feedback. Finally we end with the current directions for research on these topics in the context of the first year of NEWCOM++.

Finally, a small conclusion is provided in section 6.

2 INTEGRATION OF WPR.2 IN NEWCOM++

Here we provide a simple summary of the foreseen interactions of WPR.2 with other WPR.

2.1 WPR.1

The subject of WPR.1 is wireless channel modeling and measurement. Channel models are fundamental to accurate analysis of system performance and fundamental communication limits, whether the metric is link or system information rate or link error probability. As a result, there is a very strong relationship between WPR.1 and this workpackage, in the sense that channel modeling aspects will be used as input to analysis.

2.2 WPR.3

WPR.3 considers adaptive modulation and coding schemes and abstraction models for physical-layer processing. This work firstly relies heavily on both feedback schemes based on channel quality indicators in order to adapt coded-modulation strategies dynamically. Here WPR.2 can provide input regarding feedback protocols for next-generation wireless coding systems, primarily Hybrid-ARQ-based adaptive modulation and power-control strategies. In addition, information regarding the dimensioning of feedback channels and the granularity of the information used for feedback will be provided. The second important aspect of WPR.3 which can make use of the results of WPR.2 is physical-layer abstraction techniques. The latter requires accurate models for system performance, for instance error-rates of link. Here, WPR.2 will provide accurate characterizations of coded MIMO-OFDM channels under imperfect knowledge of channels at both ends of the communication link.

2.3 WPR.6

In WPR.6 cross-layer techniques for Cooperative communications and relaying are considered. Cooperative wireless networks are two-way in nature and thus the physical-layer results of WPR.2 are of interest to the work carried out in WPR.6. Specifically, the work carried out here in the context of feedback-based retransmission protocols for distributed relay channels is of prime importance to both workpackages. Output of WPR.6 will also be sought with respect to collaborative network models in order to properly assess the impact of imperfect resolution of the channel state on network scenarios considered there.

2.4 WPR.8

WPR.8 considers scheduling and adaptive radio resource assignment and can make use of the work here in a similar sense to WPR.3. The main difference is that system-wide (multi-user) results are pertinent as opposed to point-to-point link characterizations. This applies primarily to the WPR.2 work on MIMO broadcast techniques for single and multi-cell cellular systems.

3 IMPERFECT CHANNEL ESTIMATION

3.1 Wireless Channel Estimation

In OFDM systems, the introduction of a cyclic prefix leads in the frequency domain to a set of parallel memoryless channels at the various tones/subcarriers. In that case, optimal reception for the various tones involves per tone processing that may require accurate knowledge of the channel at the tones. Hence channel estimation is an important issue in multi-antenna OFDM transmission/reception, especially if channel knowledge is used at the transmitter.

In order to estimate the channel accurately, it is mandatory to pay close attention to all correlations between channel coefficients, such as in time, in frequency and in space. The channel response can be estimated in the time domain or in the frequency domain. On the one hand, the pilot symbols are available in the frequency domain. On the other hand, the frequency domain correlation is most easily expressed in the time domain. This and other considerations appear to suggest a time domain channel estimation approach. Such an approach requires frequent transformations between time and frequency domains, the complexity of which can be limited by pruning the FFT. The relative contribution of the various types of side information to be exploited in the channel estimation is discussed.

The methods to be discussed can be considered as "rank reduction" techniques. These techniques can be organized in terms of *a priori* and *a posteriori* techniques. A priori rank reduction techniques correspond in fact to (time-invariant) reparameterizations of the (in general) Multi-Input Multi-Output (MIMO) channel transfer function in terms of a reduced set of degrees of freedom. These a priori approaches correspond to what we shall call here deterministic parameter modeling techniques. The a posteriori rank reduction techniques correspond to Linear Minimum Mean Squared Error (LMMSE) parameter estimation approaches, taking into account a priori correlations in the channel coefficients. These a posteriori approaches correspond to what we shall call here statistical parameter modeling techniques with ensuing Bayesian parameter estimation. These techniques may be called *a posteriori* because they could be applied as a second stage to a deterministic estimate resulting from a first estimation stage. If the correlation matrices to be used in the second stage are singular with a reduced rank r, then in fact the second stage incorporates a reduction of the number of degrees of freedom to r, as the a priori rank reduction techniques do.

The end result is that without the exploitation of correlation structure in the channel, it is impossible to estimate the channel correctly so that the effect of channel estimation errors would be negligible. However, with the exploitation of frequential, temporal and/or spatial correlation, it becomes very well possible to make the channel estimation errors negligibly small. The issue in practice is which correlation to exploit (and in which way) to obtain the proper reduction in channel estimation error at the smallest computational cost.

These considerations on channel estimation are elaborated here in an OFDM setting because this simplifies the treatment for a frequency-fading channel. However, the essence of most conclusions holds in a general transmission setting.

3.1.1 SIMO OFDM Systems

The availability of multiple receive antennas leads to Single Input Multiple Output (SIMO) systems, which we shall discuss first.

Consider a radio system with a single input x_l and multiple, p, outputs (RX antennas) y_i per sample period

$$\underbrace{\mathbf{y}[n]}_{p\times 1} = \sum_{j=0}^{L} \underbrace{\mathbf{h}[j]}_{p\times 1}\,\underbrace{\mathbf{x}[n-j]}_{1\times 1} = \underbrace{H(q)}_{p\times 1}\,\underbrace{\mathbf{x}[n]}_{1\times 1} \tag{1}$$

where $H(q) = \sum_{j=0}^{L} \mathbf{h}[j]\, q^{-j}$ is the SIMO system transfer function corresponding to the z transform of the impulse response $\mathbf{h}[.]$. Equation (1) mixes time domain and z transform domain notations to obtain a compact representation. In $H(q)$, z is replaced by q to emphasize its function as an elementary time advance operator over one sample period. Its inverse corresponds to a delay over one sample period: $q^{-1}\mathbf{x}[n] = \mathbf{x}[n-1]$.

Consider an OFDM system with N samples per OFDM symbol. The introduction of a cyclic prefix of K samples means that the last K samples of the current OFDM symbol (corresponding to N samples) are repeated before the actual OFDM symbol. If we assume w.l.o.g. that the current OFDM symbol starts at time 0, then samples $\mathbf{x}[N-K]\cdots\ \mathbf{x}[N-1]$ are repeated at time instants $-K, \ldots, -1$. This means that the output at sample periods $0,\ldots,N-1$, or hence the output for OFDM symbol period 0, can be written as

$$\mathrm{Y}[0] \;=\; \mathrm{H}\,\mathrm{X}[0] + \mathrm{V}[0]\ . \tag{2}$$

where

$$\mathrm{Y}[0] \;=\; \begin{bmatrix} \mathbf{y}[0] \\ \cdots \\ \mathbf{y}[N-1] \end{bmatrix} \tag{3}$$

and similarly for X[0] and V[0], and

$$\mathrm{H} = \begin{bmatrix} \mathbf{h}[0] & & & \mathbf{h}[L] & \cdots & \mathbf{h}[1] \\ \vdots & \mathbf{h}[0] & & & \ddots & \vdots \\ \vdots & & & & & \mathbf{h}[L] \\ \mathbf{h}[L] & \vdots & & & & \\ & \mathbf{h}[L] & & \ddots & & \\ & & & \ddots & & \\ & & & & & \mathbf{h}[0] \end{bmatrix}. \tag{4}$$

The matrix H is not only Toeplitz but even circulant: each row is obtained by a cyclic shift to the right of the previous row (to be precise, the matrix is a square block matrix of course). The relation in (4) holds if the channel delay spread does not exceed the cyclic prefix length: $L \leq K$. Note also that in OFDM, the received data corresponding to the cyclic prefix time instants $(-K, \ldots, -1)$ do not get used.

Consider now applying an N-point FFT to both sides of (2) at OFDM symbol period m:

$$F_{N,p}\mathrm{Y}[m] = F_{N,p} H\, F_N^{-1} \quad F_N \mathrm{X}[m] + F_{N,p}\mathrm{V}[m] \tag{5}$$

or with new notations:

$$\mathrm{U}[m] \quad = \quad \mathscr{H} \qquad \mathrm{A}[m] \quad + \quad \mathrm{W}[m] \tag{6}$$

where $F_{N,p} = F_N \otimes I_p$ (Kronecker product: $A \otimes B = [a_{ij}B]$), F_N is the N-point $N \times N$ DFT matrix, $\mathscr{H} = \mathrm{diag}\{\mathbf{H}_0,\ldots,\mathbf{H}_{N-1}\}$ is a block diagonal matrix with diagonal blocks $\mathbf{H}_k = \sum_{l=0}^{L} \mathbf{h}[l]\; e^{-j2\pi\frac{1}{N}kl}$, the $p \times 1$ channel transfer function at tone k (frequency k/N times the sample frequency). In Orthogonal Frequency Division Multiplexing (OFDM), the transmitted symbols (belonging to a symbol constellation/finite alphabet) are in $\mathrm{A}[m]$ and hence are in the frequency domain. The corresponding time domain samples are in $\mathrm{X}[m]$. The OFDM symbol period index is m.

Taking into account the cyclic prefix also, the OFDM symbol rate is a fraction $\frac{1}{N+K}$ of the sample rate. In OFDM, we need to make a subtle difference between the sample rate that we have introduced above and the sampling rate, the rate at which the continuous-time signal gets sampled. At the transmitter, the vector of symbols $\mathrm{A}[m]$ gets inverse Fourier transformed to get the vector $\mathrm{X}[m]$ of N samples for OFDM symbol period m. A cyclic prefix of P samples gets inserted as indicated previously. The resulting discrete-time signal gets converted into a continuous-time signal via a lowpass filter (pulse shape) and gets upmodulated to the carrier frequency. At the receiver the signal gets downmodulated and sampled. This leads to the complex channel impulse response in the baseband model introduced so far. The sampling frequency employed at the receiver is normally equal to the sample frequency (no oversampling is used). This is because typically the OFDM standard puts zero valued symbols on the upper and lower tones (subcarriers) so that, even with practical transmitter/receiver filters, there is no excess bandwidth (w.r.t. (with respect to) the sample rate) to be exploited.

The components of V are considered white noise, hence the components of W are white also. At tone (subcarrier) $n \in \{0, \ldots, N-1\}$ we get the following input-output relation

$$\underbrace{\mathbf{u}_n[m]}_{p\times 1} = \underbrace{\mathbf{H}_n[m]}_{p\times 1}\, \underbrace{a_n[m]}_{1\times 1} + \underbrace{\mathbf{w}_n[m]}_{p\times 1} \tag{7}$$

where the symbol $a_n[m]$ belongs to some finite alphabet (constellation).

3.1.1.1 SIMO OFDM Reception

With circular Gaussian complex white noise in (7), $\mathbf{w}_n[m] \sim \mathscr{CN}(0, \sigma_w^2 I_p)$, the maximum likelihood (ML) estimate of the symbol $a_n[m]$ (treated as deterministic unknown) from the received signal $\mathbf{u}_n[m]$ is

$$\widehat{a}_n[m] = \frac{1}{\mathbf{H}_n^H[m]\,\mathbf{H}_n[m]}\,\mathbf{H}_n^H[m]\,\mathbf{u}_n[m] \tag{8}$$

where $\mathbf{H}_n^H[m]$ corresponds to maximum ratio combining. This ML solution also corresponds to the minimum mean squared error (MMSE) zero forcing (ZF) linear receiver output, or also to the unbiased MMSE (UMMSE) linear receiver output.

If the noise is not spatially white (has directional characteristics) due to the presence of (stationary) interferers, then we may model it as $\mathbf{w}_n[m] \sim \mathscr{CN}(0, R_w(n))$ where $\mathrm{E}\,\mathbf{w}_n[m]\,\mathbf{w}_n^T[m] = 0$, $\mathrm{E}\,\mathbf{w}_n[m]\,\mathbf{w}_n^H[m] = R_w(n)$. In this case the ML/MMSEZF/UMMSE receiver front-end becomes

$$\widehat{a}_n[m] = \frac{1}{\mathbf{H}_n^H[m]\,R_w^{-1}(n)\,\mathbf{H}_n[m]}\,\mathbf{H}_n^H[m]\,R_w^{-1}(n)\,\mathbf{u}_n[m]\ . \tag{9}$$

The implementation of this approach requires however the estimation of $R_w(n)$ at each subcarrier from e.g.

$$\widehat{\mathbf{w}}_n[m] = \mathbf{u}_n[m] - \widehat{\mathbf{H}}_n[m]\,\widehat{\widehat{a}}_n[m] \tag{10}$$

in a decision directed mode ($\widehat{\widehat{a}}_n[m]$ is the result of a decision (with or without channel decoding) on $\widehat{a}_n[m]$). Instead of estimating $R_w(n)$ at each subcarrier (independently), it can perhaps be advantageously estimated in the time domain, by imposing a limited delay spread. Related work appears in [1].

3.1.2 Pilot Based Channel Estimation

For channel estimation purposes, pilot tones are available. These are tones at which the data symbol is fixed and known. Their power may be larger than the power of unknown data tones. The details of the distribution of the pilots in time and frequency are different in every OFDM based standard. If we let $\mathscr{P}[m]$ denote the set of pilot tones in OFDM symbol m, then $\mathscr{P}[m]$ is often periodic in m.

So, from the pilot tones $n \in \mathscr{P}[m]$ we get :

$$\begin{aligned}\mathbf{u}_n[m] &= \mathbf{H}_n[m]\,a_n[m] + \mathbf{w}_n[m]\\ \widehat{\mathbf{H}}_n[m] &= \mathbf{u}_n[m]/a_n[m] = \mathbf{H}_n[m] - \widetilde{\mathbf{H}}_n[m]\\ &= \mathbf{H}_n[m] + \mathbf{w}_n[m]/a_n[m]\end{aligned} \tag{11}$$

where $\widehat{\mathbf{H}}_n[m]$ is the brute frequency domain channel estimate with estimation error variance $\sigma^2_{\widetilde{\mathbf{H}}} = \sigma_w^2/\sigma_P^2$, σ_P^2 = pilot symbol variance. We can define an overall brute frequency domain channel estimate

$$\widehat{\mathbf{H}}_n[m] = \begin{cases} \mathbf{u}_n[m]/a_n[m] & , n \in \mathscr{P}[m] \\ 0 & , n \notin \mathscr{P}[m] \end{cases} \tag{12}$$

The brute channel estimate needs to be filtered to obtain a refined estimate $\widehat{\widehat{\mathbf{H}}}_n[m]$ in which the estimation error still depends on the same noise samples.

3.1.2.1 Channel Estimation Noise

We'll assume that the noise is uncorrelated between tones in an OFDM symbol and between OFDM symbols. This is obviously true for white noise but also holds approximately in the case of mildly colored noise due to the decorrelation property of the Fourier transform and the separation of OFDM symbols by cyclic prefixes. So we'll assume that within one OFDM symbol the noise $w_n[m]$ is uncorrelated between pilot and data tones.

The (refined) channel estimation error leads to noise increase at data tones $n \notin \mathscr{P}[m]$:

$$\begin{aligned} \mathbf{u}_n[m] &= \mathbf{H}_n[m]\, a_n[m] + \mathbf{w}_n[m] \\ &= \widehat{\widehat{\mathbf{H}}}_n[m]\, a_n[m] + \widetilde{\widetilde{\mathbf{H}}}_n[m]\, a_n[m] + \mathbf{w}_n[m] \end{aligned} \tag{13}$$

where $\widetilde{\widetilde{\mathbf{H}}}_n[m]$ is the estimation error associated with the refined channel estimate. The relative noise increase (similar to the misadjustment factor in the analysis of the LMS algorithm) :

$$\mathscr{M} = \frac{\sigma^2_{\widetilde{\widetilde{\mathbf{H}}}}\,\sigma_a^2}{\sigma_w^2} \tag{14}$$

should be $\ll 1$ in order for the channel estimation error to lead to negligible performance degradation. Without filtering of the brute channel estimate we have:
$\mathscr{M} = \frac{\sigma_a^2}{\sigma_P^2}\frac{N}{P} \gg 1$ where $\frac{\sigma_a^2}{\sigma_P^2}$ is the ratio of data tone power over pilot tone power and P = number of pilot tones, $\frac{N}{P} = \frac{\#\text{ of unknowns}}{\#\text{ of equations}}$: without the exploitation of any structure, the channel coefficient at every tone is a priori an independent unknown variable.

With channel estimate filtering one can obtain

$$\mathscr{M} = \frac{\sigma_a^2}{\sigma_P^2}\frac{N}{P}\,\alpha_{F,d}\,\alpha_{F,s}\,\alpha_{T,d}\,\alpha_{T,s}\,\alpha_S\,\alpha_I \;\ll\; 1 \tag{15}$$

where any factor $\alpha \in (0,1]$, $\alpha = 1$ for a filtering aspect that is not exploited Every factor α corresponds to the exploitation of particular structure in the channel as will be analyzed in detail below. In particular, $\alpha_I = \frac{P}{P + \frac{\sigma_a^2}{\sigma_P^2}N'}$ = reduction factor due to *iterative* channel estimation and data detection (N' = number of data tones). Indeed, if all the data gets detected (roughly without error, after channel decoding) and the channel estimation gets based on the detected data also, then those data act similar to pilots for the purpose of channel estimation. Note that $P + N' < N$ due to the fact that a number of tones are left unused (e.g. for frequency separation between adjacent channels).

3.1.3 Deterministic Frequency Domain Filtering

$\alpha_{F,d} = \frac{L}{N}$ consists of the exploitation of the finite delay spread (L samples) of the channel impulse response (deterministic channel model). We assume here that this delay spread can be equal to the cyclic prefix length (worst case assumption, the CP is normally designed to exceed the delay spread).

The reduction of the channel estimation error variance by a factor $\alpha_{F,d}$ can be accomplished by transforming the brute frequency domain channel estimate $\widehat{\mathbf{H}}[m]$ into the time domain and windowing the resulting time domain estimate to keep only the portion within the delay spread (CP). Windowing in the time domain is equivalent to filtering/convolution/interpolation in the frequency domain.

The expression $\alpha_{F,d} = \frac{L}{N}$ assumes that the sampling pattern of the pilot tones is sufficient to avoid aliasing after delay spread imposition so that the estimation error is only due to noise (and not approximation error). So in fact we assume that delay spread $\leq \min\{L,P\}$. If $P > L$, the pilots provide a certain amount of oversampling. Whereas without oversampling, the interpolation filter in the frequency domain (asuming that only the regularly spaced scattered pilots would be used) would have to correspond to the Fourier transform of a rectangular window, leading to an interpolation filter that is widely spread out; if there is on the other hand a certain amount of oversampling, it can be exploited to use interpolation filters that are less spread out. Of course, one can also consider approximate interpolation filters, such as e.g. linear interpolation (triangular interpolation filter). One then has to add a certain approximation error (due to partial aliasing) to the channel estimation error.

3.1.4 Statistical Frequency Domain Filtering

The deterministic delay spread (difference between largest and smallest delays) may be quite large. On the other hand, the effective delay spread L_{eff} of the power delay profile may be much smaller (e.g. when there are no other paths between the largest and the smallest delay paths). The proper exploitation of the power delay profile leads to $\alpha_{F,s} = \frac{L_{eff}}{L}$. Whereas L is the (total) delay spread (difference between maximum and minimum delay), L_{eff} is the effective number of channel coefficients in the channel power delay profile. If $L > 1$, then $L_{eff} \geq 2$ ($= 2$ for the case of two paths, corresponding to minimum and maximum delay).

The exploitation of the power delay profile can be accomplished by weighting the time domain channel estimate, not by a rectangular window as in the deterministic exploitation of the delay spread, but by a LMMSE weighting function that depends on the power delay profile. If all channel coefficients are considered independent, then this LMMSE weighting corresponds to

$$\widehat{\widehat{\mathbf{h}}}[j] = \frac{\sigma^2_{h[j]}}{\sigma^2_{h[j]} + \sigma^2_{\tilde{h}[j]}}\, \widehat{\mathbf{h}}[j] \tag{16}$$

Here $\sigma^2_{h[j]}$ is the variance of channel coefficient $\mathbf{h}[j]$ and hence represents the power delay profile (as a function of delay j). One can easily estimate $\sigma^2_{\widehat{h}[j]} = \sigma^2_{h[j]} + \sigma^2_{\tilde{h}[j]}$ by taking the sample variance of the channel coefficient estimates in the time domain in every OFDM symbol. Also, when the noise is white, $\sigma^2_{\tilde{h}[j]}$ is simply a certain multiple of the noise variance, which can be estimated from the error signals (received signal minus channel estimate times pilot data) at the pilot tones.

There could be more statistical information to be exploited in the channel response than just the power delay profile. This occurs if the channel impulse coefficients are correlated, which may be the case if the path delays fall in between sample instants. To exploit such correlation, one should not only estimate the channel coefficient powers but also their correlations (at least between neighboring (in delay) coefficients).

3.1.5 Deterministic Time Domain Filtering

The channel $\mathbf{h}[m]$ evolves as a function of time (OFDM symbol period) m. Each channel coefficient is a finite bandwidth signal though due to the finite Doppler spread. Deterministic time domain filtering consists of ideal lowpass filtering with bandwidth equal to the Doppler spread.

This leads to $\alpha_{T,d}$ = Doppler spread expressed as a fraction of the OFDM symbol rate. If the proceesing is going to be performed in block, corresponding e.g. to a block of channel coded data, then $\alpha_{T,d} \geq \frac{1}{M}$, M is the number of OFDM symbols in the block considered.

The lowpass filtering can e.g. be performed by a first-order filter with transfer function $\frac{1-\lambda}{1-\lambda z^{-1}}$. Then $\alpha_{T,d} = \frac{1-\lambda}{1+\lambda}$ if filter bandwidth > Doppler spread (+ channel distortion otherwise).

The lowpass filtering can equivalently be done (approximately) by windowing in the frequency domain (by computing the frequency domain response of the evolution of a channel impulse response coefficient over a number of OFDM symbols).

3.1.6 Statistical Time Domain Filtering

Apart from a deterministic Doppler spread (difference between minimum and maximum Doppler frequencies), there is also a Doppler profile in which the power may be distributed unevenly over the Doppler frequencies. This leads to a reduced effective Doppler spread.

$$\alpha_{T,s} = \frac{\text{effective Doppler spread}}{\text{deterministic Doppler spread}}$$

For instance, consider the extreme case of a two path channel with one path having Doppler shift $+f_D$, where f_D is the Doppler frequency, and the other path having Doppler shift $-f_D$. Then the deterministic Doppler spread is $2f_D$ whereas the effective Doppler spread is in fact zero (for just two specular paths).

The exploitation of the statistical information in both time and frequency domain may be performed jointly by a (channel impulse response coefficient) delay dependent Wiener filter in the time domain. A first order filter would be of the form $\frac{\gamma}{1-\beta z^{-1}}$. A first-order filter though does not allow to capture the details of the Doppler profile, only its bandwidth. The use of a first-order filter appears to be insufficient to model the finite bandwidth Doppler profile at high Doppler speeds.

3.1.7 Spatial Domain Filtering

The channel impulse responses may be correlated between the different antennas. One can exploit this correlation to further reduce the channel estimation variance. This leads to

$$\alpha_S \geq \frac{1}{p} = \frac{1}{\text{\# RX antennas}} \ . \qquad (17)$$

The lower bound (reduction by p) is attained when each spatial channel impulse response coefficient $\mathbf{h}[n]$ corresponds to the contribution of only a single path at the corresponding delay. In that case, the p coefficients of $\mathbf{h}[n]$ are proportional to just a single rapidly varying complex path amplitude, the direction of the $p \times 1$ vector varies only slowly, with the physical direction of the path. It appears that the exploitation of the spatial correlation has not yet been pursued much in the literature, certainly not in the context of OFDM systems. However, it requires to estimate $L+1$ $\;p \times p$ spatial correlation matrices, one for each channel coefficient delay. Especially when the spatial correlation gets combined with the temporal correlation, this requires the estimation of a channel covariance matrix of size $p\,(L+1)$ which represents a certain complexity for estimation and for its exploitation, and which also requires the accumulation of quite a bit of data (instantaneous channel estimates) and hence sufficient stationarity of the channel evolution (so that the channel correlations only change slowly, this is the slow fading).

3.1.8 Some Complexity Considerations

Many operations get simplified if the pilots appear in a regular pattern corresponding to a certain regular subsampling of the tones, even if the position of the subsampling grid varies between OFDM symbols.

The complexity of filtering/interpolating the brute frequency domain channel estimate directly in the frequency domain is proportional to $N^{'}$ (the number of data tones) and the number of pilots involved in one interpolation operation (= filter length in number of tones spanned, divided by subsampling factor in pilot tone positioning). If the pilots appear in a regular pattern, the interpolation is frequency invariant (except for the two borders).

For the transformation of channel estimates between the time and frequency domains, no complete FFTs are required but so-called *pruned* FFTs can be used, leading to lower complexity. For the IFFT to transform the brute frequency domain channel estimate to a channel impulse response with finite delay spread, one transforms a subsampled signal (if pilots appear at a subsampling grid) into a signal with limited duration (or of which only a limited duration is of interest). (note that due to the (possible) subsampling in the frequency domain, we get a periodic signal in the time domain, so $L \leq N/(subsampling factor)$ required). So there is a double pruning aspect. Perhaps choosing $L = N/(subsampling factor)$ may lead to a particularly interesting (low) complexity.

For the FFT to transform the finite delay spread impulse response to the frequency domain at all tones (of which only the data tones are needed, but they constitute the majority of the tones), pruning can again be used due to the finite length of the signal to be transformed.

The filtering in the time domain can be time-invariant over a block (a block can be made to correspond to a channel coding block), or can be made adaptive for continuous processing. In the case of block processing, the filter can be kept time-invariant at the edges of the block if some date from neighboring blocks can be used. Or the time-invariant Wiener filtering should be replaced by time-varying Kalman filtering if optimality is desired throughout the block and no data from neighboring blocks can be used. The complexity is proportional to the order of the FIR Wiener or Kalman filter.

3.1.9 Auxiliary Parameters to be Estimated

Channel estimate filtering (refining) requires the estimation of some additional parameters:

- Noise variance σ_w^2 (see higher).
- Channel impulse response delay spread or even power delay profile: can be obtained by (non-coherently) averaging channel impulse response coefficient estimate powers in time (and correcting/thresholding for estimation noise variance, see higher).
- Doppler spread and profile, or channel impulse response coefficient temporal correlation sequence: can again be estimated by computing temporal correlations of estimated channel coefficients and correcting the correlation at lag zero for the estimation noise variance (see below).

3.1.10 Solutions Proposed in the Literature

[2]: comparison of deterministic and statistical frequency domain filtering (no temporal filtering, single antenna). Estimation/filtering performed in the time domain via weighting matrices

[3]: analysis of the MSE of refined channel estimates obtained by 2D filtering. For the brute channel estimate, the use of all data (decision directed) is assumed. For the design of the 2D LMMSE filter, a fixed power delay profile and a fixed Doppler profile are assumed and the effect on the MSE due to a mismatch in these two profiles is analyzed. It is concluded that it is more robust to do 2D deterministic filtering with an overestimated delay spread and an overestimated Doppler spread. One should note that this conclusion is reached because no attempt is made to estimate the power delay and Doppler profiles. In [4], the same analysis is performed and the same conclusions are reached when the brute channel estimate is pilot based.

[5]: delay-dependent temporal Wiener filter coefficients are determined from linear prediction coefficients for the estimated channel coefficients, plus knowledge of σ_w^2. See also [6] for related work.

[1]: the 2D channel correlation function is assumed to be separable: the power delay profile gives the channel coefficients at each delay a separate variance, but the Doppler spectrum is assumed independent of delay (not true). The Doppler spectrum is estimated via a thresholded periodogram on the estimated channel coefficients, averaged over the delays.

3.1.11 Channel Variation within an OFDM Symbol Period

Channel variations within an OFDM symbol lead to intercarrier interference (ICI) (non-orthogonality of the tones). For the ICI to be negligible, we need the Doppler spread f_d to be small compared to the intercarrier spacing $1/Ts$. It appears that the ICI problem is negligible in WLAN applications with low mobility. Nevertheless, the following references deal with ICI.

[7] computes a universal upper bound on the ICI power $P_{ICI} \leq \frac{1}{12}(2\pi f_d T_s)^2$ where f_d is the (max) Doppler frequency and $1/Ts$ is the subcarrier spacing. Note that for the channel estimation noise to have negligible impact, it suffices that $\mathscr{M} \ll 1$. However, for the ICI to have negligible impact, we require that $P_{ICI} \ll \dfrac{1}{\text{SNR}}$!

[8] introduce a non-parsimonious time-varying channel model (matrix) leading to a huge number of parameters to be estimated and the complexity of the associated equalization problem is also forbidding.

[9] perform a statistical Taylor series expansion of channel coefficients in terms of frequency f around $f = 0$. The resulting parameter estimation and equalization problems are a bit cumbersome but the technique works.

[10] express the variation of the channel impulse response coefficients over an OFDM symbol in terms of subcarriers. This leads to $\mathscr{H}$ becoming a banded matrix instead of a diagonal matrix, with the number of diagonals being the Doppler spread expressed in terms of subcarrier spacing. The channel estimation becomes one of estimating one impulse response per subcarrier component. The temporal equalization problem gets transformed into an equalization problem in the frequency domain with possibly lower spread.

3.1.12 Extension to MIMO Channels

In the MIMO transmission case with q transmit antennas, at tone n in OFDM symbol m we get the following input-output relation

$$\underbrace{\mathbf{u}_n[m]}_{p\times 1} = \underbrace{\mathbf{H}_n[m]}_{p\times q}\,\underbrace{\mathbf{a}_n[m]}_{q\times 1} + \underbrace{\mathbf{w}_n[m]}_{p\times 1} \tag{18}$$

where $\mathbf{a}_n[m]$ is a vector of q symbols belonging to some finite alphabet (constellation) when we consider a normal data transmission tone. In the case of a pilot tone, the vector $\mathbf{a}_n[m]$ can have an arbitrary value, with for instance only a single entry being non-zero. In any case, it is clear from (18) that from one pilot tone in one OFDM symbol, it is only possible to sound $\mathbf{H}_n[m]$ in a single direction, the direction of the vector $\mathbf{a}_n[m]$. Therefore, in [11] as in many other proposals, it is suggested to put and consider jointly pilot symbols in q consecutive OFDM symbols. If the channel would be arbitrarily time-varying in time and frequency it would be impossible to estimate the channel. So it is absolutely indispensable to exploit some type of correlation, in time and/or frequency and/or space, to estimate a MIMO channel.

To find the appropriate expression for the misadjustment factor in the MIMO case, let us go through the following elementary steps. Assume for a moment that a pilot tone is activated for q consecutive OFDM sybols and that the channel would not vary over that time span. Then we can write with Matlab-type notation

$$\begin{aligned}\underbrace{\mathbf{u}_n[m:m{+}q{-}1]}_{p\times q} &= \underbrace{\mathbf{H}_n[m]}_{p\times q}\,\underbrace{\mathbf{a}_n[m:m{+}q{-}1]}_{q\times q} \\ &\quad + \underbrace{\mathbf{w}_n[m:m{+}q{-}1]}_{p\times q}\end{aligned} \tag{19}$$

where the elements in $\mathbf{w}_n[m:m{+}q{-}1]$ are assumed all i.i.d. circular Gaussian with variance σ_w^2 and we assume, as suggested in [11] also, $\mathbf{a}_n[m:m{+}q{-}1]$ to be a multiple of a unitary matrix (orthogonality of the pilots for different channel inputs):

$$\mathbf{a}_n^H[m:m{+}q{-}1]\mathbf{a}_n[m:m{+}q{-}1] = q\,\sigma_P^2 I_q\,.$$

So $q\,\sigma_P^2$ is the total transmit power at a pilot tone in one OFDM symbol. Assuming $p \geq q$, the LS estimate, which is here also the deterministic ML estimate, of $\mathbf{H}_n[m]$ is

$$\begin{aligned} \widehat{\mathbf{H}}_n[m] &= \tfrac{1}{q\,\sigma_P^2}\,\mathbf{u}_n[m:m+q-1]\mathbf{a}_n^H[m:m+q-1] \\ &= \mathbf{H}_n[m] + \widetilde{\mathbf{H}}_n[m] \end{aligned} \tag{20}$$

where

$$\widetilde{\mathbf{H}}_n[m] = \frac{1}{q\,\sigma_P^2}\,\mathbf{w}_n[m:m+q-1]\mathbf{a}_n^H[m:m+q-1]\,. \tag{21}$$

The channel estimation error covariance matrix (seen from the RX side) is

$$\mathrm{E}\,\widetilde{\mathbf{H}}_n[m]\widetilde{\mathbf{H}}_n^H[m] = \frac{\sigma_w^2}{\sigma_P^2}\,I_p \;=\; \sigma_{\widetilde{\mathbf{H}}}^2\,I_p \tag{22}$$

As in (13), the (refined) channel estimation error leads to noise increase at data tones $n \notin \mathscr{P}[m]$:

$$\begin{aligned} \mathbf{u}_n[m] &= \mathbf{H}_n[m]\,\mathbf{a}_n[m] + \mathbf{w}_n[m] \\ &= \widehat{\widehat{\mathbf{H}}}_n[m]\,\mathbf{a}_n[m] + \widetilde{\widetilde{\mathbf{H}}}_n[m]\,\mathbf{a}_n[m] + \mathbf{w}_n[m] \end{aligned} \tag{23}$$

where $\widetilde{\widetilde{\mathbf{H}}}_n[m]$ is the estimation error associated with the refined channel estimate. We assume $\mathrm{E}\,\mathbf{a}_n[m]\mathbf{a}_n^H[m] = \sigma_a^2\,I_q$ so

$$\mathrm{E}\,(\widetilde{\widetilde{\mathbf{H}}}_n[m]\,\mathbf{a}_n[m])(\widetilde{\widetilde{\mathbf{H}}}_n[m]\,\mathbf{a}_n[m])^H = \sigma_{\widetilde{\widetilde{\mathbf{H}}}}^2\,\sigma_a^2\,I_p \tag{24}$$

whereas

$$\mathrm{E}\,\mathbf{w}_n[m]\mathbf{w}_n^H[m] = \sigma_w^2\,I_p\,. \tag{25}$$

Hence we obtain the relative noise increase (misadjustment factor) :

$$\mathscr{M} \;=\; \frac{\sigma_{\widetilde{\widetilde{\mathbf{H}}}}^2\,\sigma_a^2}{\sigma_w^2} \tag{26}$$

which should be $\ll 1$ in order for the channel estimation error to lead to negligible performance degradation. Now, to obtain $\sigma_{\widetilde{\mathbf{H}}}^2$ in (22), we assumed we have q pilot tones, but in fact we have on the average P pilot tones per OFDM symbol. On the other hand, without any correlation, the channel at each tone can be an independent variable. Hence, without filtering of the brute channel estimate we have: $\sigma_{\widetilde{\widetilde{\mathbf{H}}}}^2 = \frac{Nq}{P}\sigma_{\widetilde{\mathbf{H}}}^2$, which leads, with (26) to

$$\mathscr{M} = q\,\frac{\sigma_a^2}{\sigma_P^2}\,\frac{N}{P} \;\;\gg\; 1 \tag{27}$$

which is q times the value for the SIMO case. With channel estimate filtering one can obtain

$$\mathscr{M} = q\,\frac{\sigma_a^2}{\sigma_P^2}\,\frac{N}{P}\,\alpha_{F,d}\,\alpha_{F,s}\,\alpha_{T,d}\,\alpha_{T,s}\,\alpha_S\,\alpha_I \;\;\ll\; 1\,. \tag{28}$$

The reduction factors α are unchanged from the SIMO case, except for the exploitation of spatial information. The correlation between antennas at both TX and RX sides may be exploited now to obtain

$$\alpha_S \;\geq\; \frac{1}{pq} = \frac{1}{\#\,\text{RX antennas}\,\times\,\#\,\text{TX antennas}}\;\;. \tag{29}$$

The lower bound (reduction by pq) is attained when each spatial channel impulse response coefficient $\mathbf{h}[n]$ corresponds to the contribution of only a single path at the corresponding delay. In that case, the pq coefficients of $\mathbf{h}[n]$ are proportional to just a single rapidly varying complex path amplitude, and $\mathbf{h}[n]$ is a rank one matrix, proportional to the RX array response for the path considered times the transpose of the TX array response. The direction of these array response vectors varies only slowly, with the physical TX and RX directions of the path. Note that in this extreme correlation case, the reduction factor α_S can be q times smaller than in the SIMO case, which would offset the fact that the brute misadjustment is q times larger in the MIMO case compared to the SIMO case.

3.1.13 Summary Channel Estimation Challenge

The exploitation of any factor α is equivalent in reducing the excess noise $\mathscr{M}$ due to channel estimation error. In practice, it is (largely) sufficient to reduce $\mathscr{M}$ to $\mathscr{M} = 0.1$. From the previous discussion it is clear that this goal can be reached in a wide variety of ways. Theoretically, $\mathscr{M}$ can be made much smaller than 1.

Hence the challenge becomes: what is the cheapest way in terms of computational complexity (which distribution of α's) to get $\mathscr{M}$ down to e.g. 0.1 ?

3.2 Capacity limits of Non-Coherent Point-to-Point Communication

Information theoretic capacity analysis for different types of channel models started with somewhat unusual assumption that channel is perfectly known at the receiver (channel state information at the receiver (CSIR)) or sometimes even assuming that channel is known at the transmitter (channel state information at the transmitter(CSIT)). But inherently all channels are non-coherent in nature and they need some kind of estimation to get CSIR and then some kind of feedback and/or estimation to have knowledge of CSIT. The area of capacity analysis for non-coherent (no CSIR and no CSIT) fading channels has received considerable attention in recent years.

Usually block fading models are assumed for obtaining capacity bounds in the no CSIR case. In the standard version of this model [12], the fading remains constant over blocks consisting of T symbol periods, and changes independently from block to block. Capacity bounds are obtained by introducing training segments in an ad hoc fashion. For the standard block fading model, the capacity is shown [12], [13] to grow logarithmically with SNR at high values of SNR, thus log(SNR) was shown to be the dominant term of capacity. Later Liang and Veeravalli [14] allowed the fading to vary inside the block with a certain correlation matrix characterized by its rank Q and showed for SISO channels that the capacity pre-log is $(1 - Q/T)$. For block constant frequency selective channels with L taps, the pre-log was shown to be $(1 - L/T)$ in [15].

Non-coherent capacity has also been analyzed with the channel fading process taken symbol-by-symbol stationary. In this model, fading is not independent but time selective without any block structure. Surprisingly, this model leads to very different capacity results: contrary to log(SNR) capacity growth in block fading channels, here the capacity grows only double logarithmically with SNR at high values of SNR [16], [17], [18] when the fading process is non-bandlimited (the Doppler Bandwidth is over the full transmission bandwidth), in this case the channel prediction error is non-zero even if infinite past is known.

For symbol-by-symbol stationary Gaussian fading channels, if the Doppler spectrum is bandlimited (of limited support), then the fading process is called non-regular and the prediction error given the infinite past goes to zero. Lapidoth [19] studied the SISO case for this kind of fading processes showing that the capacity grows logarithmically with SNR and the capacity pre-log is the Lebesgue measure of the frequencies where the spectral density of the fading process (Doppler spectrum) has nulls.

Etkin and Tse [20] study the same channel model of bandlimited fading for MIMO systems, they show that the pre-log exists even for MIMO systems with no CSIR but they only give a lower bound of the capacity pre-log.

All of the above mentioned studies except [15] deal with flat fading channels so discrete time channel filter is of single tap at each time instant varying with the Doppler spread of the channel.

We are interested in studying non-coherent doubly selective channels where the channel has multiple taps varying in time depending upon the Doppler bandwidth. Their coherent counterparts have the pre-log of one. We will try to analyze the capacity bounds for such non-coherent doubly selective channels. Moreover we try to find schemes which show the achievability of these bounds, at least for the dominant term of the capacity. We also want to analyze the channel parameter values which govern different regimes of the capacity where log(SNR) and log(log(SNR)) become the dominant terms in the high SNR capacity expression growing with SNR.

3.3 Error-rate Analysis of Non-Coherent Point-to-Point Communication

Since wireless multiple-input multiple-output (MIMO) systems have the potential to significantly improve the communication performance without requiring extra bandwidth or power, they are considered a promising technology for future wireless communication and have already been adopted in several wireless communication standards (e.g. WiMAX, LTE, etc.). Hence, it is very important to obtain accurate and easy-to-evaluate theoretical performance results of MIMO systems under realistic conditions. Typically, however, the performance of such systems is carried out under the simplifying assumption that the channel state information (CSI) is known by the receiver. In case of receive diversity, which is realized by using one transmit antenna and multiple receive antennas, a unified approach for the computation of the exact symbol error rate (SER) of linearly modulated signals over generalized fading channels with maximal-ratio combining (MRC) is presented in [21], assuming perfect channel knowledge (PCK) at the receiver. When multiple antennas are applied also at the transmitter side, a maximum diversity order of $N_t N_r$ (with N_t and N_r denoting the number of transmit and receive antennas, respectively) can be obtained, provided that proper space-time coding is used. Since orthogonal space-time block codes (OSTBCs) [22, 23] achieve full spatial diversity, and require only linear processing at the receiver, they are a very attractive transmit diversity technique. Under the assumption of PCK, the bit error rate (BER) performance of OSTBCs has been studied extensively in e.g. [24], [25], [26], [27] and [28].

In practical wireless applications, however, the receiver has to estimate the channel response, which inevitably results in a performance penalty as compared to the case of PCK at the receiver. In [29], approximate BER expressions are given for receive diversity M-QAM with both MRC and equal-gain combining (EGC) in Nakagami fading channels with imperfect channel estimation (ICE). The performance analysis of OSTBCs has most often been carried out under the assumption of Rayleigh block fading, whereby the channel remains constant over one fading block, and changes independently from one block to another: an analytical expression for the BER of OSTBCs in case of minimum mean-square error (MMSE) channel estimation was derived in [30]; high-SNR expressions for the pairwise error probability (PEP) were derived in [31] for quite general STBCs, using an eigenvalue approach; in [32], an exact closed-form expression for the PEP of both orthogonal and non-orthogonal space-time codes in the case of least-squares channel estimation was obtained by means of characteristic functions. However, from the PEP only an upper bound on the BER can be computed, which in a fading environment does not converge to the true BER at high SNR. In [33], expressions for the exact decoding error probability (DEP) were presented for the case of square OSTBCs on arbitrary fading channels and PSK transmission.

Although the assumption of Rayleigh block fading allows in many cases a relatively simple error-rate analysis, it often represents an oversimplification of a realistic wireless channel. As realistic channels may have fading distributions that are not well approximated by a Rayleigh distribution, and change continuously from one symbol to the next, a more general error-rate analysis is required, that is valid for arbitrary fading distributions and takes into account the time-variations of the channel. Some BER results under more generalized fading assumptions are already available from the literature: the exact BER for square/rectangular QAM with MRC diversity and ICE in non-identical correlated Rician fading channels is given in [34]; an extension of this analysis for Rician fading channels with symbol-to-symbol variations is provided in [35]; in [36], (approximate) analytical BER expressions as well as the tight Chernoff bound were given for orthogonal space-time block coded systems employing M-ary phase-shift keying (M-PSK) modulation in time-varying Rayleigh fading with ICE.

3.4 Directions for research during the course of NEWCOM++

3.4.1 Frequency-selective capacity analysis

The work described in section 3.2 will be extended to the MIMO frequency-selective channel.

3.4.2 Error-rate Analysis for symbol-by-symbol time-variation

Although the effect of imperfect channel estimation on MIMO communication has already been studied to a large extent (see 3.3), a unified performance analysis of OSTBCs on (temporally correlated) arbitrary fading channels with ICE, similar to Alouini's approach for the case of MRC receive diversity with PCK [21], is still lacking. In the context of this WP, we will attempt to provide such a generalized performance analysis. As a first step, we will investigate the BER performance of OSTBCs on arbitrary block fading channels. Next, this analysis will be extended to time-varying fading channels.

As the main focus of this WP is on feedback of the channel state, we will also investigate the effect of channel estimation errors on the performance of feedback-driven communication strategies. As the assumption of PCK at the transmitter side is not valid in practice, it is essential to analyze the error-rate performance of communication systems where the CSI at the transmitter is obtained from (perfect or imperfect) feedback of the estimated CSI, extracted by the receiver (see also 5.4.3).

4 POINT-TO-POINT AND POINT-TO-MULTIPOINT TWO-WAY CHANNELS

4.1 Fundamentals of Classical Feedback-Based Systems

4.1.1 Zero-error Codes

The most important study of two-way communication began with Shannon in [37], who described the notion of *zero-error capacity*, which is defined as the as the least upper bound of rates at which it is possible to transmit information with zero probability of error. He formally showed that feedback can increase the zero-error capacity for some channels, which is not the case for the simplest channels without feedback using the conventional capacity definition which tolerates error which vanishes with coding block length. It is now known for some channels, and in particular multiuser channels, that feedback can also increase the conventional capacity. Shannon's work, although not evident at the time, essentially laid the foundations of what today is known as automatic-repeat request (ARQ) protocols which make use of one bit of feedback for every N channel uses. It is arguable that Shannon's zero-error capacity should be the measure of choice for two-way cellular communication systems, since for the many of the applications deployed on such networks, the MAC layer introduces a retransmission protocol which guarantees. More precisely in terms of 3GPP terminology, the acknowledged bearer services are more appropriately characterized using a zero-error capacity (perhaps with a minute outage event due to a finite delay constraint) rather than a conventional channel capacity. For this reason, a significant amount of our work focuses on Hybrid ARQ protocols in wireless networks. Another early important work on feedback-based systems is that of Chang [38].

4.1.2 Other Feedback Channels

The next most significant early study on feedback was that of Schalkwijk and Kailaith[39], which took on a a very different flavour. The authors showed that although feedback cannot increase capacity, it can reduce complexity of codes for a given block length. These authors showed a simple two-way protocol with a simple successive refinement estimation-based receiver which achieves channel capacity on an AWGN channel with very low complexity. The authors also make use of techniques reminiscent of automatic feedback control systems, which shows the similarity of such feedback problems to communications. This type of study was even very recently pursued by some researchers (see for example [40]). Kashyap [41] is an early example of work considering imperfect (noisy) feedback. Kadota [42, 43] was the first to rigorously show that feedback could not increase capacity on an continuous time *memoryless* channel. For AWGN channels with memory it was shown by Ebert [44] that feedback can at most double the capacity with respect to coding with lack of feedback. Tiernan [45] provides an upper-bound to the capacity for correlated noise defined by an auto-regressive process. Finally, Cover and Pombra [46] showed that feedback on a channel with memory at most increases capacity by half a bit. Both of these results leads one to believe that the use of feedback on fading channels defined by some memory process can also increase capacity. This will be one avenue for research in WPR.2 (see Section 4.6.1).

Feedback was also heavily studied in the context of multi-terminal networks. The first to consider feedback and the potential for capacity increase in a multiple-access channel were Gaarder and Wolf[47]. Cover and Leung also provided and achievable region for the multiple-access channel [48] which was later shown by Willems to be optimal for some multiple-access channels [49]. Ozarow [50] found the capacity region for a two-user Gaussian multiple-access channel with perfect feedback using a coding scheme which is a generalization of Schalkwijk and Kailaith's single-user two-way feedback system [39] which allows users to collaborate and effectively render their signals sufficiently correlated to achieve coherent combining at the receiver. This definitively showed an example of where feedback can increase the capacity of a multi-terminal network. Similar studies were also performed for the broadcast channel with feedback by El Gamal [51], Ozarow and Leung[52], and Dembo[53]. More recently, Kramer provided innovative feedback strategies for general Gaussian multi-terminal networks which highlight the potential benefits of exploiting feedback in network [54].

4.1.3 Two-way Channels

Shannon produced another landmark paper on the *two-way channel* [55], where here the characterization of the pair of rates between two communicating users across a joint channel was sought. The notion of a feedback channel was intermingled with data since both users transmit to each other. In principle, users could decide to decide how much of their resources were dedicated to helping their peer with respect to the resources used for transmitting their own information. This notion, however, is hidden in the mathematical model of the problem and only appears when searching for practical methods for two-way transmission. To this end, the much later work of Dueck and Schalkwijk [56, 57, 58, 59] provided powerful schemes approaching Shannon's outer-bounds for some channels, although not meeting them. Further tighter outer-bounds for specific channels were found by Zhang, Berger and Schalkwijk and so-called *single-output two-way channels* by Willems and Hekstra [60].

4.1.4 Channel State Feedback and Fading Channels

The majority of the most recent work concentrates on feedback of channel state information in the context of fading channels. This is of course a special case of the general feedback channel since only a piece of the received signal is fed back (i.e. a perfect channel estimate), and not the signal itself. The first study of this for a wireless channel was for single-user channels was considered in the PhD thesis of Goldsmith [61]. Here it was shown that channel side information at the transmitter when coupled with adaptive power control and potentially rate adjustment could lead to capacity increase in the low-SNR regime. Very little could be gained in the high-SNR regime. Perhaps the most important area where feedback can provide a significant capacity increase is in the case of multiuser channels and broadcast channels, for which the latter when combined with multiple transmit antennas is represented by an entire task WPR.2. (see Section 5). The first instance showing benefits of exploiting channel state information at the transmitter in a multiuser context was Knopp and Humblet [62, 63] for the case of a multiuser channel, where it was shown that feedback can virtually double the capacity for a reasonably small number of users for per-user average SNRs on the order of 0dB. This is due to the *multiuser-diversity* effect which benefits from the variation of the channel strength around its mean combined with scheduling users in an opportunistic manner. It is in fact the scheduling aspect that brings the large capacity gains and not power control. What is not clear is that how much of this benefit is lost when imperfect feedback is used, either due to channel estimation error, outdated channel estimates or more likely a combination of both. Other authors (see for instance [64]) have shown that reduced feedback induces very little penalty, and thus it is reasonable to assume that imperfect feedback should also not harm performance significantly either, provided that channels do not vary too quickly. This is proven in practice due to the success of such techniques in deployed networks (CDMA2000 EVDO,UMTS HSDPA). It was later shown that a similar characterization and the multiuser diversity effect was also possible for the downlink channel (see for instance [65] and references therein for a good summary of these studies).

4.2 Error-bounds and Coding for time-varying MIMO-HARQ (coherent)

We here present a summary of some aspects of MIMO communications in the presence of limited CSI feedback at the transmitter. We focus on the case of MIMO-ARQ (single bit of feedback per use of the feedback channel), as a simple and intuitive initiation of the reader to the effects of feedback in delay and outage limited communications. We summarize on information theoretic analysis of the error performance of MIMO-ARQ communications, and we summarize on some codes that provably perform well in the MIMO-ARQ setting. For clarity of exposition and for reasons of offering increased intuition, the information theoretic analysis is based on asymptotic approximations that are suitable for the delay and outage limited setting that is common in wireless communications. We also focus on the case where the channel state information is perfectly known at the receiver. The MIMO-ARQ protocol has recently drawn substantial attention. Recently, there has been a growing interest in MIMO ARQ schemes (e.g.,

[66]ÂŰ-[67]). It is simpler to implement and its information theoretic analysis can be used as a theoretical and practical building block on understanding how feedback affects the error performance in more involved scenarios of delay and outage limited MIMO communications, such as the case where CSI is not entirely known at the receiver, the case where long-term power control is allowed, or the virtual MIMO (relaying) case. We very briefly touch upon the time-varying channel as well.

4.2.1 The L-round MIMO-ARQ protocol

We here consider the case of the quasi-static Rayleigh fading channel, with n_t transmit and n_r receive antennas where communication takes place over T time slots. The receiver receives the $(n_r \times T)$ signal matrix Y

$$Y \;=\; \theta HX + W,. \tag{30}$$

where θX is the $(n_t \times T)$ transmitted matrix in space and time, H is the $(n_r \times n_t)$ channel matrix and W describes the Gaussian additive white noise. The energy constraint is described by

$$\mathbb{E}(||\theta X||_F^2) \leq T \text{ SNR}. \tag{31}$$

In this ARQ setting, the $n \times T$ matrix θX_1 is transmitted first, and a corresponding $(n_r \times T)$ matrix Y_1 is received. Based on the specific ARQ protocol, an ACK is passed on to the transmitter if the receiver decodes correctly. Otherwise a NACK is sent. Upon receipt of an ACK, the transmitter moves on to transmit the next message symbol. Upon receipt of a NACK however, the transmitter proceeds to transmit θX_2. Upon receipt of Y_2, the receiver again attempts to decode the message symbol, using the entire received signal $[Y_1 Y_2]$. This process is continued until the transmitter receives an ACK. At the last round (L^{th} round) the transmitter receives an implicit ACK.

4.2.2 Asymptotic approximation of error exponents in the MIMO-ARQ case

It has been shown that in the presence of quasi-static fading, allowing limited or even a single bit of CSI feedback at the transmitter can result in substantial diversity gains. The MIMO-ARQ case has been recently analyzed by El Gamal et. al. [68] who, for the delay and outage limited setting of interest, provided the fundamental error-performance tradeoff of *Diversity Multiplexing Delay Tradeoff* (DMDT) as a means of asymptotically approximating (in the high SNR regime) the probability of codeword error, or equivalently the probability of channel outage.

For the $n_t \times n_r$ quasi-static Rayleigh fading channel without feedback, it has been shown in [69] that there exists a fundamental error-performance tradeoff between diversity and spatial-multiplexing gains, referred to as the Diversity-Multiplexing Gain (D-MG) tradeoff. In the presence of limited noiseless feedback taking the form of an L-round automatic retransmission request protocol, the result is extended in [68] to the DMDT. In the above specific setting, the optimal DMDT for an L round MIMO-ARQ setting, for the $n \times n_r$ Rayleigh fading MIMO channel, is described in terms of the average (effective) spatial-multiplexing gain, denoted here as r, which is given as the normalization

$$r = \lim_{\rho \to \infty} \frac{\mathbb{E}[R(t)]}{\log \rho} \tag{32}$$

(ρ here describes SNR) of the average rate $\mathbb{E}[R(t)]$ induced by the ARQ protocol, as well as in terms of the optimal diversity gain

$$d_{\text{opt},L}(r) = -\lim_{\rho \to \infty} \frac{\log P_e(r)}{\log \rho}$$

describing the optimal probability $P_e(r)$ that a message symbol is decoded incorrectly in an L round ARQ.

In this setting, the work in [68] shows that

$$d_{\text{opt},L}(r) = d_{\text{opt},1}(\frac{r}{L}), \tag{33}$$

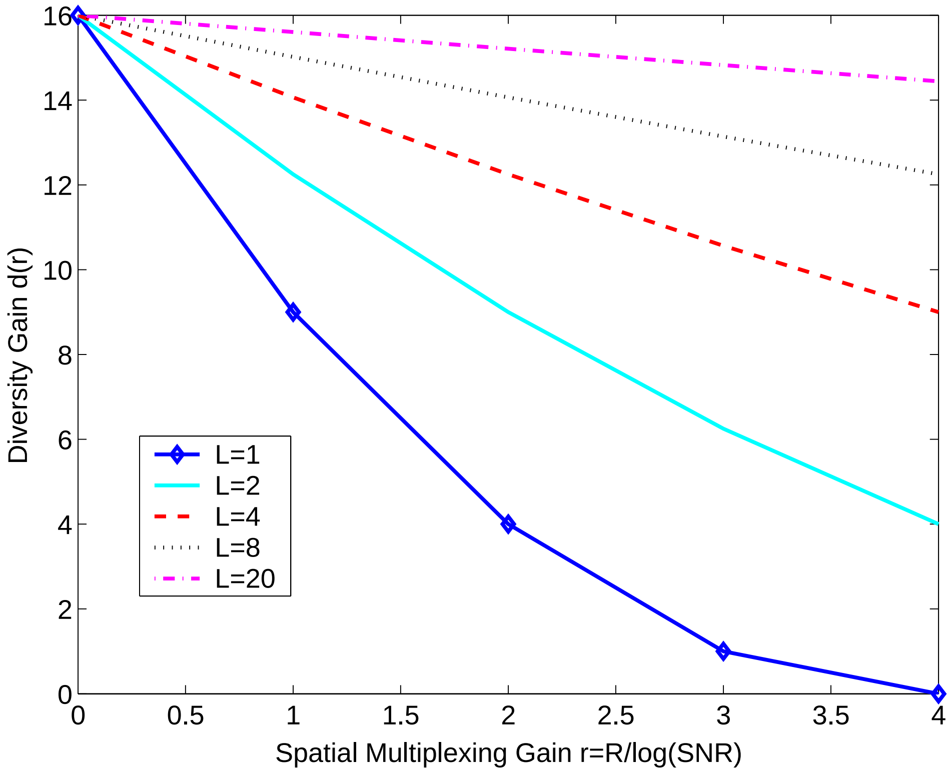

Figure 1: DMDT vs DMT. The diversity gain increase describes the effect of channel feedback

where $d_{\text{opt},1}(r)$ describes the optimal DMT for the corresponding MIMO channel without feedback. Consequently, for the case of the quasi-static fading channel, the DMDT tradeoff takes the form (at integer points)

$$d_{\text{opt},L}(r) = (n_t - r/L)(n_r - r/L), \quad r = 1,2,\ldots,\min(n_t,n_r). \tag{34}$$

The effect of feedback is made even clearer from a graphical representation of the above expression, depicted in Figure 1.

An example of the role of feedback in time-varying channels can be viewed through the above for the case where feedback is sent every time the channel changes, i.e., where an ARQ round has duration equal to the coherence time of the channel. In this case the DMDT takes the form

$$d_{\text{opt},L}(r) = L(n_t - r/L)(n_r - r/L), \quad r = 1,2,\ldots,\min(n_t,n_r). \tag{35}$$

The above results allow for intuition on issues such as:

- how many feedback bits are required for a specific target error performance for different channel statistics (see main DMDT expression (33) which holds for a variety of fading statistics)
- the effect of feedback when the channel is time-varying ((35))
- the effect of feedback on the effective communication rate r (32) (at high SNR, this tends towards the rate of the single ARQ round).

4.2.3 Space-time coding for the MIMO-ARQ channel

We here summarize on a specific family of MIMO-ST codes that achieve that above optimal performance in the MIMO-ARQ setting. The summary of the codes is based on the work in [70] and is focused on a single ST-code family whose simplicity of construction allows for clear insight on how information needs to be distributed in space and time, and on what coding gain requirements can guarantee proper utilization of feedback. The specific family of codes manages to optimally utilize feedback, not only in terms of error performance (i.e., in terms of DMDT), but also in terms of time delay. This summary can be used as a first step towards understanding the workings of more involved codes which can in fact achieve DMDT optimality for all statistical descriptions of the channel, including time-varying channels [70].

4.2.4 Space-time signaling in the MIMO-ARQ setting

In the MIMO-ARQ setting each message symbol from the source is associated with a unique block

$$\theta[X_1\ X_2 \cdots X_L]$$

of matrices, each $X_i \in \mathbb{C}^{n_t \times T}$, in such a way that it is possible to uniquely decode the message symbol given $\theta[X_1 X_2 \cdots X_l]$ (here θX_l is the matrix sent during the lth ARQ round) for any $1 \leq l \leq L$, in particular, given just θX_1. θ is such that the energy constraint

$$\mathbb{E}\left[\theta^2 \|X_l\|_F^2\right] \leq T\ \text{SNR},\ 1 \leq l \leq L \tag{36}$$

is met.

The ARQ ST code $\mathscr{X}_{\text{ARQ}}$ is the collection of matrices $\theta[X_1\ X_2 \cdots X_L]$. At the end of the lth round, the receiver 'sees' an lth round ST code $\mathscr{X}_{\text{ARQ},l}$, which is $\mathscr{X}_{\text{ARQ}}$ truncated to l rounds. The codes define equivalent rates R_l seen at the receiver at the end of each round. Specifically R_l is the rate associated with the lth ARQ code $\mathscr{X}_{\text{ARQ},l}$, and r_l the corresponding normalized rate given by

$$R_l\ =\ r_l \log_2(\text{SNR}).$$

Finally let $P_{e,l}(r_l)$ be the probability of error of the space-time code $\mathscr{X}_{\text{ARQ},l}$, and let $P_e(r)$ be the overall probability of error.

4.2.5 A Sufficient Condition for Achieving the DMD Tradeoff

Let $\mathscr{A}_l$ ($\overline{\mathscr{A}}_l$) denote the event that the received matrix $\mathbf{Y}_l$ at the end of the l^{th} round of transmission is such that a ACK (NACK) is sent. The sufficiency condition then is as follows.

Theorem 1 *[70] Let $\mathscr{X}_{ARQ}$ be an ST code designed for the MIMO-ARQ channel having normalized single-round rate r_1 and average multiplexing gain $r = R/\log\rho$. If the code $\mathscr{X}_{ARQ}$, in conjunction with receiver decoding algorithms $\{\mathscr{D}_l\}$ satisfies the following,*

1. *$Pr(\overline{\mathscr{A}}_1) \doteq \rho^{-\mathscr{T}}$, where $\mathscr{T} > 0$ for all $0 \leq r_1 < \min(n_t, n_r)$.*
2. *The full-length ST code $\mathscr{X}_{ARQ,L}$ is a D-MG optimal ST code for multiplexing gain $r_L = \frac{r_1}{L}$ and*
3. $$P_{e,l}(r_l)\ \dot{\leq}\ P_{e,L}(r_L) \quad 1 \leq l \leq (L-1)\ . \tag{37}$$

Then

$$\lim_{SNR \to \infty} r\ =\ r_1$$

and the ST code $\mathscr{X}_{ARQ}$ achieves the DMD tradeoff for all r, $0 \leq r < \min\{n_t, n_r\}$.

In other words, if $\mathscr{X}_{\text{ARQ}}$ satisfies:

1. that with high probability there will be just a single ARQ round and

2. the full-length ST code $\mathscr{X}_{\mathrm{ARQ},L}$ is optimal with respect to the D-MG tradeoff of the channel,

3. the error probability of the lth decoder applied to the task of decoding the ST code $\mathscr{X}_{\mathrm{ARQ},l}$ is no larger than that incurred by the ML decoder applied to the task of decoding the ST code $\mathscr{X}_{\mathrm{ARQ},L}$

then $\mathscr{X}_{\mathrm{ARQ}}$ will achieve the DMD tradeoff.

Such codes (that meet the above criteria) were constructed in [70]. Here is a quick summary of the construction [1].

4.2.6 Construction of DMDT optimal codes for the Case $L|n_t$

For block length T such that $T = n_t/L$, the DMDT optimal construction is derived from a square $(LT \times LT)$ *perfect space-time codes* $\mathscr{X}_{(LT\times LT)}$, with every codeword $X \in \mathscr{X}_{(LT\times LT)}$ taking the form

$$X = \begin{bmatrix} \ell_0 & \gamma\sigma(\ell_{n-1}) & \cdots & \gamma\sigma^{n-1}(\ell_1) \\ \ell_1 & \sigma(\ell_0) & \cdots & \gamma\sigma^{n-1}(\ell_2) \\ \ell_2 & \sigma(\ell_1) & \cdots & \gamma\sigma^{n-1}(\ell_3) \\ \vdots & \vdots & \ddots & \vdots \\ \ell_{n-1} & \sigma(\ell_{n-2}) & \cdots & \sigma^{n-1}(\ell_0) \end{bmatrix}_{n\times n}$$

where the elements ℓ_i are restricted to be of the form

$$\ell_i \;=\; \sum_{k=1}^{n} e_{i,k}\beta_k, \;\; e_{i,k} \in \mathscr{A}_{\mathrm{QAM}}, \tag{38}$$

with

$$\mathscr{A}_{\mathrm{QAM}} = \{a + \imath b \;\mid\; -M+1 \le a,b \le M-1, \;\; a,b \;\; \text{odd}\;\}$$

and where $\beta_k, k = 1,2,\ldots,n$ is a proper integral basis corresponding to the division algebra of the perfect codes. The underlying M^2-QAM-alphabet $\mathscr{A}_{\mathrm{QAM}}$ is such that $|\mathscr{A}_{\mathrm{QAM}}| = M^2$. and the code has cardinality $[M^2]^{n^2}$. If it is desired to communicate at a rate $r\log_2(\mathrm{SNR})$ bits/channel use, then one must choose the size M^2 of the underlying $\mathscr{A}_{\mathrm{QAM}}$ alphabet accordingly, i.e., set

$$M^2 \;=\; \mathrm{SNR}^{\frac{r}{n}}. \tag{39}$$

To guarantee that $R = r\log_2(\mathrm{SNR})$ is the desired rate of the ARQ scheme, we set

$$|\mathscr{X}_{(LT\times LT)}| = \mathrm{SNR}^{Tr}, \tag{40}$$

and in each round of the ARQ transmission we transmit T successive columns from the matrix X, i.e., during the l^{th} round, we transmit $\theta X_l = \theta[\underline{c}_{(l-1)T+1}\cdots\underline{c}_{lT}]$ for $l = 1,2,\ldots,L$, where $\underline{c}_i$ denotes the i^{th} column of X.[2]

Below we see (directly from [70]) an indicative simulation example of the ARQ ST-code, for the case of $L = 2$ (2-round ARQ), and observe that the average ARQ rate approaches a rate that is twice the data rate of the full-length code while maintaining a comparable error probability.

[1]The codes require minimum delay and are DMDT optimal for the case of $L|n_t$ and $n_r \ge n_t$. Note that other families of codes, including codes for time-varying channels, as well as codes that are optimal irrespective of n_r, n_t, L and irrespective of channel statistics, can be found in [70]

[2]Note the that theoretical average rate will vary depending on SNR.

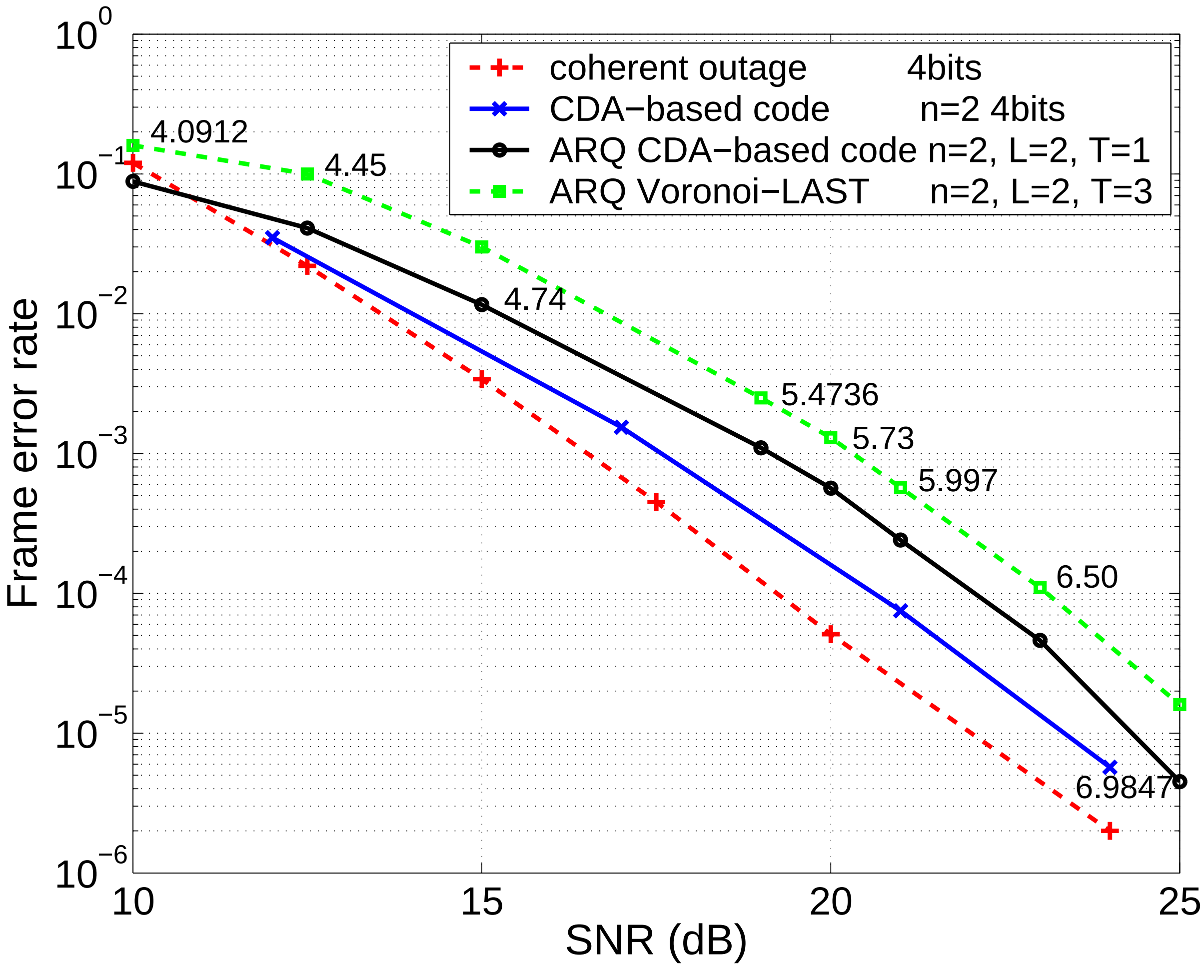

Figure 2: Average probability of error of CDA based ARQ ST code: $n_t = n_r = L = 2$, $T = 1$ [70]

4.3 Metrics and Methods for Represensation of Channel State Information for Feedback Channels

4.3.1 Single antenna systems

The channel state information (CSI) in a single antenna system consists of channel SNR levels, which must be fed back to the scheduler to exploit the gains from adaptive modulation and coding or multiuser diversity. Direct feedback of the continuous-valued SNR values is not feasible; some quantization scheme must be applied first.

The simplest approach is direct scalar quantization of the SNR values using as few bits as possible. This approach has been followed in e.g. [71] [72], where 1-bit quantization has been shown to give an overall system throughput of more than 90% of the throughput with unquantized feedback . By individually adapted quantization thresholds [73] [74], the performance can be further improved.

In a multiuser system, an approach denoted *SNR-limited feedback* is possible. With a scheduling policy where the base station only schedules users with high SNR, it is unnecessary for users with low SNR to send feedback; the probability that such a user is scheduled is very low. This can be exploited to reduce the feedback rate, a fact that many reports have proposed [64, 75, 76, 77, 78, 79, 80].

Another approach is to exploit the properties of the channel fading process. The channel SNR values are correlated in both time and frequency (depending on the Doppler effect and the delay spread), and the feedback bit rate can be considerably reduced by exploiting those correlations. Many reports suggest the use of lossless coding methods, such as differential coding [81], run-length coding [82], Huffman coding [83] or Lempel-Ziv coding [84]. Other reports suggests lossy methods based on transform coding and downsampling [85].

An overview of methods for compression of feedback for adaptive transmission and multiuser scheduling can be found in [86].

4.3.2 Multiple antenna systems

Wireless systems using multiple antennas have attracted substantial interest of research for their potential to increase the spectral efficiency and robustness against fading. These benefits of multiple antenna systems can be further enhanced when the transmitter customizes the transmitted waveforms to the fading environment based on the knowledge of channel state information (CSI). In contrast to single antenna systems, the necessary channel state information required at the transmitter consists of complex unit norm beamforming vectors in Multiple Input Single Output (MISO) systems or matrices in Multiple Input Multiple Output (MIMO) systems, and of SINR values to exploit multiuser diversity in point to multiple point transmission scenario. To attain the necessary CSI for transmitter optimization, limited feedback was first proposed in [87] where the CSI was conveyed to the transmitter via a finite rate feedback link. In order to meet this rate constraint, various techniques have been proposed for quantizing CSI such as Grassmannian line packing [88], [89], vector quantization [90], [91], [92] and random vector quantization [93], [94], [95]. The performance improvement is proportional to the amount of feedback bits available. However, the penalty paid for it is the overhead that feedback introduces as it consumes bandwidth on the feedback link.

In order to increase the spectral efficiency of the feedback link, some researchers have worked on feedback rate reduction by exploiting the temporal correlation of fading channels. Intuitively, the data compression technique [96], [97] can be employed. However, there is performance loss due to the delay introduced by the block processing in these techniques. Alternatively, Roh and Rao [98] utilized adaptive delta modulation to quantize smoothly changing parameters extracted from the CSI. For this algorithm, the minimum required amount of feedback bits increases with the number of transmitter and receiver antennas. Banister and Zeider [99] proposed a stochastic gradient adaptation algorithm by using one bit feedback to maximize the received SNR. However, this requires the transmitter to broadcast channel subspace matrices which decreases the spectral efficiency of the forward link transmission. The approach introduced by Huang et al. in [100] reduces the amount of feedback bits by ignoring transitions between Markov states of the channel that occur with small probabilities. The performance loss is negligible in slowly fading channels. This algorithm is most efficient in case of a high quantization algorithm.

In future wireless systems, an important practical issue that arises is the proper labeling of the overall quality of a multi-channel scenario, independently of the specific architecture and signal processing techniques currently available. A proper labeling helps to decide on which transmission strategy to use (e.g. with or without Channel State Information, to optimize diversity or rate,...) and, thus, on the amount of feedback that is required in transmission.

In MIMO systems the feedback should provide the transmitter with information about $\mathbf{H}^H\mathbf{H}$, being $\mathbf{H}$ the MIMO channel matrix. The set of matrices $\mathbf{H}^H\mathbf{H}$ lie in the manifold of semidefinite positive matrices, which is a cone, and only locally looks like a flat Euclidean Space. Therefore, points are not connected by straight-lines but by exponential paths and the minimum distance between points is given by geodesic curves.

We propose to work on the generalization of the Euclidean ideas to Riemannian manifolds [101] in order to gain insights into the feedback mechanisms and study if geometric properties can be exploited for solutions developed in the following subsections:

- 4.6.2 Compression strategies for feedback channels exploiting temporal, frequential and spatial correlation,
- 4.6.4 Multi-user feedback channel.

4.4 Review of Recent Results on Analog Feedback

Of late, analog feedback as an alternative to quantized feedback is attracting more and more attention [102, 103, 104, 105, 106, 107]. In following text, we will present the main results in several groups.

4.4.1 SISO Channel

It is well known that for a white Gaussian source whose bandwidth is equal to the AWGN channel bandwidth, the uncoded unquantized transmission is the optimal with respect to mean-squared error (MSE) [108, 109]. For a time-continuous AWGN channel, this optimality can be achieved by single-sideband (SSB) modulation [110, 111]. For a time-discrete AWGN channel, this optimality can be achieved by single-letter codes and MMSE receivers [112, 105].

In [105], considering downlink and uplink estimation errors, Samardzija and Mandayam propose an analog scheme for CSI feedback in the case of independent Rayleigh fading AWGN SISO channel. Their criterion is to minimize the mean-squared error on the received channel estimate at the transmitter with respect to the downlink and uplink estimation errors, noise in the feedback phase, and channel distributions.

4.4.2 Vector Channel

So far, many discussions about analog CSI feedback are focused on the scenario of vector channel for multiple users. That is, the base station has multiple antennas, each user has only one antenna, and the CSI feedback is from users to the base-station. Feedbacks from different users are supposed to be distinguished by code-division or some other ways [103, 104].

The discussion in the scenario of vector channel can be traced back to [102], where Visotsky and Madhow propose to use analog feedback for the Rayleigh fading vector channel. They propose two approaches to do analog feedback, *direct approach* and *innovations approach*. They present the innovations approach performs better than the direct approach when the channel is correlated while both of them are suboptimal in that case.

In [103], Thomas *et. al.* propose to apply analog feedback in OFDM systems via a vector channel. They do CSI feedback and feedback channel estimation in one OFDM symbol period by utilizing features of OFDM. And, a MMSE combing weight is used to recover channel estimate for each user at the base-station.

In [104], Marzetta and Hochwald suppose a zero-forcing receiver for the direct analog CSI feedback and discover that although the total amount of channel information increases with the number of antennas at the base station, the burden of leaning this information at the base station paradoxically decreases.

4.4.3 MIMO Channel

In the case of quasi-static multi-input multi-output (MIMO) channels, space-time coding considerations with issues of diversity and spatial multiplexing arise also for analog transmission.

In [113], Chen and Dirk suppose to use spatial-multiplexing space-time block coding (SMSTBC) to do analog channel feedback via a peer-to-peer MIMO channel and compare the channel-estimate analog feedback scheme to the received-signal analog feedback scheme in terms of MSE on CSIT. Both schemes employ zero-forcing (ZF) receivers. They show that although the channel-estimate-based scheme has advantage over the other but the difference is trivial. Such a full-multiplexing coder without exploitation of transmit diversity is fast but is not optimum with respect to mean-squared error (MSE).

In [114], they propose to use rate 1/2 complex orthogonal space-time block coding (COSTBC) to do analog feedback. It is proved that by COSTBC, the matched-filter bound on SNR of a MIMO channel can be achieved. The COSTBC-MRC scheme is shown to have advantage over SMSTBC schemes and uncoded RVQ schemes with respect to MSE.

4.4.4 Analog vs. Quantized

Gastpar *et. al.* have given a fundamental analysis on when the analog transmission is optimal and when is not [115].

It is seen that in a bandwidth expansion system, any linear analog single-letter codes cannot be optimal (e.g., [112, 116]).

In [106], in the scenario of the SISO AWGN channel, Tejera and Utschick show that if D channel uses per source output are allowed, at high SNR, the delay limited digital approaches can achieve a distortion decay of at least $D/2$ dB per dB of SNR whereas the decay rate for the linear analog transmission is 1 regardless D. Therefore, when the system is in bandwidth expansion ($D > 1$), the linear analog transmission is inferior with respect to the decay rate.

In [107], in the scenario of multiuser vector channel, Caire $et.al.$ analyze and compare upper bounds of rate gaps of RVQ feedback schemes and linear analog feedback schemes. They show that even an uncoded RVQ (no channel coding and labeling) has the smaller rate gap upper bound than the analog schemes when the system is in bandwidth expansion. However, in [114], it is shown by simulation results that the COSTBC-MRC analog transmission scheme has the smaller MSE than uncoded RVQ schemes.

Although analog transmission is not optimal in bandwidth expansion systems, its advantage of low complexity and instantaneousness is deniable.

4.4.5 HDA and "Soft" Transmission

Furthermore, there are other interesting transmission methods different to traditional digital transmission and analog transmission. They can be considered as a mixture of quantized transmission and analog transmission.

In [117], Shamai *et. al.* propose to parallel use digital channel and analog channel for transmitting source. They assimilate this method to systematic codes and call it as *systematic communication*. They show the analog transmission can be used to lower the digital channel bandwidth required to achieve a given degree of fidelity.

In [118], Mittal and Phamdo propose a detailed hybride digital-analog (HDA) scheme for SISO channels which inherits the philosophy of [117]. In [119], Caire and Narayanan propose a HDA scheme for MIMO channels and show that it achieves a fairly good distortion SNR exponent.

While HDA schemes allocate different power and bandwidth resources to analog and digital source representations, the "soft" transmission proposed by Kochman and Zamir in [116] treats the source and the channel in the time domain. It is a analog-basic scheme with modulo operations and can achieve Shannon's optimum attainable performance $R(D) = C$.

4.5 Use of Feedback in Cooperative Wireless Networks

In cooperative communications, several cooperative protocols have been proposed with the purpose of combating fading by producing diversity gain. For the very common scenario (see Fig. 3), where $n-1$ relays help in the communication between source node (denoted as S) and destination node (denoted as D), several cooperative communication protocols have been proposed within the context of delay and outage limited communications, with the aim to provide gains in error performance by describing the exact manner with which the different participating nodes can communicate between themselves.

If the channel is randomly changing, the network's error performance is heavily affected by the event of channel outage as well as by the channel statistics. As in the MIMO case, it is also the case that in the virtual-MIMO (relaying) case, feedback plays a major role towards efficient communications, and a substantial volume of work was recently dedicated to looking for ways for efficient utilization of feedback in such relay networks. For example the work in [120] explores some instances of relay networks, where knowledge of the network channel state can result in well performing power control strategies. Feedback can provide for proper temporal power control or rate control that can respectively reduce the probability of outage and maximize throughput. The effect of feedback depends on the amount of knowledge on the

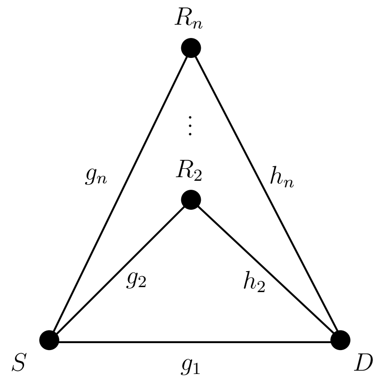

Figure 3: Cooperative diversity in networks. Relays $R_2, ...R_n$ assist in the communication between source node S and destination node D.

CSI at the transmitting nodes (source and relays), and the challenge is to find such efficient power and rate control policies. It is often observed (see for example [120]) that such policies perform well even with only a few bits of feedback.

Along the same lines, the work in [121] reveals the large gains, in terms of outage performance in different cooperative protocols, of having partial channel side information at the transmitters and the receiver. Furthermore the work in [122] discusses coding protocols related to relaying and the work in [123] deals with the issue of reducing packet outage probability for fading channels by exploiting queuing delay and transmitter channel information. Information theoretic analysis on the probability of outage was performed in [123], assuming perfect CSIT, and different rate and power adaptation policies were constructed considering buffer occupancy and channel conditions. The same work further explores the outage effect of an increasing transmitter channel information and of larger delays.

Focusing on the protocol of dynamic decode and forward (DDF) and on a hybrid ARQ feedback scheme, the work in [124] studies the performance of the automatic repeat request-dynamic decode and forward (ARQ-DDF) cooperation protocol for the multiple access relay channel and the cooperative vector multiple access channel. In the first case, a single relay simultaneously assists two users wishing to communicate with a common receiver, whereas in the second case, two users cooperate in delivering their messages to a destination equipped with two receiving antennas.

In [125] the authors present an efficient protocol for the delay-limited fading ARQ single relay half-duplex channel where the source uses an ARQ protocol to communicate to the destination with the help of the relays, and analyze using the DMT tool how the proposed protocol exploits spatial diversity and ARQ diversity obtained by leveraging the retransmission delay to enhance the reliability.

4.6 Directions for Research During the Course of NEWCOM++

4.6.1 Performance Bounds for Time-Varying Channels under a feedback rate constraint

Here we will consider at first very simple channel model, namely a unit-gain multiplicative phase-noise channel with additive Gaussian noise

$$y_n = e^{j\phi_n} x_n + z_n. \tag{41}$$

The phase-noise ϕ_n varies according to some pre-defined process which is slowly varying compared to the bandwidth of the signal, x_n, and is unknown to the receiver and transmitter. We will assume that the receiver can feedback quantized versions of y_n which are limited by some rate of R bits/sample. This feedback rate models a constraint on the amount of feedback that can be tolerated in the system. The recent work of Lapidoth [126] characterizes the performance of this channel *without feedback*. We will study a similar characterization with feedback and assess the amount of feedback that is required to approach AWGN performance where both transmitter and receiver collaborate perfectly to resolve the unknown phase process. Depending on the success in the analysis of this simple channel model, we will aim to extend the results to the more complex broadband MIMO setting with multipath-fading.

4.6.2 Compression strategies for feedback channels exploiting temporal, frequential and spatial correlation

When the receiver feeds back the channel to the transmitter (FDD systems), the geodesic curves can be parameterized and exploited to minimize the required feedback, for instance in adaptive and predictive feedback (e.g. differential quantization for feedback) for time-varying channels.

Also, if the transmitter obtains knowledge of the channel by a training sequence (TDD systems), good channel covariance estimates with low sample support or Signal to Noise Ratio (SNR) are desirable. The sample covariance matrix estimate is a lousy estimate and subspace an ad-hoc methods like "diagonal loading" are used. Covariance geodesics can be used to obtain better channel estimates under this scenario conditions.

4.6.3 Use of Feedback in Cooperative Wireless Networks

Two studies will be carried out with respect to the use of feedback in the context of cooperative wireless networks. The first will build upon the results of Tabet *et al* [125] and [124] to analyze practical schemes for two-way HARQ communication via relays and in particular schemes for power-control exploiting 1-bit feedback as in [68] and [125] for the single-relay case. The study will also focus on the throughput of short block-length codes with both HARQ type-I and type-II protocols which was not considered in the DMT analysis of [125] and [124].

The second study deals with distributed synchronization in collaborative networks which is a means for using two-way communication to globally resolve a time-reference in a network. This is also what is known as *firefly synchronization* [127, 128, 129, 130] which is a well-known biological process which can be used in digital communication networks. This subject has been treated primarily in the context of short-range pulse-based networks for the purpose of synchronizing nodes in a distributed fashion (i.e. without a common external time reference or from one particular node in the network, such as a basestation). The use of similar distributed synchronization mechanisms for mesh networks or even cellular networks could have far reaching implications. In the context of cellular networks, basestations could synchronize (roughly) their transmission frames using the joint transmission of terminals instead of relying on external references such as GK'S or Gallileo. This would in turn allow for advanced collaborative resource allocation and distributed channel coding to be used.

4.6.4 Multi-User Feedback Channel

Feedback load is especially critical in Multiuser MIMO (i.e. point-to-multipoint two-way channel) systems because of its higher dimensionality and a proper MIMO channel labelling is of main interest. The challenge of extending the single-user MIMO designs to Multiuser transmission is that of combating interference since, for instance, the transmitter for one user depends on the channel of the other users. Also, multiuser systems pose the additional problem of user selection or scheduling. So far, the existing solutions resort to partial channel state information that is based either on quantization based techniques (e.g. Grassmanian codebook) or on dimension reduction and projection-based techniques. Geometric properties of the covariance manifold, as for instance, taking into account that they are not a vector space and that they lie in a Riemannian manifold, can be exploited for new solutions and insights on both of these directions. Also, the design of the feedback resources among users to optimize the performance of the whole system will be studied.

5 PRECODING FOR MIMO BROADCAST CHANNELS

This task focuses on what is most likely the key practical problem in wireless communications today, namely precoding for the multi-antenna broadcast channel. The MIMO broadcast channel is a model for the downlink in a cellular network, where multiple-antennas (at least 4) will surely be used in evolving standards such as UMTS-LTE and 802.16m combined with feedback channels for channel state information and decoding capacity indicators. Here a capacity boost by a factor equal to the number of antennas at the basestation, due to spatial processing, can be attained even without requiring multiple-antennas at the user terminals, which is the case in a MIMO point-to-point scenario. This is only possible, however, with the help of a feedback channel which allows the basestation to perform various forms of precoding. Because of this capacity boost, it has become a key technological target for evolving cellular network standards and the search for efficient low-complexity techniques is on.

5.1 Fundamental limits of MIMO Broadcast Channels

Design and analysis of multi-element communication systems under uncertainty conditions have recently attracted considerable attention in both theoretical and engineering aspects. Text books, special issues of professional journals, and technical meetings are focused on these important subjects. Future communication systems such as the 4G Cellular system and modern wireless LANS will include multiple users and access points using multiple transmit antennas as well as multiple receive antennas. Moreover, these systems will be combined with ad hoc cooperative networks, wireless sensor networks, the newly emerging cognitive radio and other state of the art communication networks (See the recent special issue [131] and references within). All of these fit together in the framework of multi-element communications for which a full theoretical framework is still missing. More importantly, one of the main obstacles in practically employing such systems is the lack of perfect knowledge of the environment conditions. This proposal focuses on the design and analysis of such systems under uncertainty conditions.

The multi-antenna application demonstrates the immense potential of multiple input multiple output (MIMO) techniques to dramatically enhance the performance of communications systems, motivating extensive theoretical research (See [132] and references within). A large portion of this research is done via information theoretic tools, which proved themselves as central not only in assessing the ultimate potential of these techniques but also in providing fundamental insight into the structure of practical communications systems with performance approaching the ultimate limits. The great potential of MIMO systems, in the wireless arena, realized by multiple transmit and multiple receive antennas, has been recognized in the last decade [133, 134] and those works based on information theoretic approaches have spurred an immense interest in the field. Dozens of papers are published every year both in the information theory literature as well as in the signal processing literature.

Most of the research on communication channels with multiple elements is dedicated to the use of multiple antennas in point to point single user communication under the availability of perfect channel state information (CSI). Information theory suggests that in this scenario the multiple dimensions of the MIMO channel can provide diversity gain and/or multiplex gain [135]. During the last decade, these potential gains were practically exploited using advanced signal processing and the problem is well understood [132, 133, 134]. This success along with the growing use of wireless communications has brought special attention also to multiuser MIMO systems. The single user results were generalized in a rather straight forward manner to the multiple access channel (MAC) where the capacity regions and the concept of degrees of freedom were fully characterized. On the other hand, many of the questions regarding the MIMO broadcast channel (BC) remained elusive until recently. The problem was first introduced in the pioneering paper by Caire and Shamai [136] which laid the foundations to the use of the dirty paper coding [137]. Based on these results, and the exciting notion of BC-MAC duality the MIMO BC capacity region was characterized [138, 139, 140]. The full capacity region was finally established in [141].

From a signal processing perspective the single user and multiuser MIMO channel also received considerable attention in the last years. Here, the main difference between the different MIMO channels involves the concept of cooperative processing. In the single user MIMO channel it is clear that the processing should be cooperatively implemented on both sides of the MIMO link, i.e., joint receive and transmit processing. In the MAC, signal processing at the transmitter side is impossible as the transmitting users cannot cooperate. Hence, the design is restricted to receive processing. e.g., via successive or joint decoding [142]. On the other hand, in the BC, the receiving users cannot cooperate and all the signal processing must be done at the transmitter side, e.g., via successive or joint encoding. Clearly, in future multi-element communication networks the problem of cooperation (and partial cooperation between multiple cells) will be much more involved.

Information theory deals with the theoretical limits of communications, but in practice suboptimal yet efficient signal processing algorithms are vital. Linear precoding schemes have been considered in [143, 144, 145]. Among these, the subset ZF precoding techniques [146, 147, 148, 149, 150, 151] showed promising performance when the number of users is sufficiently high. In fact, [152, 153] demonstrate that as this number grows, beamforming is sufficient for attaining the maximal throughput. In another line of works [154, 155] the gain of DPC over time sharing precoding methods was examined and upper bounds under different settings were derived. Other more intricate techniques based on superposition and beamforming which do not incorporate DPC were recently reported in [156].

The previous information theoretical results and signal processing algorithms are all based on the assumption that perfect channel state information is available to all the elements in the network. Clearly, this assumption is not too practical and therefore a considerable body of work addressed the problems under uncertainty conditions. In this case, the problems and solutions depend on the available information to each element. The underlying assumption is that only the receiver has access to the the channel and may estimate it. On the other hand, transmit elements can only obtain the information through feedback or through delayed estimates that originate from a previous stage in which these elements acted as receivers [157]. As a consequence, single user and MAC MIMO communication systems under channel uncertainty are relatively easy to handle, whereas there are much fewer theoretical (and practical) results on the BC channel and more complex multi element networks.

The problem of single user MIMO communication under uncertainty conditions is currently well understood. A large body of information theoretical literature focuses on the impact of the knowledge of the channel state information (CSI). It was shown that the capacity of the MIMO channel does not degrade as long as the coherence interval is sufficiently long [158, 159]. Most of the research in this area is devoted to stochastic models of the CSI in which the channel is modeled as a random variable. Here, the mutual information is also a random quantity and must be treated appropriately, either by considering its ensemble average known as the *ergodic capacity*, or by considering its cumulative distribution function (CDF) via the *outage mutual information*. Depending on the available feedback, optimal capacity achieving strategies were derived as well as conditions for the optimality of beamforming [160, 161, 162, 163]. Other results can be found in [164, 165]. Recently, an extension of these results to the MAC of a multiuser system was presented in [166]. A different approach for describing partial CSI is using a deterministic model for the channel, i.e., assuming that the channel is a deterministic variable within a known set of possible values. The use of deterministic CSI models is common in the signal processing community for designing algorithms which are robust to the uncertainty [167]. In the context of information theory, the maximal achievable rate of reliable communication over such channels is the *compound capacity* [168, 169, 170]. Under different uncertainty sets, uniform power allocation maximizes the compound capacity [171, 172]. In another work [173], the compound capacity was analyzed and bounded under the assumption of an isotropically distributed specular component. In [174] upper and lower bounds on the capacity of the compound channel with side information were derived. Recently, in [175] we considered the compound MIMO capacity and proved that under a similar setting to that in [162] beamforming is always optimal. The same results hold also for the compound MAC [176].

The problem of communication through a MIMO BC channel under uncertainty conditions is still immature and there are many open questions. Unlike the previous results, it has been shown that the degrees of freedom in the BC setting depend critically upon the availability of channel knowledge at the transmitter [136, 177, 178]. Moreover, the capacity achieving strategy both in the stochastic and the compound setting is still unknown. From an information theory perspective, it is still unclear how should one generalize the idea of dirty paper coding to the case of channel uncertainty. A few recent attempts at this difficult problem can be found in the method of carbon copying (robust dirty paper coding) [179], the achievable region of the linear assignment strategies [180] and the recently obtained bounds on the compound capacity with side information [174].

The inherent difficulty in addressing the fundamental informational quantities in the BC channel under uncertainty conditions led many researchers to follow a different approach. The idea is to propose simple suboptimal signal processing methods and to analyze their performance. Among these, the opportunistic beamforming strategies presented in [65, 181] proved that the sum rate due to complete CSI can be asymptotically achieved using only partial CSI. These studies as well as the channel hardening result in [182] demonstrate the significant potential gains in combining the MIMO technology with simple scheduling algorithms. In this context, an eigen-based scheduler using partial feedback information is proposed [183]. Robust zero forcing and MMSE linear precoders have been proposed [184, 185, 186], as well as non-linear THP based robust precoders [187, 188, 189, 190].The throughput degradation due to finite rate feedback in zero forcing precoding is examined in [191]. See also [192] for the impact of CSI on multiuser MIMO systems. Another approach for reliably transmitting in MIMO channels using partial CSI involves the concept of antenna selection [193, 194, 195, 196, 197].

We have included here only a minute fraction of the vast literature currently available, and brought mainly recent references, as to reflect the updated art in this vibrant field of massive research. Many aspects of MIMO techniques have not been mentioned, as to keep the background section concise. The impression this may give is that we deal here with a mature well-understood subject. Even though massive fruitful research efforts have been invested and much fundamental understanding has been gained, the subject is yet far from maturing, and in fact certain important aspects of information theoretic treatment of MIMO systems, are at their infancy. Our research objectives, as detailed in the following section, are directed towards some of these specific areas in an effort to achieve a deeper information theoretic understanding and to gain also insight into the practical implications of the theory.

5.2 Precoding Techniques for MIMO Broadcast Channels Without Channel Uncertainty

It is important to emphasize the interplay between information theory and signal processing in the context of multi element communications. As explained, the core informational problems of the different network topologies originate due the practical cooperative processing constraints. The solutions are also highly related. The main tool in characterizing the BC capacity region is successive encoding via dirty paper coding (DPC). In practice, the signal processing methods are based on MIMO Tomlinson Harashima Precoding (THP) [198, 199, 200, 201, 202] and the state of the art vector perturbation schemes [203, 204]. All of these methods can be interpreted as practical DPC approaches. Another important example is the well known BC-MAC duality in information theory and its complementary downlink-uplink beamforming in signal processing [205, 206, 207, 208, 209, 210, 211, 143].

5.2.1 Vector Perturbation Non-Linear Transmission

As a method to combat the noise enhancement of the ZFB a vector perturbation technique was introduced in [[203], [204]]. This is accomplished by considering data symbols from a cyclic extension of the constellation alphabet and adding a non-linear perturbation vector to the transmitted data vector. Part of the appeal of such an approach is that the processing required at the receivers decouples, making the technique suited for the broadcast scenario. The perturbation vector may be chosen in a number of ways, and in particular so that it minimize the transmit energy over all possible perturbation vectors. The possible gains of this transmission technique have also been characterized analytically in the large system limit using replica analysis [212]. Following the publication of [[203, 204]], a number of authors proposed improvements to the basic transmission scheme. These included the use of lattice reduction

(LR) techniques [[213, 214]] to reduce the computational complexity of obtaining the perturbation vector while maintaining close to optimal performance with respect to the transmit energy. The near optimality of the LR approach have also recently been proved analytically [6].

5.3 Opportunistic Beamforming Techniques with Limited-Feedback

Notice that all the previous transmission techniques require for full CSIT for their correct performance, so that even their complexity is lower than DPC but their integration within commercial systems is handicapped by their full CSIT requirements, as a high load on the feedback channel is required from all the users in the system. Therefore, a search for realistic transmission schemes in the multiuser MIMO scenario is required, where such schemes can benefit from the multiuser availability through an integrated users selection process, while a low complexity transmitter processing is accomplished together with the feedback load consideration.

The multiuser MIMO scenario is highly dependent on the amount of available channel knowledge, where if no CSIT is present, then the multiuser and multiplexing gains within this scenario are wasted [215], therefore, some quantity of CSIT is required. Partial CSIT systems seem to be the proper choice for the feedback load, to balance between the effects of feedback requirement, and the restrictions on the feedback load within commercial systems.

One of the main transmission techniques based on partial CSIT is the Multibeam Opportunistic Beamforming (MOB) [215], where only the channel modulus information is sent through the feedback channel from each user and towards the BS scheduler. This decreases the feedback load in comparison with the full CSIT (phase and modulo with respect to each antenna) by a factor higher than 2, depending on the feedback strategy. The MOB scheme benefits from the partial CSIT that is available from all the users, to extract the system multiuser gain, as an implicit users selection is included in its performance.

The amount of feedback reduction depends on the used metric for the channel modulus feedback process, where several metrics are proposed for the MOB scheme:

1. The initial strategy [215] decides on the use of the Signal to Noise Interference Ratio (SNIR) metric with respect to all the generated beams, thus decreasing the total amount of feedback to the 50% of the full CSIT case.

2. An upgrade also provided by [215] reports an alternative metric where the maximum SNIR value over all the beams received at each user is the only feedback to the BS scheduler, which further decreases the feedback load, while a small performance loss is resultant.

3. Interesting studies [64][216] show that the modulus feedback information from each user is not required and therefore, feeding back the modulus information only when it is larger than a certain threshold obtains similar performance as the initial scheme in [215].

4. Additional metrics for the different environments are supplied in [217], which provides the system operator with the freedom to choose among the available metrics for the most convenient one, based on its requirements and constraints.

While the initial scheme in this paragraph enables a power loading, the threshold application decreases the feedback load to minimal values, to the extent that power loading is not possible.

One of the most important features of MOB is the simplicity in its operation and feedback load requirement Also its capabilities in taking advantage of any further information regarding the channel conditions. Notice that for the MIMO implementation in any communication system, the commercial operator can ask for Quality of Service (QoS) minimum allowed rate requirements rather than maximum sum rate, and while such requirements are difficult to accommodate in the optimal DPC strategy (and other suboptimal schemes), MOB offers this possibility with minor changes in its operation.

5.4 Directions for Research During the Course of NEWCOM++

5.4.1 Information Theoretic Treatment of Gaussian Multi-Antenna Broadcast Channels

The central objective of this endeavor is to deepen the body of available information theoretical results and practical signal processing algorithms for the important class of multi element communications models under different uncertainty conditions. Our primary goal is to develop theoretical results that address systems and scenarios which are of practical importance on one hand and which are yet simple enough that they lend themselves to a rigorous analytical attack. We envision a twofold goal. First we plan to set up novel ultimate theoretically achievable limits of performance of MIMO systems in the different uncertainty conditions. Next we hope to harness the theoretical results and the enhanced insight acquired in the information theoretical research to derive practically appealing signal processing algorithms for communication systems which strive to perform in the vicinity of the ultimate limits. Some concrete issues to be addressed in our research in which preliminary efforts show promising results are stated below and further elaborated on in the ensuing section.

5.4.1.1 MIMO Broadcast Capacity Region under Uncertainty Conditions

Our central objective is to characterize bounds on the theoretical capacity region of communications through a multiuser BC under different uncertainty conditions. We will address both stochastic and deterministic uncertainty models. In the stochastic models we will concentrate on the ergodic and outage capacities, whereas in the deterministic setting, we will focus on the compound capacity (which can also be viewed as the zero outage capacity).

5.4.1.2 Uncertainty Modelling and Broadcast-MAC Duality

In the prospect of eventually analyzing the BC, we plan on filling gaps in existing results on the more simple single user channel, as well as the MAC. From a theoretical information perspective this problem is basically understood. However, there are still many unsolved issues concerning the practical derivation and evaluation of the optimal transmit schemes. There are numerous works on transmitter design using stochastic uncertainty models, and we ourselves have successfully confronted the problem in the context of deterministic uncertainty models as detailed in our work [176, 175]. Yet, most of the results are on rank one MIMO channels (which basically transform the problem into a simplified single output channel) and it is not clear what are the optimal transmit strategies in channels with higher ranks which are considerably more interesting. We hope that by introducing novel uncertainty models and performance measures a different kind of BC-MAC duality may be established, that will account also for mismatched decoding capturing the uncertainty in the channel state information.

5.4.1.3 Practical Scheduling Policies

One of our central objectives is to identify efficient and information theoretic inspired scheduling methods which use the available CSI to optimize the system's performance subject to different practical constraints. The idea is that in a rich user environment, it is possible to identify, with large probability, a selected number of users which are jointly 'good' in the sense of exhibiting channels with almost orthogonal propagation vectors, while enjoying separately good channel conditions. This allows the use of simple signal processing techniques with relatively low feedback (and consequently bad uncertainty conditions) while compromising little on performance to the ultimate theoretical bounds.

5.4.1.4 Co-operative Multi-Cell Systems under Uncertainty

We plan on studying yet not fully understood theoretical problems, such as the basic tradeoffs and impact of MIMO approaches in a multi-terminal scenarios. The availability of multiple dimensions is used to combat both natural channel random fluctuations as well as time varying nature (fading) on one hand

and on the other trying to mitigate the inherent interference effect emerging in systems which communicate simultaneously sharing the same degrees of freedom among different users. Mitigating interference which emerge in multi cellular systems by joint multi-cell site processing has demonstrated significant advantages [218]. This concept of cooperation has recently attracted considerable attention in the signal processing community and involves difficult design problems [219]. We propose to attack these state of the art optimization tools, and analyze their performance using modern random matrix theory [220]. Of course, here too the problem of channel uncertainty is crucial and must be taken into consideration. In fact, the assumption of perfect channel state information in the cooperative setting is even less realistic than in the standard BC setting. Thus, we will also focus on the theoretical achievable rates under uncertainty conditions and practical robust cooperative signal processing algorithms, as well as mismatched approaches.

5.4.2 Empirical Analysis of MU-MIMO Precoding Strategies Using Channel Measurement Databases

The performance of precoding for MIMO broadcast channels is often assessed using simplified channel models, such as the i.i.d. model. The assumption of an i.i.d. channel is often justified using the argument that the users are spatially separated and thus the signals arriving at different users will be independent even in the presence of a line of sight (LOS) component. However, it turns out that this assumption is not always true [221].

In this workpackage we will use real channel measurements to study the performance of precoding for MIMO broadcast channels. The channel measurements will be obtained using Eurecom's MIMO Openair Sounder (EMOS) [222]. The EMOS can perform real-time channel measurements synchronously over multiple users moving at vehicular speed. The measured channels are stored to disk for offline analysis.

This work will be done in close cooperation with WPR1, where the main goal is the acquisition, the characterization and the modeling of the channel. See deliverable DR1.1 for a more detailed description of the hardware platform.

One of the conclusions in [223] is that there is a very large gap between full feedback and quantized feedback. One possible extension/improvement of the work is to exploit time and frequency correlation in the analysis. The final objective is to take into account the correlations of the channel in different domains to minimize the quantity of the feedback information, or, what is equivalent, to increase the quality of the channel information and, consequently, also of the global system performance, with a constraint over the maximum load in the feedback link (for example, using differential quantization strategies in channels with temporal correlation).

In order to validate the designs developed in this framework, it is proposed to use realistic and measured data from real channels to reduce the gap between the real performance and the one predicted by mathematical models that, in general, are not able to accommodate all the possible effects and non-ideal behaviour that may arise in a practical deployment.

5.4.3 Effects of Imperfect CSI on Practical Precoding Strategies

The success in the practical implementation and deployment of any communications system will be directly linked to the assumptions taken into account during the theoretical development of the design of the transmission scheme. More concretely, special attention should be paid to the issues related with the quantity and the quality of the channel information available during the design. In a realistic and practical scenario, this information will not be perfect and, therefore, the corresponding degradation should be taken into account.

The problems arising from having an imperfect CSI are specially significant in the case of multi-antenna multi-user systems, basically due to the fact that the number of parameters that describe the channel is much higher than in the single-user or single-antenna configurations. The imperfections in such channel knowledge have an impact on the increase of the levels of inner multi-user interference. Besides, in the multi-user case, the qualities of the information coming from different users may be quite different and therefore, the system should be able to accommodate in the design this heterogeneous information. Consequently, in multi-user multi-antenna systems, the adoption of a proper design strategy able to cope with imperfect CSI is critical to obtain an acceptable performance in a practical deployment.

Concerning the multi-user broadcast MIMO channel, there are several possible transmission architectures, going from linear to non-linear schemes and with different capabilities of adaptation. Each of these schemes has a different computational complexity and requires a different degree of channel knowledge. Some examples are the linear beamforming design [224], the non-linear approach based on the spatial Tomlinson-Harashima precoding [225], and the multi-beam opportunistic beamforming scheme [215, 226]. The channel information required for each case is different (for example, the adaptive linear scheme requires a complete channel estimation, whereas the opportunistic solutions requires only some SNR values).

In all the previous cases, although the needed quantity of information is different, this information is expected to be imperfect, i.e., to have some error. For example, in the case of adaptive linear precoding, the channel response has to be estimated. In this estimation process some noise will appear. Besides, in some cases this estimation has to be quantized by the receiver before feeding back that information to the transmitter, including an additional quantization noise. In the opportunistic scheme the SNR values have also to be quantized.

Taking all this into account, the proposal is to work on the development of robust schemes and designs that are able to jointly fulfill the constraints derived not only from the complexity, but also from the channel imperfections. These solutions will control the increased levels of multi-user interference due to the imperfections in the channel knowledge. The designs, that will be directly related to the quality of the channel estimation, will also derive design criteria for the definition of proper feedback mechanisms. In other words, the challenging work to be done will focus on the interactions between the multi-user robust schemes and the feedback mechanisms, that, from an optimal point of view, should be designed jointly.

The effect of imperfect CSI on vector perturbation techniques such at those proposed in [5] have not yet been studied in the literature. Initial findings suggest that the impact of the CSI imperfections on the performance of the system, e.g. in terms of detection error probability, may be significantly affected by the specific power allocation policy applied (c.f. [227, 228]). Thus, it is of interest to understand and characterize the complex interplay between the power allocation polices, the CSI imperfections and overall system performance. Thus, we propose to study the performance vector perturbation pre-coding over under different power allocation polices and different channel models. Further, it is anticipated that this will aid the practical design of such systems by indicating under what conditions the multiuser interference becomes a limiting factor in the overall system performance.

5.4.4 Cross-Layer Techniques for Precoding with limited-feedback including QoS constraints

A main target behind the proposal of any transmitting scheme for its implementation in realistic systems, relates to its performance in terms of the QoS of the users in the system. To fulfill the QoS objectives, the scheduler needs information about system parameters, such as current application requirements, queue size or power restriction among others; thus, a Cross Layering (CL) between the different layers of the OSI structure [229] is mandatory for QoS achievement.

As already commented, partial-CSIT MIMO schemes need to consider the (partial) channel instantaneous characteristics to increase the system throughput through the most efficient way, but notice that the administrator requirements are usually set in the higher layers of the communication process, where some QoS measures are fixed for the system performance in terms of maximum number of active customers and/or a certain level of fairness among them. On the other hand, additional per user QoS requirements are imposed in terms of minimum allowed rate and/or maximum allowed packets delay. Therefore, for the integration of the partial-CSIT MIMO schemes in realistic systems while satisfying the users' QoS demands, a Cross Layering between the Physical layer channel characterization and the higher layer QoS demands is needed.

Along this Work package, a down-up philosophy is regarded, as the optimization process is mainly treated at the Physical layer with the higher layers requirements operated as QoS constraints, that are desired to satisfy through some processing mainly carried out at the Physical layer. Therefore, several scenario simplifications can be considered (i.e. Forward Error Coding (FEC), Automatic Repeat reQuest (ARQ) and users mobility, among others). Cross Layer schemes are also considered in the Work package 8, but with the focus of the higher layers of the communication process, so that an up-down philosophy is considered. Cooperation between the two philosophies can lead to interesting results about the Cross Layering among a larger number of layers, and with the different approaches of people working in the Physical layer and higher layers. But such large Cross Layering has to be done with caution, as it can lead to a system instability due to the large number of variables and their cross effects.

Hence, within the CL approach in this Work package, several resources are employed to optimize the system performance, and from all the layers. Some of these resources are now listed

- Number of active users (access control).
- Number of simultaneously transmitting users.
- Transmitting power.
- Modulation.
- Multiple access (Ex. spatial separation, frequency bins, codes, ...).
- Quantity of channel knowledge.
- ...

showing the resources that have to be jointly optimized to achieve the system objectives in terms of sum throughput and users' QoS satisfaction.

Therefore, the research objective within this task is to study the impact of the different variables that affect the CL process in partial-CSIT MIMO Multiuser scenarios, and optimize them to increase the system throughput and guarantee the QoS requirements for the users. A heterogeneous scenario is considered, where each user asks for different QoS demands on the basis of its running application, thus matching the realistic systems with several coexisting applications in the system. A sequential study of the optimization variables is individually carried out, together with a gradual consideration of the QoS demands. As a final result, the joint consideration of all the variables is envisaged to increase the system throughput together with the satisfaction of all the QoS demands.

6 CONCLUSION

This deliverable presented an overview of the activities pertaining to feedback and imperfect channel estimation and the interplay between the two in the form that will be considered in NEWCOM++. Their relation stems from the the fact that feedback of channel state information has become an integral part of modern wireless networks (UMTS-HSDPA, UMTS-LTE, 802.16m), both from the point-of-view of resource scheduling and advanced multi-antenna signal processing and therefore the ability to resolve the channel at the transmitter is ultimately related to what can be estimated at the receiver. Moreover, we strongly believe that the analysis of such systems should be considered in the two-way setting since bandwidth for feedback does not come without a price.

We gave an overview of the state-of-the-art in relation to the three tasks of WPR.2 namely, Imperfect channel estimation, Point-to-point and Point-to-multipoint Two-Way Channels, and Precoding Techniques for the MIMO Broadcast Channel. We included the initial planning for joint research research activities during the first year of NEWCOM++.

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