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State of the Art

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Editor(s): J. Sayir (FTW), C. Novak (VUT), C. Poulliat (CNRS), G. Lechner (FTW), N. Noels (UGent))
Participating institutions: Bilkent/Kadir Has, Technion, NKUA, CNIT, CNRS, LNT, RWTH Aachen, UCL/UGent, FTW/VUT, PUT, Chalmers, AAU
Contributors: F. Alberge (CNRS), A. Alvarado (Chalmers), E. Arikan (Bilkent), J. Bhatti (UGent), D. Declercq (CNRS), P. Duhamel (CNRS), B. Fleury (AAU), S. Godtmann (RWTH Aachen), T. Gucluoglu (Kadir Has), G. Kirkelund (AAU), G. Lechner (FTW), G. Matz (VUT), M. Moeneclaey (UGent), G. Montorsi (CNIT), Z. Naja (CNRS), N. Noels (UGent), C. Novak (VUT), E. Panayirci (Kadir Has), S. Papaharalabos (NOA/NKUA), C. Poulliat (CNRS), J. Sayir (FTW), H. Sneessens (UCL), H. Steendam (UGent), P. Tyczka (PUT), L. Vandendorpe (UCL)
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Abstract:

This deliverable lays out the state of the art within the research areas tackled by WPR.4 of NEWCOM++. It is structured in 4 technical sections that resulted out of a consolidation and prioritization of the 8 tasks originally defined in the Technical Annex of NEWCOM++. These sections are: theoretical framework for iterative processing; code design; low complexity and implementation; and synchronization. The sections were selected so as to address a significant portion of the main challenges facing the use and implementation of iterative "turbo" processing algorithms in wireless applications. In each of the sections, the state of the art is described and the plans for further collaborative research in the field is established.

Keyword list: Iterative Processing, Turbo Codes, LDPC Codes, Convex Optimization, Information Geometry, Factor Graphs, Polar Codes, Non-Binary Codes, Bit-Interleaved Coded Modulation, Quantization, EXIT Charts, Density Evolution, Synchronization

Table of Contents

1 INTRODUCTION

The use of algorithms based on the "turbo" principle underlies every aspect of modern receiver design. Since the invention of turbo codes [BGT93] and re-discovery [Gal62, Gal63, Mac99] of low-density parity-check (LDPC) codes in the 1990s, it has been established that iterative processing is necessary for communication devices to achieve an optimal power-throughput-bandwidth balance. This is particularly true of wireless communications, where power and bandwidth are scarce resources.

In 1993, the initial turbo coding paper [BGT93] demonstated that it was possible in a simulated communication environment to achieve a rate, power and bit error rate regime within 1 dB of the theoretical Shannon limit [Sha48]. After the initial shock triggered by this realization, the communications research community set out to understand the theory underlying these results, aiming to reproduce this type of performance for a wide range of channels and in realistic environments. Within 10 years, some of the principles behind the "turbo" effect had become well understood:

- the connection between iterative decoding and graph-based models had been established and explained [MMC98, KFL01, AM00];
- alternative techniques like LDPC and repeat-accumulate codes had been (re-)invented [Mac99, JKM00] that exhibited the same type of performance as Turbo codes;
- techniques had been developed to track the convergence of an iterative decoder in the asymptotic regime, i.e., when the code block length tends to infinity, notably density evolution [RU01a] and EXtrinsic Information Transfer (EXIT) charts [tB99];
- using these techniques, it became possible to design binary codes [RSU01, tB00b, BCV$^+$02] that come extremely close to the Shannon limit. However, these designs were limited only to the asymptotic regime of infinite block length, and only to specific channels, notably the Binary Erasure Channel (BEC) and the Additive White Gaussian Noise (AWGN) channel with binary anti-podal modulation;
- initial attempts at combining decoding and other receiver components (detection, demapping, demodulation, synchronization) in an iterative fashion had been proposed heuristically and tested by simulation [AGR98, GA00, AS99].

In the past five years, some progress has been made towards resolving the remaining issues standing in the way of a successful translation of the "turbo" principle from theory to practice. In particular, LDPC and related codes have been designed and optimized for Multiple-Input Multiple-Output (MIMO) fading channels [tBKA04] (e.g., multiple antenna wireless communications); "turbo" designs have been proposed for some multi-user scenarios, in particular multiple access channels; and some design methodologies have been developed for non-binary codes [BB06].

There are still tremendous challenges in the way of making iterative processing practical for the wide range of scenarios that are relevant for wireless communications. Iterative processing promises to deliver increased data rates, extended reach, and reduced power and bandwidth requirements. These are all crucial contributions towards improving the business case for the deployment of new technologies over the wireless medium, and the difference they make goes beyond the predictive capabilities of most current business analysts in telecommunications. In particular, there is need for:

- new theories to tackle the analysis of iterative systems without asymptotic assumptions;
- the design of moderate length codes that fulfill realistic delay constraints;
- low complexity alternatives for some of the more complex components that are necessary for iterative decoding;

- constructive code designs that can be described and implemented without specifying large matrices, and where the decoder can take advantage of the known code structure without trading in performance in exchange;
- low complexity encoders;
- non-binary codes and techniques that achieve the performance bounds for non-binary signalling;
- a theoretical framework to account for implementation constraints, such as the fixed-point constraint of processors, and design and optimization of codes and decoders given these constrints;
- more theory to account for other receiver imperfections, such as synchronization and channel estimation failings, and optimal iterative processing techniques that include these stages of the receiver.

In this report, we will present our plans to tackle some of these challenges within the resources available to the workpackage.

The institutions participating in NEWCOM++ have pioneered significant advances in the design of iterative algorithms in the past. They have accumulated a rich experience in the field, partly through the predecessor Network of Excellence NEWCOM. Since iterative processing has become an enabling technology for many of the applications considered in other research workpackages within NEWCOM++, WPR.4 is a workpackage of central significance, with potential interactions with several other packages, in particular WPR.1, 3, 5, 6, 7 and A. With 12 out of 17 NEWCOM++ partners participating and 48 researchers having registered their interest for the workpackage, WPR.4 is currently one of the largest workpackages within NEWCOM++.

While it is a bonus for this workpackage to stand at the crossroads of the shared interests of the majority of NEWCOM++ partners, it makes it more difficult to define a streamlined program of collaborative research that is not simply the sum of the diverging interests of the partners. In the NEWCOM++ technical annex, we defined 8 tasks resulting out of preliminary discussions between partners:

Task 1 Analytical Bounds

Task 2 Information Combining

Task 3 Code Design

Task 4 New Decoding Algorithms

Task 5 Mismatched Decoding

Task 6 Fixed Point Decoding

Task 7 Turbo Synchronization

Task 8 Joint Design and Optimization

The primary goal of the workpackage kick-off meeting held in Munich on the 26-27 February 2008 was to consolidate and prioritize this extensive task list, so as to create a reduced set of four focal points to concentrate the collaborative research within the workpackage. This reduced set of topics is reflected in the structure of the deliverable. It does not formally replace the original task list: some of the tasks are now included within a topic (Tasks 3, 6, 7) or split between topics (Tasks 1, 4, 8), while some are expected to be tackled later in the life of NEWCOM++ (Tasks 2, 5). The current structure was established to foster collaborations between partners where common interests were identified, in many cases accross the tasks that had been originally defined. The sections and contents of this deliverable will be structured as follows:

Theoretical Framework for Iterative Processing This section englobes all efforts within the workpackage to investigate the mechanisms that underlie iterative processing in relation to several other disciplines, e.g., information geometry, convex optimization, graph-based models, and network information theory. It tackles graphical models for iterative decoding and applications of iterative decoding to multi-user scenarios involving relaying and cooperation.

Code Design Much has been achieved in the design of binary LDPC and turbo codes using methods such as density evolution and EXIT charts. We will study new code designs that transcend known methods, in particular a fundamentally new approach known as binary "polar codes", as well as the design of non-binary codes that is well suited for applications with delay constraints where classical design methodologies tend to fail, and the design of codes for application with stringent bandwidth constraints that are common in wireless applications.

Low Complexity and Implementation With much of the theory behind capacity- achieving codes developed over the past decade, research interests are shifting towards the problems that underlie the implementation of iterative processing in the real world. This chapter will set out our research intentions regarding the implementation of iterative algorithms under quantization constraints that occur in a fixed-point implementation, the use of simplified decoder and other receiver components, and low-complexity decoder algorithms for polar codes.

Synchronization This section englobes all research into receiver design when there is no coherence assumption, and synchronization must be provided as part of the iterative processing. We consider both the uncoded case and the coded case where synchronization can rely on feedback from the decoder.

Each section documents the state of the art in the field and elaborates on our collaborative research intentions within WPR.4.

1.1 Abbreviations

Throughout this report, abbreviations are defined in the text where they are used. For convenience, we also provide a list of abbreviations used:

APP A-Posteriori Probability

APPA A Priori Probability Aided

AWGN Additive White Gaussian Noise

BCJR Bahl Cocke Jelinek Raviv

B-DMC Binary-input Discrete Memoryless Channel

BEC Binary Erasure Channel

BER Bit Error Rate

BICM Bit-Interleaved Coded Modulation

BPSK Binary Phase Shift Keying

BSC Binary Symmetric Channel

CA Code Aided

CDMA Code Division Multiple Access

DD Decision Directed

DFE Decision Feedback Equalization

DM Divergence Minimization

DMC Discrete Memoryless Channel

EKF Extended Kalman Filter

EM Expectation-Maximization

EXIT EXtrinsic Information Transfer

FS Frequency Selective

GEXIT Generalized EXtrinsic Information Transfer

GF Galois Field

I-EXIT Input-to-input EXtrinsic Information Transfer

IRA Irregular Repeat Accumulate

ISDD Iterative Soft Decision Directed

ISI Inter Symbol Interference

JED Joint Estimation Detection

KL Kullback Leibler

LDPC Low-Density Parity Check

LLF Log Likelihood Function

LLR Log Likelihood Ratio

LMMSE Linear Minimum Mean Square Estimation

LogMAP Logarithmic Maximum A Posteriori

LUT Look-Up Table

MaxLogMAP Maximum Logarithm Maximum A Posteriori

MAP Maximum A Posteriori

MIMO Multiple Input Multiple Output

MSE Mean Squared Error

MSEE Mean Square Estimation ERror

NB-LDPC Non-Binary Low Density Parity Check

NDA Non Data Aided

O-EXIT Output-to-output EXtrinsic Infmormation Transfer

OFDM Orthogonal Frequency Division Multiplexing

OFDMA Orthogonal Frequency Division Multiple Access

PB Pilot Based

PCCC Parallel Concatenated Convolutional Coding

PSK Phase Shift Keying

QPSK Quadrature Phase Shift Keying

RM Reed Muller

SAGE Space-Alternating Generalized Expectation-maximization

SCCC Serially Concatenated Convolutional Coding

SDD Soft Decision Directed

SISO Soft Input Soft Output

SMA Sum Minimum Algorithm

SMC Sequential Monte Carlo

SNR Signal to Noise Ratio

SOVA Soft-Output Viterbi Algorithm

SPA Sum Product Algorithm

STC Space Time Codes

VB Variational Bayesian

VBEM Variatonal Bayesian Expectation Minimization

VFEM Variatonal Free Energy Minimization

2 THEORETICAL FRAMEWORK FOR ITERATIVE PROCESSING

2.1 Introduction

While iterative algorithms are used in a wide range of areas, only some understanding has been gained as to how and why they work so well. For example, maximum likelihood bounds [SS06] are helpful in bounding the performance of iterative algorithms. In this section, we combine some promising approaches to gain further understanding of iterative algorithms.

In section 2.2 we introduce some basic concepts of information geometry, which use methods of differential geometry to interpret iterative algorithms. As an example the Blahut-Arimoto algorithm [Ari72, Bla72] is given an information geometric interpretation, and progress on information geometric interpretation of BICM [Zeh92, CTB98, LCR02] is reported.

Section 2.3 introduces (iterative) divergence minimization approaches to the problem of receiver design. Starting with the EM-algorithm [DLR77], several variants and their applications to communication problems, mostly joint channel estimation and detection problems, are shown.

In section 2.4 the framework of graphical models is introduced and the message passing algorithms are presented, which allow a unified view of many signal processing problems.

Finally, section 2.5 gives an overview of distributed processing in the case of relay channels. Being conceptually different than coding for point-to-point links, the distributed aspect allows application of concepts like message passing and scheduling, questions that we believe can be tackled with the help of previous sections.

2.2 Geometrical interpretation of iterative algorithms

Iterative turbo-like algorithms, as pioneered in [BGT93], have been a major breakthrough in digital communications. The idea underlying turbo codes and iterative decoding in general is to split the overall optimal receiver into mutually dependent subblocks. Each subblock provides each other subblock with extrinsic soft information in an iterative fashion. Considerable efforts have been expended to understand theoretically the near optimal performance of iterative decoding.
Information geometry is a powerful tool for the study of iterative estimation procedures. One starting point for information geometry was Csiszar's paper [Csi75] that provides a generalized Pythagorean theorem based on the Kullback-Leibler divergence. In [CT84], information geometry was exploited to analyze alternating minimization procedures. The Expectation Maximization (EM) algorithm [DLR77], the Blahut-Arimoto algorithm [Ari72, Bla72] for evaluation of capacity and rate-distortion curves were demonstrated to be special cases of alternating minimization procedures based on information divergence. In this section, we focus on the various interpretations of the iterative procedure of the Blahut-Arimoto algorithm. Then, the connection between information geometry and iterative decoding is reviewed. Finally, we point out some directions for further research within NEWCOM++.

2.2.1 The Blahut-Arimoto algorithm

In 1972, R. Blahut and S. Arimoto [Ari72, Bla72] received the Information Theory Paper Award for their independent work on how to compute numerically the capacity of memoryless channels with ***finite*** input and output alphabets.

In [CM03], an information geometric interpretation of the Blahut-Arimoto algorithm in terms of alternating information projection was provided. Based on this last approach, Matz [MD04] proposed a modified Blahut-Arimoto algorithm that converges significantly faster than the standard Blahut-Arimoto algorithm. This formulation was obtained by rephrasing the classical Blahut-Arimoto algorithm as a proximal point method [Vig95]. New elements regarding this interpretation are reported in [NAD]. By using those two approaches in practical implementations, excellent performance (in fact, superlinear convergence) are observed in the numerical experiments.

The Blahut-Arimoto algorithm was recently extended to channels with memory and finite input alphabets and state spaces [DYW04]. An algorithm was also proposed for computing the capacity of memoryless channels with ***continuous*** input and/or output alphabets [Dau05].

This is a first step towards a full understanding of iterative algorithms via information geometry. The next section is dedicated to turbo-like algorithms.

2.2.2 Iterative decoding

The introduction of turbo-codes in [BGT93] began a new era in communication systems achieving performance at rates extremely close to the Shannon limit. The turbo-decoder and more generally iterative decoding was not originally introduced as the solution to an optimization problem rendering the analysis of its convergence and stability very difficult.

Among the different attempts to provide an analysis of iterative decoding, the EXIT chart analysis and density evolution have permitted to make significant progress [tB01, GH01] but the results developed within this setting apply for large block length. Another tool of analysis is the connection of iterative decoding to factor graphs [KFL01] and belief propagation [Pea88]. Convergence results for belief propagation exist but are limited to the case where the corresponding graph is a tree which does not include turbo codes or LDPC codes. A link between iterative decoding and classical optimization algorithms has been made recently in [WRJ06] where the turbo decoding is interpreted as a nonlinear block Gauss Seidel iteration for solving a constrained optimization problem. In parallel, a geometrical approach has been considered and provides an interesting interpretation in terms of projections.

The particular case of BICM-decoding (Bit Interleaved Coded Modulation) [Zeh92, CTB98, LCR02] has been studied by Muquet in [MDdC02] where the decoding sub-block is interpreted as two successive projections. The interpretation of the demapping sub-block in terms of projection remains unachieved. In [Ric00], the turbo-decoding is interpreted in a geometric setting as a dynamical system leading to new but incomplete results. The failure to obtain complete results is mainly due to the inability to efficiently describe extrinsic information passing in terms of information projection. Without extrinsic propagation (*ie* with APP propagation), the iterative decoding would be the alternate projection algorithm of Csiszár [CT84] or more generally the method of Bregman retractions [BC02a]. The convergence of these algorithms is well established for projection onto closed convex sets but not in the general case including turbo-codes. Dykstra's algorithm [Dyk83] is a well-known algorithm for solving the best approximation problem. It has been recently proved [Wal07, Alb08] that the propagation of extrinsic rather than APP is connected with Dykstra's algorithm.

2.2.3 Intended collaborative research within NEWCOM++

- Use the twofold interpretation of the Blahut-Arimoto algorithm (alternate projections/proximal point method) as a first step towards an analysis of the iterative decoding.
- Use the analogy between the Dykstra's algorithm and iterative decoding to produce new convergence results or a modified iterative structure with proved convergence.
- Extension of the interpretations in terms of projection for the classical alternatives to the BCJR like Log-MAP, SOVA or Max-Log-MAP.

2.3 Receiver design and analysis via (iterative) divergence minimization approaches

Traditionally, receivers are based on a structure, which consists of three elements: channel estimation, multi-user interference cancellation and a bank of single-user a-posteriori probability (APP) channel decoders, see e.g. [GA00, KBC01, Lam02, WM06, LBP07]. Although this makes sense, the structure needs to be placed in a unified framework to perform joint optimization. By applying EM-like frameworks to the receiver [KLF05, KF03, KLF07] some optimization can be performed. However EM/SAGE-like frameworks do not allow the passing of probability distributions for both the parameter of interest and the nuisance parameters, the latter being included in the complete data. For instance, while these frameworks may admit soft symbols from a soft-output decoder as inputs, the channel parameter estimates they return can only be in the form of hard decisions, i.e., they can not convey any uncertainties in the estimation of the channel parameters back to the decoder. Soft-value EM relies on a modification, which thus violates the framework [CV01, HLPF07, HKP+04], making it impossible to incorporate the APP directly as a part of the optimization.

The variational Bayesian (VB) framework proposed by [Att99] is obtained based on variational Bayesian inference, for the case where the structure of the data-set is unknown. Variational inference algorithms were originally introduced in machine learning from graphical models. The variational optimization framework links the APP demodulators and soft input soft output equalizers. As shown in [LL05], the methods mentioned in [AGR98] and [WP99] can be seen as solutions to the variational energy minimization.

The VB framework was applied to the joint channel estimation and multi-user detection problem in [SK07] for OFDM in a time-varying multi-path fading channel. In this reference, the receiver iterates between a data detection part and a Kalman-like tracking part for the time-varying channel. In [Nis07], a VB-framework is deployed to estimate the phase noise in the system. A similar scheme was proposed by [LL07]. In [NP07], it was argued that in the presence of Rayleigh fading, many soft-input soft-output equalizers are instances of a variational optimization problem.

The VB framework is elaborated and extended in [Bea03] to the variational Bayesian EM algorithm (VBEM) by introducing the notion of the complete data-set, known from the EM algorithm [DLR77]. An iterative receiver based on the VBEM algorithm for the joint channel estimation and multi-user problem for a single-user MIMO system was proposed in [CL06]. In the VBEM framework, the only constraint is put on the auxiliary function, which is limited to a class of auxiliary functions, which factorize in a given way. Furthermore, the factorization dictates the structure of the receiver. This factorization provides an approximation, where the original joint posterior distribution is approximated by an auxiliary probability density function, that leads to a simpler solution for the marginalization. It can be shown that the EM algorithm and the space-alternating generalized EM (SAGE) algorithm [FH94] can be seen as special cases of the VEM/VBEM (variational EM and variational Bayes EM) algorithms. The EM/SAGE-frameworks represents formal optimization frameworks [NP07].

For a multi-user CDMA system, an extension of the VBEM algorithm, named the Divergence Minimization (DM) algorithm, was presented in [HLR+08]. In the DM framework, the Kullback-Leibler divergence is minimized at each iteration. APPs of the code-symbols are passed to the single-user APP decoder, instead of extrinsic probabilities. Because the DM framework minimizes the KL divergence at each iteration, the algorithm is guaranteed to converge. This result is in contrast to receiver structures utilizing soft symbols without a formalized framework. Another advantage of divergence minimization is that each updating stage produces a probability distribution of a given estimate. This distribution in particular represents the uncertainty of this estimate, e.g. in form of a variance (one-dim. parameter) or a covariance matrix (multi-dim. parameter). Thus, the quality of the estimates is also passed in constituents of the resulting iterative scheme. Furthermore the framework allows handling of the residual interference cancellation in the expression for the noise covariance estimation, which is in contrast to e.g. LMMSE filtering.

The variational free energy is similar to the Kullback-Leibler divergence in the sense that it also measures the divergence between an auxiliary function and the true posterior. Frameworks for design of multi-user detectors based on variational free energy have been proposed [LL05], where belief propagation is applied for message passing combined with a variational inference multiuser detector for coded CDMA. The variational free energy minimization (VFEM) framework was formulated in [Nis08] and applied to receiver structures. When the postulated auxiliary distribution belongs to a class of factorable functions, the minimization of the variational free energy becomes particularly tractable. If the Bethe approximation is considered as the variational free energy [Bet35], originally deployed for statistical physics, the zero-gradiant points of the Bethe free energy are the stationary points of the belief propagation algorithm [YFW00, YFW05]. However, because the Bethe free energy is not a convex function, the zero-gradient is not guaranteed to minimize the free energy.

We propose to engage in collaborative research on these topics according to the following time plan:

Short Term To exploit the potential of applying DM/VBEM-based framework to joint channel estimation and multi-user detection, coding and synchronization, especially for OFDM systems, like LTE.

Long term To develop a unified theoretical framework, with a unique appropriate optimization criterion, for iterative information processing (or message-passing processing), which integrates currently proposed iterative schemes derived based on intuitive argument or disparate principles (Bethe approximation, free energy minimization). The attribute "appropriate" means here that optimum information exchange between the different constituents of an iterative system will be identified that maximize the overall convergence and performance behaviours of the iterative solution obtained by using the criterion. Applications of the theoretical framework to concrete problems (see short-term research), will be investigated and the solutions obtained will be compared to current ones.

2.4 Receiver Design and Analysis using Graphical Models

2.4.1 Introduction

Graphical models like Bayesian networks, Markov random fields, junction graphs, and factor graphs have been used in a variety of different fields like coding, signal processing, machine learning, statistics and many others. In this section we present factor graphs, which are bipartite graphs. They allow to visualize a factorization of a complicated function of many variables into a product of simple functions, each of which depends on a subset of the variables.

Closely connected to factor graphs is a generic message-passing algorithm, called the sum-product algorithm, which operates on the factor graph and computes various marginal functions associated with the global function. The sum-product algorithm efficiently computes marginal functions by i) exploiting the way in which the global function factors ii) uses the distributive law to simplify the summations, and iii) reuses intermediate values by using partial sums. In [AM00] a so called generalized distributive law is introduced, which extends the sum-product algorithm to sum-min and other algorithms.

If the factor graph is cycle free, the sum product algorithm terminates after a finite number of steps and the computed marginal functions are exact. For example, the factor graph describing trellis-like structures is cycle-free and in [KFL01] the sum-prodcut algorithm is applied to such a factor graph, thereby (re)obtaining the BCJR algorithm [BCJR74]. A factor graph modeling a system of state space equations is also cycle-free and the application of the sum-product algorithm leads to the Kalman filter [KFL01].

In case of factor graphs with cycles, the sum-product algorithm no longer terminates after a finite number of steps, but becomes an iterative algorithm, which requires suitable scheduling of the exchanged messages. Furthermore, the computed marginal functions are only approximations [KFL01, AHM98].

2.4.2 Coding

In coding theory the factor graph is equivalent to the Tanner graph [Wib96] of the code. LDPC codes are defined in terms of a bipartite graph, where the code bits correspond to variable nodes, and the parity checks correspond to check nodes. Application of the sum-product algorithm to this (cyclic) factor graph yields an iterative decoding algorithm, which is also known as belief propagation: Messages are exchanged between variable and check nodes, and although the computed marginal functions are not exact, the performance of LDPC codes comes very close to capacity [RU01a].

LDPC codes have been studied in great depth [RU01a, RSU01, RU01b, CRU01]. Their degree distribution is analyzed by the so called density evolution method (among other methods), where the form of the messages is tracked as the iterations progress. A wealth of results about fixed points of belief propagation, concentration results and others exist [FKF$^+$].

2.4.3 Communications

In [WS01, CG05] receivers are designed by means of the factor graph approach. The optimal receiver in terms of bit-error probability is the MAP receiver, where the bit decision is designed to maximize the posterior probability mass function (pmf) of the bit given the received sequence. This pmf is expressed as marginalization of a product of functions, describing constraints induced by the channel code, the symbol mapping and by the channel probability density function (pdf) obtained from the system model. The factor graph corresponding to this factorization can then be used to solve the marginalization by means of the sum-product algorithm. For parts of the receiver, like the decoder of the channel code or the symbol demapper, well-known algorithms like belief propagation for decoding are reobtained, while e.g. the exchange of extrinsic information between different sub blocks follows naturally from the sum-product algorithm. In [WS01] the passing of messages of complicated form is avoided by restricting the messages to belong to some family of 'canonical' distributions, which can be parametrized. Instead of tracking these distributions, only their parameters are tracked, leading to reduced requirements in exchanging, updating and saving messages.

In [BC02b] factor graphs are applied to the problem of multiuser detection in a CDMA system. Since the factor graphs have cycles, different schedulding strategies are possible, yielding to different types of receivers. By approximating messages low-complexity detectors can be obtained. Finally, density evolution is employed for performance assesment and several concentration results about the underlying factor graphs are obtained.

In [NMH08] the factor graph framework is applied to pilot-assisted interleave-division multiple access (IDMA) schemes. IDMA systems are multiple access schemes, where user separation is achieved by use of user-specifiec interleavers and low-rate channel coding [PLWL06]. From the system model, the factor graph is obtained and then the sum-product algorithm is used to develop a receiver that performs joint data detection and channel estimation. One of the keypoints are suitable approximations to the messages passed between the nodes of the factor graph to yield an implementation with a complexity that scales linearly with the number of users.

2.4.4 Intended collaborative research

Short-term
Extend the algorithms presented in [NMH08] to more general channel models (OFDM, AR,...) and to MIMO systems. Enhance spectral efficiency by employing higher-order modulation.

Long-term
To understand the behaviour of message-passing algorithms on factor graphs with cycles, derive guidelines for the approximations of messages passed between the nodes and gain insight on how the scheduling of the message passing affects performance. Because of the inherent iterative nature of message passing algorithms, we see close connections between graphical models and information geometry.

2.5 Distributed turbo-processing for relay networks

Error-correcting coding for networks is, as information theory shows, intrinsically different from coding for point-to-point communication. In the case of relay networks, the best known achievable rates clearly require distributed coding and processing techniques.

The objective of this task is to provide additional insight on how to bring the performance of turbo-coding and processing to wireless relay networks.

We consider a simple relay network where a source node transmits information wirelessly to a destination node, with the help of a relay node.

Practical relay techniques for such wireless networks can be classified in three categories named after the operation carried out by the relay : amplify-and-forward (AF), decode-and-forward (DF) and compress-and-forward (CF) [CEG79, KGG05, LTW04]. Choosing between these strategies is dependent both on channel conditions and complexity requirements for the devices in the network.

The DF strategy attracted a lot of interest, because of its optimality when the relay node is close enough to the source node but also thanks to its simplicity. Relays using this strategy decode the signal received from the source, re-encode it and forward it to the destination.

The design of the codes used by the source and by the relay is a central issue in the design of practical DF strategies. Current research directions are presented in the following.

A first technique proposed in [HN06], for the case where the source and the relay use orthogonal signaling to the destination, makes use of rate-compatible punctured convolutional codes [Hag98]. The source transmits one set of parity bits and the relay, after reception and decoding of these, transmits a complementary set. The destination then makes use of both sets to decode the signal. This technique was later optimized for transmission on block fading channels, by choosing the code so as to reduce the probability of outage [SE04].

A slight modification of this technique gives rise to so-called distributed turbo-coding, which enables a significant performance gain [JHHN04, VZ03]. The additional step is letting the relay interleave its decoded signal before re-encoding. By doing so the two sets of parity bits received by the destination make up a parallel turbo-code, that require an iterative receiver at the destination but lower the bit and block error rates substantially. The complexity at the relay remains low, whereas the use of a turbo-code directly from the source would require an iterative receiver at the relay too.

When the source and the relay do not use orthogonal signaling, a turbo-coded strategy using the idea of block-Markov coding of [CEG79] was shown to achieve performance close to the channel capacity [ZD05]. The source and the relay use the same channel at the same time, thus the destination receives a superposition of the source and relay codewords. Since the relay operates in a causal fashion, a codeword sent by the source in block k is forwarded by the relay in block $k+1$; superposition of these codewords enforces the destination to perform an heavy joint decoding of all the blocks altogether.

Coding techniques based on LDPC codes have essentially been developed with a focus on optimizing the code for known channel parameters. For orthogonal signaling schemes, *bilayer* codes have been developed in [RY06a]; whereas other codes were designed for half-duplex relay channels, where the source and the relay do not use orthogonal signaling but where the relay transmits half of the time only [CBSA07, LKY$^+$07]. These codes are optimized for given instantaneous channel conditions, so the extension to fading channels is still open.

All the preceding coding techniques require that the relay decodes the signal of the source perfectly. However, releasing this constraint can bring an additional performance gain. Two coding techniques were designed to take advantage of this : the first is based on *soft-encoding* at the relay [SV05, LVWD06] while the other addresses the problem at the receiver side [SLV08].

In the soft-encoding strategy, the relay decodes the signal but does not make hard decisions; instead it uses the soft values on the decoded bits to produce soft values on the re-encoded bits. These soft values are sent to the destination in an analog fashion; the sign of each symbol represents thus the hard estimate of the bit and its amplitude is proportional to its reliability.

The technique [SLV08] that also releases the constraint of perfect decoding at the relay was presented more recently. In that case the relay makes hard decisions, which might contain decoding errors, then re-encodes and forwards the signal to the destination. The receiver at the destination has to be modified to take into account the uncertainty at the relay.

A general framework for these last two techniques, where the relay deals with a signal it is not able to decode, cannot be developed in a strict decode-and-forward setting; it should rather be considered more generally as an integration of both decode-and-forward and compress-and-forward strategies.

In order to understand and further develop the advantages of this coding approach, this tasks aims at defining a theoretical framework and developing capacity-approaching schemes for it.

3 CODE DESIGN

3.1 Introduction

Under iterative decoding, it is possible to communicate arbitrarily close to the capacity of binary-input output-symmetric memoryless channels for sufficiently large information block lengths by a proper design of the coding scheme using powerful tools such as EXIT charts [tB00a] or density evolution [RSU01]. These tools have been then successfully applied or extended to the design of coding schemes for a wide class of other channels, such as, for example, inter-symbol interference channels [KMM03] or non-binary input channels [BB06], etc... In parallel, finite length design and optimization have been also addressed to develop efficient coding schemes having both good waterfall performance as well as low error-floor in the high SNR regime to ensure good performance for practical standardized communication or recording systems. However, performance in waterfall or in the error floor regions are not the only performance requirements for modern communication systems. Indeed, for example, due to high data rate applications such as video communications and to the heterogeneity of the required services, coding systems have to handle bandwidth efficiency and have to support rate adaptivity for practical transmitter implementation purposes.

In the task “code design" of the WP4, NEWCOM++ participants aim at addressing particular open research issues for different topics related to the code design. The different interests are related to the study of some classes of codes, binary or non binary, as well as the design of bandwidth efficient and rate adaptive coding schemes.

In the following, we review some research topics of interest for future collaborations within the WP4 of NEWCOM++. For each research topic, we intend to give a brief overview of the state-of-the-art in the research area and we try to present some open research issues that can be addressed within NEWCOM++. To this end, we present first a new binary coding scheme based on polarization codes [Arı07], that are a class of capacity achieving codes that are explicitly defined by a construction rule and have low-complexity encoding and decoding algorithms. Then, we present research topics related to iteratively decodable non-binary coding schemes. We finally end with research issues for bandwidth efficient coding schemes such as bit-interleaved coded-modulations.

3.2 New binary coding schemes

In this section, we present a new binary coding scheme called *polar coding*. Polar codes are a class of codes that achieve the symmetric capacity $I(W)$ of any given binary-input discrete memoryless channel (B-DMC) W [Arı07]. The symmetric capacity $I(W)$ is the highest rate achievable subject to using the input letters of the channel equiprobably.

Throughout the following discussion, let $W : \mathscr{X} \to \mathscr{Y}$ denote an arbitrary B-DMC with input alphabet $\mathscr{X} = \{0,1\}$, output alphabet $\mathscr{Y}$, and transition probabilities $[W(y|x)]$, $x \in \mathscr{X}$, $y \in \mathscr{Y}$. Let W^N denote the DMC consisting of N independent copies of W. An important subclass of B-DMCs that we will focus on is the class of symmetric channels. A B-DMC W will be called *symmetric* if there exists a permutation π of its output alphabet $\mathscr{Y}$ such that $W(y|1) = W(\pi(y)|1)$ for all $y \in \mathscr{Y}$. Examples are the binary erasure channel (BEC) and the (binary symmetric channel) BSC. For symmetric channels, $I(W)$ equals the channel capacity $C(W)$.

An important channel parameter in polar coding is

$$Z(W) = \sum_{y \in \mathscr{Y}} \sqrt{W(y|0)W(y|1)} \tag{1}$$

This parameter is used to gauge the reliability of W. Note that $Z(W) \in [0,1]$ with $Z(W) = 0$ corresponding to a perfect channel and $Z(W) = 1$ to a useless channel.

We will use the notation a_1^N to denote an arbitrary vector $(a_1,\dots,a_N)$, and a_i^j to denote the sub-vector $(a_i,\dots,a_j)$. The operation $\oplus$ will denote addition in $GF(2)$.

3.2.1 Channel polarization

Polar coding rests on a phenomenon called *channel polarization*. This refers to the fact that it is possible to synthesize, out of N independent copies of a given B-DMC W, a different set of N binary-input channels such that the capacities of the latter set, except for a negligible fraction of them, are either near 1 or near 0. This second set of N channels are well-conditioned for channel coding: one need only send data at full rate through channels with capacity near 1 and at 0 rate through the others.

Channel polarization consists of channel combining and channel splitting. The channel combining part is a recursive method that transforms the underlying raw channels W^N into a synthetic channel $W_N : \mathscr{X}^N \rightarrow \mathscr{Y}^N$. The recursion begins by combining two copies of W as shown in Fig. 1 to obtain the channel $W_2 : \mathscr{X}^2 \rightarrow \mathscr{Y}^2$ with transition probabilities

$$W_2(y_1 y_2 | u_1 u_2) = W(y_1 | u_1 \oplus u_2) W(y_2 | u_2) \tag{2}$$

The next level of recursion is shown in Fig. 2 where two independent copies of W_2 are combined to create

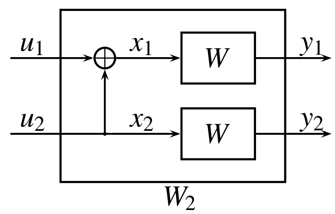

Figure 1: The channel W_2.

the channel $W_4 : \mathscr{X}^4 \rightarrow \mathscr{Y}^4$ with transition probabilities $W_4(y_1^4 | u_1^4) = W_2(y_1^2 | u_1 \oplus u_2, u_3 \oplus u_4) W_2(y_3^4 | u_2, u_4)$. In Fig. 2, Π_4 is the permutation $a_1^4 \mapsto (a_1, a_3, a_2, a_4)$.

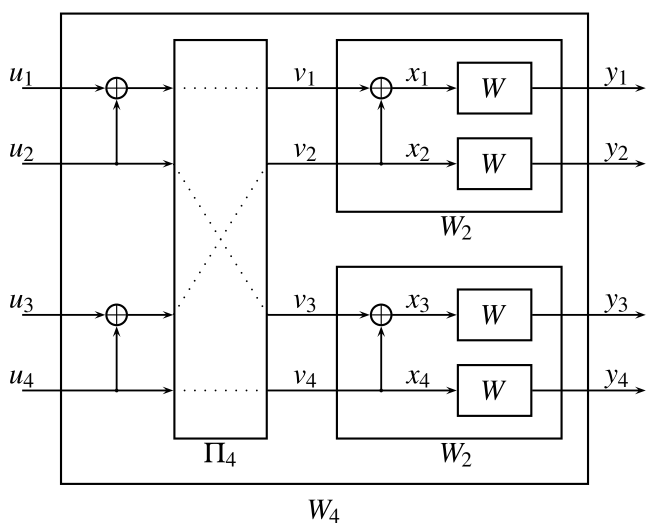

Figure 2: The channel W_4 and its relation to W_2 and W.

The general form of the recursion is shown in Fig. 3 where two independent copies of W_N are combined to create $W_{2N} : \mathscr{X}^{2N} \rightarrow \mathscr{Y}^{2N}$. The input vector u_1^{2N} to W_{2N} is first transformed to a vector s_1^{2N} such that $s_{2i-1} = u_{2i-1} \oplus u_{2i}$ and $s_{2i} = u_{2i}$ for $1 \leq i \leq N$. The permutation Π_{2N} acts on s_1^{2N} to generate v_1^{2N} such that $v_i = s_{2i-1}$ and $v_{N+i} = s_{2i}$ for $1 \leq i \leq N$.

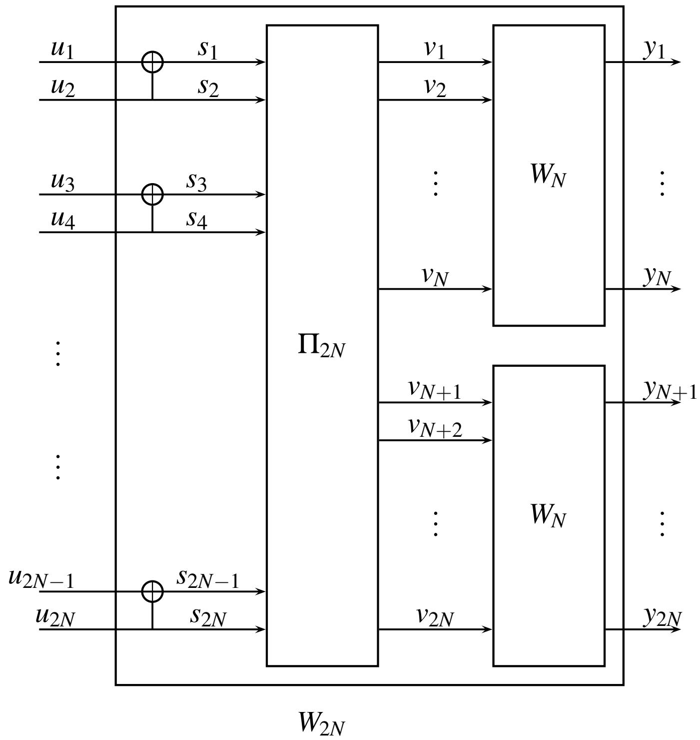

Figure 3: The relation between the channels W_{2N} and W_N.

The channel splitting phase splits the combined channel W_N back into a set of N binary-input channels $W_N^{(i)} : \mathscr{X} \to \mathscr{Y}^N \times \mathscr{X}^{i-1}$, $1 \le i \le N$, defined by the transition probabilities

$$W_N^{(i)}(y_1^N, u_1^{i-1}|u_i) = \sum_{u_{i+1}^N \in \mathscr{X}^{N-i}} \frac{1}{2^{N-1}} W_N(y_1^N|u_1^N). \tag{3}$$

The channels $W_N^{(i)}$ exhibit a polarization effect in the sense that the fraction of indices i for which the symmetric capacity $I(W_N^{(i)})$ is inside the interval $(\delta, 1-\delta)$ goes to zero as N goes to infinity for any fixed $\delta > 0$. This is illustrated in Fig. 4 for W a binary erasure channel (BEC) with erasure probability $\varepsilon = 0.5$.

3.2.2 Polar coding

The basic idea of polar coding is to create a coding system where one can access each synthesized channel $W_N^{(i)}$ individually and send data only through the subset of them for which $I(W_N^{(i)})$ is near 1.

Let G_N denote the *generator matrix* for the channel combining operation of order N, i.e., the matrix that describes the transformation of the input vector u_1^N to W_N to the vector x_1^N that actually gets transmitted over W^N, the underlying product-form channel. For example,

$$G_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \qquad G_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

A general formula for G_N can be found in [Arı07].

For any set $\mathscr{A} \subset \{1, \ldots, N\}$, let $G_N(\mathscr{A})$ denote the submatrix of G_N consisting of rows with indices in $\mathscr{A}$. Let $|\mathscr{A}|$ denote the cardinality of $\mathscr{A}$, and $\mathscr{A}^c$ its complement in $\{1, \ldots, N\}$. For any such $\mathscr{A}$ and

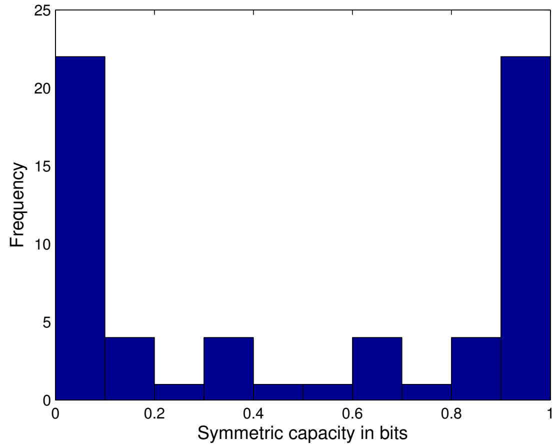

Figure 4: Histogram of $I(W_N^{(i)})$ for $N = 2^6$ and W a BEC with $\varepsilon = 0.5$.

any binary vector $f_1^{|\mathscr{A}^c|}$ of length $N - |\mathscr{A}|$, the pair $(\mathscr{A}, f_1^{|\mathscr{A}^c|})$ will parametrize a block code that maps a data vector $d_1^{|\mathscr{A}|}$ to a codeword vector x_1^N by the transformation

$$x_1^N = d_1^{|\mathscr{A}|} G_N(\mathscr{A}) \oplus f_1^{|\mathscr{A}^c|} G_N(\mathscr{A}^c) \tag{4}$$

We will refer to $\mathscr{A}$ as the *information set* and to $f_1^{|\mathscr{A}^c|}$ as the *frozen vector.*

Polar code construction rule. To construct an (N,K) polar code, select the information set $\mathscr{A} \subset \{1,\ldots,N\}$ so that (i) $|\mathscr{A}| = K$ and (ii) for each $i \in \mathscr{A}$ the value $Z(W_N^{(i)})$ is among the smallest K values in the set $\{Z(W_N^{(j)}) : j = 1,\ldots,N\}$. Select the frozen vector $f_1^{|\mathscr{A}^c|}$ as any vector. $\Box$

This rule pays special attention to the selection of the information set, but does not discriminate any choice for the frozen vector. On symmetric channels, polar code performance does not depend on the choice of the frozen vector, and it may be taken as the all-zero vector.

The rule uses $Z(W_N^{(j)})$ for selecting the information set, instead of the more natural $I(W_N^{(j)})$. This is because the Z-parameter has the advantage of being computationally simpler and it also readily provides a bound on decoding error probability of the constructed code.

It is shown in [Arı07] that, if W is a symmetric B-DMC, polar codes achieve any rate $R < C(W)$ with P_e, defined as the probability of decoding error under successive cancellation decoding, bounded in terms of the code block-length N as

$$P_e \leq \mathscr{O}(N^{-1/4}). \tag{5}$$

This performance is achieved by encoding and decoding algorithms of complexity $\mathscr{O}(N \log N)$. Thus, polar codes are a class of codes that (i) are explicitly defined by a construction rule, (ii) *provably* achieve channel capacity on symmetric B-DMCs, and (iii) have low-complexity encoding and decoding algorithms.

The use of a successive cancellation decoder for polar coding is for analytical tractability only. Polar codes are suitable for decoding by more powerful iterative message-passing algorithms without increasing the decoding complexity beyond $\mathscr{O}(N \log N)$. The complexity-performance issue for polar codes will be discussed more fully in a later section of this document.

3.2.3 Research problems on polar coding

Some open research problems on polar coding can be stated as follows.

- The bound (5) on error probability of polar coding is known to be loose [Arı07]. Obtaining better bounds is an important problem.
- It is conjectured that channel polarization is a commonplace phenomenon, which is almost impossible to avoid as long as channels are combined with a sufficient density and mix of connections, whether chosen recursively or at random. The study of channel polarization in such generality is an interesting theoretical problem.
- Polar coding for non-binary channels is a natural direction to extend the present results.
- A thorough performance evaluation of polar coding using iterative decoders has not yet been done.
- Combining polar coding with turbo coding/decoding techniques is an idea that needs to be explored.

3.3 Non binary LDPC codes

3.3.1 Definition

Binary LDPC codes can be easily generalized to non binary LDPC codes (NB-LDPC) [DM98]. The parity-check equations are written using symbols in the Galois field of order q, denoted GF(q), where $q=2$ is the particular binary case. The parity-check matrix defining the code has only a few nonzero coordinates which belong to GF(q), and a single parity equation involving d_c codeword symbols has then the form:

$$\sum_{i=1}^{d_c} h_{j_i}.c_i = 0 \tag{6}$$

where $\{h_{j_i}\}$ are the nonzero values of the $j-th$ row of H.

3.3.2 On the benefit from NB-LDPC codes.

In terms of algebraic properties and error correcting capabilities, there is not much difference between non binary and binary codes, and the question whether it is useful to consider NB-LDPC codes is a valid question. If we leave aside the better behavior of non binary codes for correcting bursts of errors, the principal reason of using NB-LDPC codes lies in the fact that the practical decoder is sub-optimal, which is the case of the belief propagation (BP) decoder, or its reduced complexity derivatives. Resulting from this sub-optimality, the main drawbacks of Belief Propagation decoding of binary LDPC codes come from the dependence of the messages in the Tanner graph representation of the code. This dependence comes from very specific topological structures in the Tanner graph of the code, e.g. cycles, stopping or trapping sets. The bad behavior of the BP decoder on these topological structures is even enhanced if the log-likelihood ratio (LLR) messages that initialize the decoder are already correlated by the channel.

In the following two examples, the use of NB-LDPC codes helps to bypass correlation effects of the messages:

- *short/moderate length codes:*
  The Tanner graph of the NB-LDPC code is much sparser than the one of a binary code with same parameters. This has been pointed out by several authors [DM98, MD99, HE04, PFD06a]. As a consequence, the higher girth of NB-LDPC graphs helps to avoid the short cycles and also mitigates the effect of stopping or trapping sets, making the BP decoder closer to maximum likelihood decoding (MLD). Actually, when q is larger than $q \geq 2^6$, best error rate results on binary input channels are obtained with the lowest possible variable node degree, that is $d_v=2$. Those codes have been named *cycle-codes* in the literature, or also *ultra-sparse* LDPC codes [DM98, MD99]. For example, the girth of a binary irregular LDPC code with length $N=848$ bits and rate $R=0.5$ is at most $g_b=6$ for the good degree distributions, while the girth of a NB-LDPC code with same parameters is $g_{nb}=14$ when a good graph construction is used [HE04].

- *high order modulation (M-QAM):*
  For binary LDPC coded modulations, the output of the Bayesian maximum *a posteriori* demapper gives correlated probability weights, which means that the initialization of the BP decoder will experience correlated messages even without any short cycle. Of course, there are several ways of fighting this effect, by using an interleaver (BICM-LDPC), or using multilevel coding. However, if the LDPC code is build in a field with order equal or higher than the modulation order, the non binary LDPC decoder is initialized with uncorrelated vector messages, which helps the BP decoder to be closer to MLD. This way, the code operates in the modulation signal set, like in the Trellis-coded modulations. The application of NB-LDPC codes to high order modulations has been proved very efficient both with analytical approaches and in simulations [SF02, DCG04, BB06].

Therefore, if one accepts to increase the decoding complexity of the receiver, it is possible to expect a significant performance gain in the above described cases.

3.3.3 Non-binary decoders: a brief review.

The performance improvement of NB-LDPC codes is achieved at the expense of increased decoding complexity. As in all practical coding schemes, an important feature is the complexity/performance trade off, it is very important to try to reduce the decoding complexity of NB-LDPC codes, especially for high order fields GF(q) with $q \geq 64$.

The base decoder of non binary LDPC codes is the BP decoder over the factor graph representation of the code. The main difference with the binary BP decoder is that for GF(q) LDPC codes, the messages from variable nodes to check nodes and from check nodes to variable nodes are defined by q probability weights, or $q-1$ log-density-ratios. As a result, the complexity of non binary LDPC decoders scales as $\mathscr{O}(q^2)$ per check node [WSM04b], which prohibits the use of codes build in high order fields. Computing the check node in the Fourier-domain reduces the complexity to $\mathscr{O}(q \log q)$ per check node [DM98, BD03], but adapting the Fourier-domain decoder to practical implementation is tedious due to complicated operators like exponentials or real multiplications.

In [SC03], the authors present a log-domain BP decoder combined with a FFT at the check node input. However combining log-values and FFT requires a lot of exponential and logarithm computations, which may not be very practical. To overcome this issue, the authors propose the use of a look-up table (LUT) to perform the required operations. Although simple, this approach is of limited interest for codes over high order fields since the number of LUT accesses grows in $q \log_2(q)$ for a single message. As a result, for fields of high order, unless the LUT has a prohibitively large size, the performance loss induced by the LUT approximation is quite large. In [SC03], the authors present simulation results for LDPC codes over fields up to GF(16), in which case the LUT approach remains manageable.

Recently, sub-optimum decoders based on generalization of the min-sum decoder have been developed [WSM04b, WSM04a, DF05, DF07]. The algorithm that proposes the best complexity/performance trade off is the one in [DF05] for which the complexity scales as $\mathscr{O}(n_m.q)$ with $n_m << q$ and a very small performance degradation compared to the BP decoder. This algorithm is called extended min-sum (EMS) decoder. However the complexity of the EMS decoder is still too large to compete with current binary hardware implementations of LDPC codes. Therefore in [VDV+07] [VDV+08a], the authors propose an efficient implementation of the EMS algorithm, extending the message truncation used in the original EMS. In that implementation, the order of complexity is reduced to $\mathscr{O}(n_m \log_2(n_m))$ and shows to be reasonable enough to compete with binary decoders.

3.3.4 NB-LDPC codes optimization.

- Code profile optimization:

  As in the binary case, it has been shown that the irregularity of the code profile can be optimized with respect to the considered channel to approach the capacity of the channel under asymptotic assumptions (ie. infinite codeword length and infinite number of iterations). To this end, some authors have generalized the methods based on density evolution used in the optimization of binary LDPC codes. However, all convenient optimization methods are based on a Gaussian approximation of the densities of the log-density ratio (LDR) messages carried out along the graph edges, also referred to as EXIT charts for LDPC codes [BB06, LFK03, BT05]. These methods have been applied to both binary input and non-binary inputs channels and then were further extended to the design of concatenated systems implying NB-LDPC codes as in [PC06b] for multiple-inputs multiple-outputs (MIMO) systems. Even if exact density evolution is quite difficult to handle in the non binary case for general channels, exact density evolution has been derived for the binary erasure channel (BEC) in [RU05] and the stability condition for non binary code ensembles has been stated in the general case for any binary input memoryless symmetric channels. Evaluating the thresholds for different codes ensembles, the authors show in particular that the BEC threshold is not a monotonic function of the field order for a given code profile.

Note that for the binary input additive white Gaussian noise channel , the obtained profiles have in general a large proportion of degree 2 variable nodes and this proportion tends to increase as q increases, confirming the fact that for large field orders the optimization process tends to have a graph as sparse as possible.

- Finite length and structured code design:

However, the obtained irregularity profiles only apply to very long codeword lengths and consider that non-binary values are randomly and uniformly distributed. As in the binary case, finite length design has to be properly addressed in order to ensure good performance in both the waterfall and the error floor region. To this end, the graph structure and the choice of the non binary values have be considered jointly in the finite length context.

For finite length codes, the optimization problem has been first solved in a disjoint manner. First, the positions of the nonzero entries of the parity-check matrix H associated with the non binary code are optimized in order to have good girth properties and to minimize the impact of cycles, when using the BP algorithm on the associated Tanner graph. This can be efficiently done using the progressive edge growth (PEG) algorithm [HEA05]. Then, the nonzero entries can be selected randomly from a uniform distribution among nonzero elements of GF(q). However, in their seminal works [DM98][Mac], the authors show that rows with d_c coefficients can be carefully selected in order to improve the waterfall. Based on a Monte Carlo method, the proposed method starts with a choice of channel model, after which they search for the d_c-tuples that maximize the marginal entropy of the syndrome for a given number of iterations, where d_c is the number of non zero values in the row. Row optimization has been also considered in [PFD06b] using the binary images of the rows and considering the maximization of the minimum distance as an optimization criterion. Interestingly, both methods lead to exactly the same results, and some properties of the optimized rows have allowed [PFD06b] to propose an accelerated search procedure to optimize rows for field up to GF(256).

When considering cycles codes (ie. regular $(2, d_c)$ non binary LDPC codes), it has been shown independently in [PFD06b] and [KGP06] that the performance in the error floor region can be improved by properly selecting the non-binary coefficients to avoid low-weight codewords that can be induced by short length cycles in the graph. Avoiding low-weight codewords induced by a cycle is finally possible if the square matrix associated with a cycle is full rank. In [KGP06], a PEG-like construction is proposed, where the coefficients are chosen in order to fulfill the full rank condition for a maximum of cycles related to the added edge. In [PFD06b], the optimization is done recursively to ensure that the cycles influence is cancelled for almost all shorter length cycles. It has been then applied to quasi-cyclic graphs in [PC07]. Unfortunately, cancelling cycles is not sufficient since inherent topological structures composed of at least 3 imbricated cycles give you codewords of the overall code, defining a sub-code whose influence can not be “cancelled". Note that these sub-structures are abusively denoted “stopping sets" due to their topological definition.

Finally, [PFD06b] include in their optimization the maximization of the minimum distance over all identified "stopping sets" to ensure a good minimum distance.

Other types of constructions have been considered for the magnetic recording channel as in [SC03], where a PEG construction is considered jointly with the Minimum Space Distance (MSD) maximization for the parity-check matrix. The MSD criterion applied to the parity-check matrix is related to the capacity of correcting erasure bursts and corresponds to the minimum distance between non-zero entries of the parity-check rows.

Good results are also obtained, especially in the error floor region, with structured non binary LDPC codes such as quasi-cyclic non-binary LDPC codes [SZLAG06, LSL$^+$06, ZLT$^+$08b, ZLT$^+$08a] or as in [PC06a] for QAM channels.

3.3.5 *Beyond* classical *NB-LDPC codes.*

Other non-binary schemes have been proposed in the literature. For example in [SD06], the authors propose an extension of non binary LDPC codes named "hybrid non-binary LDPC codes", where variable nodes belong to finite sets of different orders. As a result, this class of codes is not defined in a finite field anymore, but in finite groups. Considering groups indexed by k whose cardinality q_k is a power of 2, the class of hybrid LDPC codes is thus defined on the product group $(\frac{\mathbb{Z}}{2\mathbb{Z}})^{p_{\min}} \times \ldots \times (\frac{\mathbb{Z}}{2\mathbb{Z}})^{p_{\max}}$, where $p_k = \log_2(q_k)$ and $p_{\min}$ (resp. $p_{\max}$) is related to the group with the minimum order (resp. with the maximum order). Asymptotic analysis and optimization is carried out in [SD07]. These codes have shown to have good performance for small to moderate codeword length and seem to be suitable for low-rate code design. More recently, a new class of codes has also been proposed in [VDV$^+$08b] named "split non-binary LDPC codes". These codes consider that the check nodes and the variable nodes are not defined in the same fields, ie. variable nodes have field orders less than the check node field order. These codes exhibit a good trade off performance-complexity, since they have both a good waterfall behavior and a low error floor due to an enhanced minimum distance and they overcome the memory space requirements problem of classical NB-LDPC codes. Non-binary LDPC codes defined over multiple Galois Fields have been also proposed and can be adapted to the profile of a frequency selective channel and show good performance when considering multi-carrier with QAM modulations [BGYW06].

Apart from these extensions of NB-LDPC codes, some extensions of binary Generalized LDPC (GLDPC) codes have been proposed in [WF06, PFC06] , named "doubly-generalized LDPC codes". For these codes, not only row extension is considered using columns of the parity-check matrix of some binary components codes, but also column extension using rows of some generator matrices of binary components codes. Thus, they extend the concept of LDPC codes by allowing some variable and check nodes to be generic linear block codes instead of repetition and single parity-check codes respectively. This allows to overcome the problem of rate loss of GLDPC. These codes can be seen through their super-variable nodes as an instance of non-binary codes, in the sense that the Tanner graph associated with that kind of codes has variable nodes that can carry more than one bit. They exhibit good performance in both the waterfall and the error floor.

3.3.6 *Open research issues.*

Some open research problems on non-binary coding can be stated as follows:

- Finite length design and optimization of irregular NB-LDPC codes: finite length design is well understood for regular NB-LDPC codes, but even if they performs fairly well at finite length, they have a minimum distance that scales with $\log(N)$ [PFD06a]. Moreover, for small field orders (and thus for a reasonable decoding complexity), performance are from far better when considering irregularity. Therefore, finite length design and optimization of irregular NB-LDPC codes appear as interesting issues for having codes with good waterfall and minimum distance for field orders up to $GF(16)$.

- NB-LDPC codes in iterative systems: the benefit of having NB-LDPC codes in an iterative concatenated system has not been widely considered up to now and it should be interesting to explicitly evaluate the real gain of having symbol-wise iterative systems compared with the state-of-the-art iterative receivers (for example in turbo-detection, turbo-equalization or turbo-synchronization as well as joint-source and channel decoding).

3.4 Bandwidth efficient coding schemes

3.4.1 Bit-interleaved coded modulations

Bit-interleaved coded modulation (BICM) was first introduced by Zehavi in [Zeh92], and later analyzed from an information theory point of view in the landmark paper of Caire *et al.* [CTB98]. BICM is a bandwidth efficient scheme which owes its popularity to the fact that the channel encoder and the modulator separated by a bit-level interleaver may be chosen independently allowing for a simple and flexible design [CTB98, Sec. V]. BICM is considered the dominant technique for coded modulation in fading channels [Gol05], and it only introduces a small penalty when compared to the coded modulation capacity [CTB98, Ung82]. BICM schemes have been proposed in IEEE wireless standards such as IEEE 802.11a/g [IEE99] (wireless local area network) and IEEE 802.16 [KR02] (broadband wireless access). Other examples include the low complexity receivers proposed by the IEEE for the multiband orthogonal frequency-division multiplexing (OFDM) ultra wide-band transceivers [B$^+$04], and the wireless world initiative new radio consortium [L$^+$07]. An additional advantage of BICM compared to other schemes such as trellis coded modulation is that due to the flexibility imposed by the bit-level interleaver, the implementation of adaptive modulation and coding schemes is straightforward [OLGW01].

In order to increase the spectral efficiency, BICM can be combined with quadrature amplitude modulation (QAM) or phase shift keying (PSK). However, evaluation of performance of BICM for high-order modulations was usually limited by the lack of formal description of the metrics used in the decision process. This problem was partially palliated by bounding techniques, e.g., [Zeh92] or the so-called expurgated bounds in [CTB98]. Important contributions addressing this issue have been published recently by Martinez, Guillén i Fàbregas and Caire who presented in [GMC04] a method to approximate the binary-input soft-output BICM channel by a BPSK channel with scaled SNR. Another approach presented by the same authors to tackle this problem is the so-called saddle-point approximation [MGC06], which has also been used in [MGC07] for computing bounds on the BER for BPSK over fading channels. Nowadays, the probability density functions of the metrics used in BICM-PSK and BICM-QAM for fading and non-fading channels are known, cf. [BSA06, SB06, ASFA07, SAF07, SBF06, SAF08]. It is well known however, that when high-order constellations are used, the binary mapping makes the BICM channel not symmetric, i.e., the bit mapping causes unequal error protection (UEP) [Le 02, CTB98]. Consequently, different bits get different levels of protection when transmitted through the channel. The UEP was already observed by Caire *et al.* in [CTB98] where, for simplicity of the resulting analysis, a random scrambler was proposed to make the channel symmetric, and thus overcome this “problem”.

In general, the coded bits can be classified into two groups: systematic and parity bits. When BICM is used with BPSK both systematic and parity bits are transmitted with the same protection level. However, it has been recognized that if an unequal power allocation for these two groups is imposed, some performance improvement can be achieved. This idea was first pointed out for turbo codes (TC-BICM) in [MK97] and later analyzed in [SRTB00, ZW04, DS97, HM96, MK99]. Puncturing can also be seen as another instance of unequal power allocation to the two groups of bits, and it has been analyzed in [KM02, LH00a, AR99] for turbo codes. It is worth mentioning that the literature is somehow contradictory respect to what the optimum puncturing/power allocation strategy is: According to [HM96] and [KM02], the performance of the system for high SNR values can be improved if the systematic bits are more protected. In [SRTB00], [DS97] and [MK99] it is shown that if the energy assigned to the parity bits is increased, the performance of the system can be improved in the error floor region. If the waterfall region is the optimization criterion, according to [LH00a], puncturing systematic bits (protecting more the parity bits) improves the performance. In [SRTB00] it was shown that for short block lengths more power should be allocated to systematic and parity bits in the low and high SNR region respectively. In [CL99] it is claimed that in fact the optimal power allocation depends on the block length and the code rate.

When the mapping inherently causes UEP, instead of seeing this as a "problem" as in [CTB98], it can be take into account when selecting/designing the code and the interleaver. It has been shown in [Le 02, WV05] that in order to improve the performance of TC-BICM, the systematic bits must be assigned to the most protected bit positions. The problem of designing LPDC codes in this context has been analyzed in [SSN04, vDS07, LHWS05]. More specifically, in [SSN04] LDPC codes are designed blending the concepts of multi level coding and BICM, and in [vDS07] a modified decoding algorithm is proposed to take into account the UEP caused by the mapping. In [LHWS05] it is recommended that for LDPC codes and channels with UEP, the best strategy is to protect the systematic bits. In this context, it would be desirable to have a general analysis tool, that could be used to decide whether protect more the systematic or the parity bits.

Following the framework set in [CTB98], random (RN) interleavers are most often applied in BICM. This simplifies the analysis of the resulting system, but leads to sub-optimality already noted in the literature [AS99]. For example, in the original paper of BICM [Zeh92], the application of independent interleavers between each of the encoder's output and the corresponding modulator's input (e.g., using three interleavers for a 2/3-rate encoder, each of them feeding bits to one of the bits' positions in the 8-PSK symbol) was postulated. Similar interleavers have been used for BICM in [HA96, AS99], and also in [LR97] where the idea of BICM with iterative demapping (BICM-ID) was introduced. In [NA05] similar modular (MD) interleavers were proposed in the context of serially concatenated systems, and recently in [SF07], this kind of interleavers were used for BICM-OFDM. When such MD interleavers are used, the performance gains will strongly depend on the bit assignment between the encoder's output and the bit positions in the complex symbol. Interleaver design for TC-BICM has been analyzed in [RY06b] where a greedy design algorithm was proposed, and in [MB06] for LDPC-BICM. Thus, an open problem here is the design of interleavers to connect the channel encoder and the mapper taking into account the UEP and the code's properties.

It is worth to mention that for BICM-ID in the error floor region, the so-called Euclidean distance spectrum (EDS) [Sch07] of the mapping can be used for performance analysis. If the EDS is calculated in a per-position basis, it allows us to analyze the performance of MD interleavers, and consequently, it can be used as a tool to design good mappings. Although good mappings for BICM-ID are well-known in the literature, there are no systematic methods to construct them. Consequently, the construction of good mappings for BICM-ID in the error floor region, and in particular when MD interleavers are used, is still an open problem. In a more general context, it is well-known that the optimum BICM-ID design can be achieved if the code and the mapping are jointly designed. In the literature however, this approach is considered too difficult, and in general the design of mappings is tackled for a given code and vice versa. Moreover, the interleaver design in this case is also crucial, but in most of the cases, it is completely ignored.

3.4.2 *Open research issues.*

Some open problems in BICM with QAM constellations are the following:

- The problem of the assignment between the coded bits and the bit positions in the unequally protected QAM symbols has not been completely solved. In particular, when MD interleavers are used, the simple question of assigning different protection levels to systematic/parity bits, has no clear answer. Moreover, the problem could be tackled from a more general point of view so the methodology could be applied to other analogous problems.

- It is not clear which mappings are the ones which maximize the BICM capacity. Consequently, an open problem is to find the optimum mapping for any constellation size. Based on the results available in the literature, which are available only for small constellation sizes where it is still possible to do a full search, it seems that there is no unique optimum mapping. However, the BRGC is a good candidate since numerical results have shown that this mapping is the optimum for most of the analyzed cases and SNR values of interest.

- Although good mappings for BICM-ID are well-known in the literature, there are no systematic methods to construct them. All the good mappings are found based on some algorithmic search which has obvious limitations when the constellation size increases. When using MD interleavers and based on the EDS, systematic construction methods could be developed.

4 LOW COMPLEXITY AND IMPLEMENTATION

4.1 Introduction

Complexity issues are an important aspect for the success and deployment of iterative algorithms. In contrast to the theoretical analysis, practical implementations have strict constraints like numerical stability, quantization of variables, memory requirements and convergence behavior to mention just a few.

Therefore, the research area of low complexity implementation is a broad field with a variety of proposed methods and techniques. Many of them, like quantization issues and approximations of algorithms, are based on heuristic approaches which lead to sub-optimal solutions in general. This section gives an overview of concepts that are used to reduce the implementation complexity of iterative receivers and puts heuristic approaches on an information theoretic scientific basis.

Section 4.2 deals with the quantization of variables. An analysis of reduced complexity algorithms for iterative receivers is given in Section 4.3 and 4.4. Finally, general concepts and novel algorithms are presented in Section 4.5, 4.6 and 4.7.

4.2 Quantized Receivers

The theory underlying receiver design mostly deals with real infinite-precision quantities that are meant to be passed from one receiver component to another. This is true for "classical" receiver design as well as for "turbo" iterative receiver architectures. In reality, most receiver algorithms are implemented using fixed-point arithmetic, where quantities can only take one of a finite number of values. There is no satisfying theory to motivate the passage from the optimal theoretical real-valued algorithms to their fixed-point finite precision versions. In most communication devices that are implemented, this passage is realized in a heuristic manner, and designed using simulate-and-tune approaches.

We propose to follow two paths to tackle the practically significant design of quantized receiver components: on one hand, we will develop a theoretical foundation for the design of receiver components directly in the quantized domain; on the other hand, we will generalize fixed point algorithms that exist in the literature, e.g. Gallager's binary message-passing algorithms "A" and "B" for LDPC decoding, so that they can be implemented with any finite message set rather than use just binary messages. These two approaches to quantized receiver design are described in the following two sections.

4.2.1 Information Theory of Quantized Receivers

Consider two scenarios for quantized receiver design:

Quantized De-Mapper Complex observations are received from the channel in the form of I and Q components of 8 bits each, generated by the A/D converter. The modulation is 16-QAM. The decoder requires "soft" inputs of 3 bits for each binary code digit. The de-mapper must generate 4 soft inputs of 3 bits each per received observation.

Quantized Decoder A regular (3,6) binary LDPC decoder operates using 4-bit quantized messages and 3-bit quantized channel observations. Each variable node in the decoder receives a 3-bit channel observation and three 4-bit graph messages, and must produce three 4-bit extrinsic messages to return to the graph. Each check node receives six 4-bit messages and must produce six 4-bit extrinsic messages.

The common denominator in these two scenarios is the quantization constraint on the output of the receiver components. The classical approach in both cases is as follows:

1. re-construct the inputs as real-valued messages;
2. apply Bayesian calculation to compute extrinsic real-valued log-likelihood ratios

3. quantize the resulting values in order to satisfy the quantization constraint on the output.

This classical approach has two drawbacks:

- internal calculations are designed in the real domain, and need to be implemented in high precision even when performed on fixed-point processors;
- there is little theory regarding the design of the re-construction and quantization steps for this type of scenario. These steps are usually designed using heuristic rules, and optimized by simulation.

The second drawback in particular suggests that it may be possible to improve performance significantly (i.e., reduce the number of quantization levels required to achieve a certain performance) by studying the theory behind receiver design with quantization constraints.

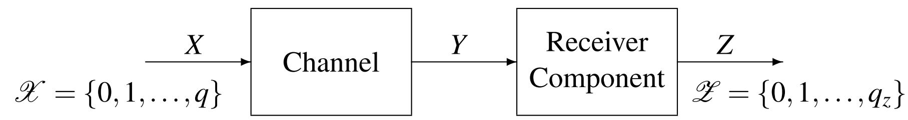

Figure 5: Information-theoretic setup for quantized receiver component design

Most cases of practical interest can be brought down to the setup represented in Figure 5. Let X be a random variable containing the quantity of interest. Let Y be an observation related to X by a known joint distribution P_{XY}. In the quantized de-mapper, Y contains the received I and Q components, while in the LDPC decoder node, it contains the messages obtained from the graph and possibly a channel observation (for the variable node.)

Our aim is to produce a random variable Z such that

- X - Y - Z forms a Markov chain, i.e., Z obtains all its knowledge about X from Y and has no other sources of information about X;
- the alphabet $\mathscr{Z}$ of Z is given and satisfies the quantization constraint;
- Z gives as much information about X as possible.

Note that we do not ask for the alphabets $\mathscr{X}$ and $\mathscr{Z}$ of the random variables X and Z to coincide. Indeed, if they do, then classical Bayesian decision theory provides an excellent framework for designing the receiver component, and the optimal solution is to pick the maximum a-posteriori decision for the reconstructed value of X. When $\mathscr{Z} \neq \mathscr{X}$, Bayesian decision theory still applies in the wide sense [Ber80], but its performance hinges upon the choice of a cost, loss or utility function to measure the success of "reconstruction" (the term is sometimes used in Bayesian theory for this case as well, but loses its meaning, since the reconstruction is defined over a different alphabet from the quantity that needs reconstruction.) Therefore, Bayesian theory fails to provide an absolute reference for our problem: any "optimal" design will be relative to the cost function chosen, and there is no prevalent theory guiding the choice of a cost function.

Since our interest is in communication receiver design, mutual information suggests itself as a gain function to define optimality and serve as a design criterion in our problem. There are two arguments for this:

- The data processing theorem dictates that information will be lost at every processing stage. Therefore, any information lost by a receiver component cannot be recovered by later processing stages. By optimizing a receiver component with respect to mutual information, we can ensure that the information loss is minimized, i.e., that Z conveys as much information about X as possible to later processing stages.

- In an iterative setup, the success of EXIT chart methodology [tB99, AKtB04] demonstrates the power of mutual information as a design criterion: by optimizing a receiver component relative to mutual information, we are in effect maximizing its EXIT transfer function. This implies that we are optimizing the achievable threshold if the component is to be used in an iterative setup.

Once we agree to pick mutual information as the gain function, the problem can be formally stated as follows:

> Given P_{XY}, construct a random variable Z over the given alphabet $\mathscr{Z}$ such that
>
> $$P_{Z|Y} = \arg\max_{P_{Z|Y}} I(X;Z) \tag{7}$$
>
> and X - Y - Z form a Markov chain.

Within NEWCOM++, we propose to pursue the study of this information-theoretic problem along the following lines:

- Investigate numerical methods to solve the maximum in Equation 7. It has been shown in [SLG07] that the maximum is achieved when Z is a function of Y. Therefore, the maximization can be expressed as an integer programming problem.

- Establish theoretical justifications for the use of mutual information as a utility function in this setup. In particular, show how maximizing mutual information for a receiver component contributes to the overall performance of the receiver.

- There are similarities between the problem proposed and the problem giving rise to rate-distortion theory [Yeu02]. The main difference is that quantization in rate distortion theory is a consequence of the distortion or rate constraint, whereas our approach proposes to maximize rate as a consequence of a quantization constraint. The resulting information-theoretic optimization is different, but we expect some parallels between the two problems and between the techniques to solve them.

- Maximizing mutual information can also be used as a criterion to optimize the classical approach described above, where Y is converted to a probability distribution, and Z is the quantized a-posteriori distribution of X. By re-interpreting the problem as a mutual information maximization, design criteria follow for the design of the re-construction and quantization steps. The question arises whether it can be shown that the classical method and the direct design in the quantized domain yield essentially equivalent or distinct solutions when both are optimized by maximizing mutual information.

4.2.2 Decoding of LDPC Codes using Binary Messages

When Gallager introduced LDPC codes [Gal62, Gal63], he also presented message-passing decoding algorithms that exchange only binary messages between the variable and check node processors. These algorithms are called Gallager A and Gallager B and we refer to [Gal62, Gal63] for a description of them. The advantages of these algorithms are the reduced memory requirements and the low complexity implementation, especially of the check node decoder, making them promising candidates for high-speed applications. However, these advantages come with the cost of a significant loss in performance. Binary message-passing algorithms were studied in [AK05] where the authors proved that optimum algorithms must satisfy certain symmetry and isotropy conditions.

For binary message-passing decoders, the extrinsic channel [AKtB04] of the variable and check node decoder is represented as a binary symmetric channel (BSC) with crossover probability ε which we assume to be smaller than or equal to 0.5. Since there is a one-to-one mapping between mutual information

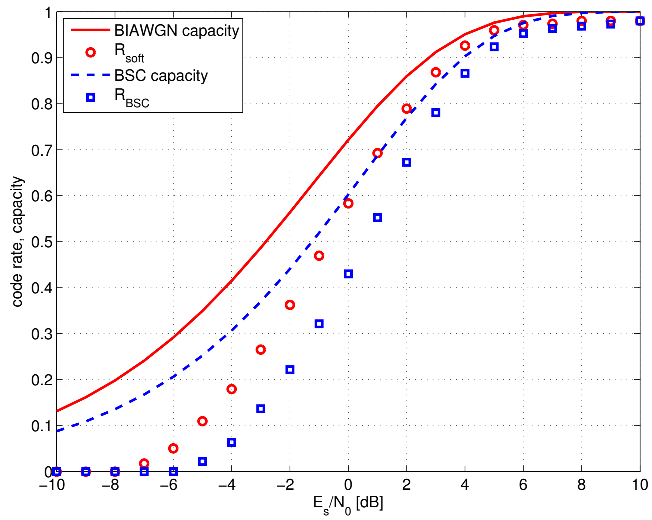

Figure 6: Thresholds of optimized Codes for Soft Channel Information and Hard Decision Channel (BSC).

and crossover probability for the BSC and the model of the a-priori channel is not an approximation, EXIT chart analysis provides exact results of the performance of these decoders.

Since every node is only allowed to send a binary message, there is no choice of the quantization as described in Section 4.2.1. For the analysis and design of such decoding algorithms we will therefore assume that the variable node decoder converts all incoming messages to L-values [HOP96], performs decoding in the L-value domain and applies a hard decision on the result. This general approach assures that the symmetry and isotropy conditions are satisfied and we are able to extend the algorithms for systems where the channel provides more information than hard decisions, while the variable and check node decoder still exchange binary messages only.

This extension of the binary message-passing algorithm was presented in [LPK07] where the authors also showed how these codes can be optimized. In Figure 6, the capacity and the decoding threshold of optimized codes is shown for the case of an AWGN channel that provides either soft information or hard decisions. Especially for high rates, the gap to capacity becomes small which makes binary message-passing algorithms attractive for such applications.

In practice, *true* soft information from the channel is never available due to some quantization of the analog received values. In [LPK07] it is shown that even a quantization using two bits (instead of just one bit for a hard decision) can significantly improve the decoding performance. Using just two bits for the quantization of the channel observations allows the use of simple majority based decision algorithms which allow for a small and fast implementation of the variable node decoder as well.

4.3 Simplification of Component Decoders

This section deals with appropriate algorithms with the aim of reducing the decoding complexity of turbo [BGT93] and LDPC [Gal63] codes. In most of the cases, this is achieved as a trade-off between the observed bit error rate (BER) performance and complexity. That is, the less complex the decoding algorithm is, the greater performance degradation it has when compared to the optimal solution, i.e. Maximum A posteriori Probability (MAP) algorithm [BCJR74] and Sum-Product Algorithm (SPA) [Mac99], respectively. Over the past years several decoding algorithms have been proposed for both turbo and LDPC codes [RVH95, VF00, GG98, CO00, CBD02, CDE+05] with small BER performance degradation, i.e. in the order of a fraction of decibel, compared to the MAP algorithm and SPA, respectively. Apart from this, an advantage of reduced complexity decoding algorithms is in principle that they are composed of much simpler operations, therefore they simplify decoder hardware implementation.

In the subsequent sections, the most popular reduced complexity decoding algorithms dealing with both turbo and LDPC codes are given in brief.

4.3.1 Simplified Max* Operation - Turbo Codes

In this section the optimal MAP algorithm in the logarithmic domain (Log-MAP) for turbo decoding is introduced. Then, the most popular simplifications of it are described in brief.

Let us consider a random sequence of N bits denoted as $\bar{u} = [u_1, u_2, \ldots, u_N]$. This sequence is turbo encoded and then modulated using binary phase shift keying (BPSK) signals taking equally probable values from the alphabet $\{\pm 1\}$. After transmission over an additive white Gaussian noise (AWGN) channel with noise spectral density N_o, the received sequence $\bar{y}$ is passed to a turbo decoder, which estimates the transmitted sequence of bits.

Let us now consider a trellis transition occurring from a state s', at time instant $k-1$, to a state s, at time instant k. The forward recursion (α_k) and the backward recursion (β_k) can be computed recursively as [RVH95], [Vit98]

$$\alpha_k(s) = \ln \sum_{s'} \exp\left\{\alpha_{k-1}(s') + \gamma_k(s',s)\right\} = \max_{s'}{}^* \left\{\alpha_{k-1}(s') + \gamma_k(s',s)\right\} \tag{8}$$

$$\beta_{k-1}(s') = \ln \sum_{s} \exp\left\{\beta_k(s) + \gamma_k(s',s)\right\} = \max_{s}{}^* \left\{\beta_k(s) + \gamma_k(s',s)\right\}. \tag{9}$$

Note that both α_k and β_k have to be initialized first. The decoder soft-output value of the transmitted bit u_k, in terms of the log-likelihood ratio (LLR), can be computed as

$$\begin{aligned} L(\hat{u}_k) &\triangleq \ln \frac{P(u_k = +1|\bar{y})}{P(u_k = -1|\bar{y})} = \ln \frac{\sum\limits_{(s',s):u_k=+1} \exp\left\{\alpha_{k-1}(s') + \gamma_k(s',s) + \beta_k(s)\right\}}{\sum\limits_{(s',s):u_k=-1} \exp\left\{\alpha_{k-1}(s') + \gamma_k(s',s) + \beta_k(s)\right\}} \\ &= \max{}^*_{(s',s):u_k=+1} \left\{\alpha_{k-1}(s') + \gamma_k(s',s) + \beta_k(s)\right\} \\ &\qquad - \max{}^*_{(s',s):u_k=-1} \left\{\alpha_{k-1}(s') + \gamma_k(s',s) + \beta_k(s)\right\}. \end{aligned} \tag{10}$$

In the above equations, γ_k is the branch metric associated with the corresponding trellis transition, taking also into account the *a priori* information in the iterative process. The max* operation in (8)-(10) is defined as [Vit98]

$$\max{}^*(x_1,x_2) \triangleq \ln\{\exp(x_1) + \exp(x_2)\} = \max(x_1,x_2) + \ln\{1 + \exp(-|x_1 - x_2|)\}. \tag{11}$$

It is the increased computational complexity of the non-linear logarithmic function

$$\ln\{1 + \exp(-|x_1 - x_2|)\}$$

that makes the Log-MAP algorithm impractical for hardware implementation. Below, the most popular algorithms used to reduce the complexity of the Log-MAP algorithm are listed.

1. `LUT Log-MAP`. For the look-up-table (LUT) Log-MAP algorithm, the non-linear logarithmic function is implemented with LUT consisting of eight values. The range of the absolute difference between x_1 and x_2 is typically $0 \leq |x_1 - x_2| \leq 5$ [RVH95]. Therefore, (11) becomes

$$\max{}^*(x_1,x_2) \approx \max(x_1,x_2) + \text{LUT}. \tag{12}$$

2. `Max-Log-MAP`. If LUT is omitted, then the LUT Log-MAP simplifies to the Max-Log-MAP algorithm [RVH95]. Therefore, (11) becomes,

$$\max{}^*(x_1,x_2) \approx \max(x_1,x_2). \tag{13}$$

3. `Constant Log-MAP`. This simple algorithm consists of smaller LUT with two values, instead of the more assumed eight values [GG98]. Therefore, (11) becomes

$$\max{}^*(x_1,x_2) \approx \max(x_1,x_2) + c \tag{14}$$

where

$$c = \begin{cases} 3/8, & \text{if } |x_1 - x_2| < 2 \\ 0, & \text{otherwise} \end{cases}. \tag{15}$$

4. `Linear Log-MAP`. In this algorithm the correction factor, which is added after the max operation, is linearly dependent on the absolute difference between x_1 and x_2 [CO00]. Therefore, (11) becomes

$$\max{}^*(x_1,x_2) \approx \max(x_1,x_2) + \max\{(4\ln 2 - |x_1 - x_2|)/4, 0\}. \tag{16}$$

5. `Average Log-MAP`. This algorithm depends on the average value between x_1 and x_2 [CBD02]. Therefore, (11) becomes

$$\max{}^*(x_1,x_2) \approx \begin{cases} \max(x_1,x_2), & \text{if } |x_1 - x_2| > 2\ln 2 \\ (x_1 + x_2 + 2\ln 2)/2, & \text{otherwise} \end{cases}. \tag{17}$$

The above algorithms are sub-optimal, in the sense that the $\max^*$ operation is approximated by a simpler expression. As a consequence, a performance degradation is expected when compared to the optimal Log-MAP algorithm. An exception is LUT Log-MAP algorithm, as performance evaluation results have indicated that it has almost the same performance compared to the Log-MAP algorithm. Due to the sub-optimality of the decoding algorithms, a scaling factor is used in most of the cases to improve performance. That is, the extrinsic information extracted from the component decoders at each iteration is multiplied with a constant value that is less than one [VF00].

4.3.1.1 Proposed Research Work

So far, reduced complexity decoding algorithms have been obtained by approximating the non-linear logarithmic function $\ln\{1 + \exp(-|x_1 - x_2|)\}$ of (11) by different methods. Instead, we aim at approximating the $\max^*$ operator from the same equation directly by formulating a mathematical problem in an appropriate way. It is expected that novel reduced complexity decoding algorithms will be obtained with near optimal and sub-optimal BER performance.

4.3.2 Sum-Product Algorithm Simplifications - LDPC Codes

In this section optimal check node update rules based on the SPA for LDPC decoding are reviewed. Then, the most popular simplifications of SPA are described in brief.

Let us assume a binary LDPC code with block size (N,K) and sparse parity-check matrix H of size $M \times N$ where $M = N - K$. This code can be represented by a bipartite graph, e.g. *Tanner* graph, with M check-nodes in one class and N bit or variable nodes in the other. A *regular* LDPC code is assumed, denoted by (d_s, d_c), in which every bit node is connected to d_s check-nodes and every check-node is connected to d_c bit nodes.

Let (λ) represent the log-likelihood ratio (LLR) of the message that bit node n sends to check-node m, indicating the probability of bit u_n being zero or one, based on all checks involving n except m, i.e. $\lambda_{n\to m}(u_n) = \ln\{q_{n\to m}(0)/q_{n\to m}(1)\}$. Similarly, let (Λ) represent the LLR of the message that the mth check-node sends to the nth bit node, indicating the probability of bit u_n being zero or one, based on all bits checked by m except n, i.e. $\Lambda_{m\to n}(u_n) = \ln\{r_{m\to n}(0)/r_{m\to n}(1)\}$.

For two statistically independent binary random variables U and V, the *tanh rule* [CDE+05] is defined as

$$L(U \oplus V) \triangleq 2\,\tanh^{-1}\left\{\tanh\left(\frac{L(U)}{2}\right)\tanh\left(\frac{L(V)}{2}\right)\right\}. \tag{18}$$

Based on this rule, the check-node update can be computed from [CDE+05]

$$\Lambda_{m\to n}(u_n) = 2\ \tanh^{-1}\left\{\prod_{n'\in N(m)\backslash n} \tanh\left[\lambda_{n'\to m}(u_{n'})/2\right]\right\} \tag{19}$$

where $N(m)\backslash n$ is the set of bit nodes that participate in the mth parity-check equation, i.e. the position of ones in the mth row of the parity-check matrix H, excluding n.

In another approach by Gallager [Gal63], it can be shown that (19) is simplified to

$$\Lambda_{m\to n}(u_n) = \left\{\prod_{n'\in N(m)\backslash n} \mathrm{sign}\left[\lambda_{n'\to m}(u_{n'})\right]\right\} \times \phi\left\{\sum_{n'\in N(m)\backslash n} \phi\left[\left|\lambda_{n'\to m}(u_{n'})\right|\right]\right\} \tag{20}$$

where

$$\phi(x) = \ln\left(\frac{\exp(x)+1}{\exp(x)-1}\right),\ x>0 \tag{21}$$

Alternatively, the *tanh rule* from (18) can be expressed as [CDE+05]

$$L(U\oplus V) = \ln\left(\frac{1+\exp\{L(U)+L(V)\}}{\exp\{L(U)\}+\exp\{L(V)\}}\right). \tag{22}$$

Using the *Jacobian* logarithm [Vit98] twice, the above equation becomes

$$\begin{aligned} L(U\oplus V) = \mathrm{sign}\{L(U)\}\ \mathrm{sign}\{L(V)\}\ \min(|L(U)|,|L(V)|) \\ +\ln(1+\exp\{-|L(U)+L(V)|\})-\ln(1+\exp\{-|L(U)-L(V)|\}). \end{aligned} \tag{23}$$

It can be shown that $|L(U\oplus V)| \le \phi\{\phi(\min(|L(U)|,|L(V)|)\}$ or $|L(U\oplus V)| \le \min(|L(U)|,|L(V)|)$ as the ϕ function from (21) is monotonically decreasing [CDE+05].

Since the magnitude can be approximated as

$$|L(U\oplus V)| \approx \min(|L(U)|,|L(V)|), \tag{24}$$

the corresponding check-node update from (20) can be approximated as

$$\tilde{\Lambda}_{m\to n}(u_n) = \left\{\prod_{n'\in N(m)\backslash n} \mathrm{sign}\left[\lambda_{n'\to m}(u_{n'})\right]\right\} \times \min_{n'\in N(m)\backslash n} \left|\lambda_{n'\to m}(u_{n'})\right| \tag{25}$$

which is known as the min-sum algorithm. Note that the same expression results from (23), if the two correction terms, i.e. non-linear logarithmic functions, are omitted. It is noted that the magnitude of $\tilde{\Lambda}$ in (25) is always greater than that of Λ in (20). This necessitates a search for further improvements to the updating rule of the min-sum algorithm, so that more accurate soft values are computed.

One straightforward application is to use a scaling factor α that is greater than one [CDE+05]. Therefore, (25) becomes

$$\tilde{\Lambda}_{m\to n}(u_n) = \left\{\prod_{n'\in N(m)\backslash n} \mathrm{sign}\left[\lambda_{n'\to m}(u_{n'})\right]\right\} \times \min_{n'\in N(m)\backslash n} \left|\lambda_{n'\to m}(u_{n'})\right| \times 1/\alpha. \tag{26}$$

This approach is also known as normalized min-sum algorithm [CDE+05].

Another variation of the min-sum algorithm using simple correction is the offset min-sum algorithm [CDE+05]. The resulting check-node update is

$$\tilde{\Lambda}_{m\to n}(u_n) = \left\{\prod_{n'\in N(m)\backslash n} \mathrm{sign}\left[\lambda_{n'\to m}(u_{n'})\right]\right\} \times \max\left\{\min_{n'\in N(m)\backslash n} \left|\lambda_{n'\to m}(u_{n'})\right| - \beta, 0\right\}. \tag{27}$$

where β is positive constant. In this case, the incoming messages λ with magnitude less than β are eliminated from the next iteration of check-node update computation.

The above mentioned algorithms, i.e. min-sum, normalized min-sum and offset min-sum algorithms are sub-optimal, in the sense that the SPA is approximated by a simpler expression. This is similar to the algorithms resulted from the approximation of the $\max^*$ operator for turbo decoding.

Both improvements of the min-sum algorithm (scaling factor and offset) are heuristic methods which turned out to work well in practice. The best values for these correction terms are evaluated by simulations or by density evolution. In [LS04], it is shown that the optimal post-processing function, i.e. the correction of the min-sum algorithm, is a non-linear function that can be derived analytically. An approximation of this non-linear function by a scaling factor or an offset term, leads to the same results that were found by simulations. The analytical expression of the post-processing function not only justifies these correction methods, but it also allows to derive improved correction functions that are useful especially for irregular codes [LS06, Lec07].

4.3.2.1 Proposed Research Work

Novel reduced complexity LDPC decoding algorithms can be obtained by selecting either the sub-optimal normalized min-sum (or offset min-sum) algorithm or the optimal SPA. In this hybrid solution, an appropriate predefined criterion will be used based on the incoming message values λ entering the LDPC decoder. In principle, these decoding algorithms are sub-optimal but with smaller complexity compared to the optimal SPA.

4.4 Low Complexity Equalization

The use of multiple antennas with space-time coding (STC) [TSC98] techniques have become popular since they can provide offer high data rates [Tel99] while achieving low error rates. When the transmission rates are increased by making the symbol periods smaller, the wireless channels are more likely to become frequency-selective (FS) which results in intersymbol interference (ISI) and hence increased error rates. ISI can be canceled with the use of an equalizer [Pro00] which may provide multipath diversity in wireless channels. Next generation communication systems can have STC transmission over frequency selective fading channels [Ton00, VB03].

The major drawback of coded or uncoded transmissions over FS fading channels with single or multiple antennas is that the optimal receivers for them require prohibitively high computational complexity which depends exponentially on the number of ISI taps and the number of transmit antennas. With the motivation of reducing the hardware and software complexity of coded or uncoded single or multiple antenna systems, several simplified approaches have been studied in the literature.

A low-complexity equalization approach is the soft-input soft-output stack equalization algorithm which is based on the sequential decoding algorithm. Two new metrics for a previously designed soft-input soft-output stack detection algorithm [SM01] is proposed in [GD05] to obtain a soft equalizer for single or multiple antenna system over frequency-selective fading channels. Unlike the MAP equalizer, the complexity of the new equalizers does not depend on the memory of the channel, hence the soft-input soft-output stack equalizer are quite desirable for channels with large ISI spans. Furthermore, it is shown that the modified soft-input soft-output stack equalization methods are effective low complexity techniques for coded transmission over MIMO frequency-selective fading channels.

Decision feedback equalization is also well known low complexity equalization technique based on minimizing the mean squared error (MSE), and it can be extended to MIMO systems [ADS00, TAS01]. With the motivation of using DFE in coded systems over ISI channels (or, frequency selective fading channels) as part of a turbo equalization scheme, an iterative receiver with MIMO DFE can be designed [Guc06] in which the MIMO DFE is modified to utilize the soft inputs from the decoder [MSPB01].

Antenna selection technique [Mol03] can be used to decrease the hardware complexity of multiple antenna systems. Since this technique reduces the number of signals to be processed, it also results in reduced computational complexity for STC transmissions over frequency selective fading channels [GDG04]. Although antenna selection is usually applied at the receiver side, recent works show that it is also advantageous to use it at the transmitter and/or receiver [GD07a, GD07b]. Even though the number of RF chains can be reduced considerably, the receive and/or transmit antenna selection based on received power levels does not degrade the achievable diversity order when full rank STCs are employed.

Based on our previous works, we would like to develop iterative receivers with low complexity soft equalization techniques for single or multiple antenna systems with or without antenna selection. Future works may include M-ary transmissions, fast fading channels, iterative estimation with equalization and/or decoding.

4.5 Pragmatic vs. Iterative Decoding

4.5.1 Introduction

A general problem that is encountered in the design of the physical layer of modern transmission systems is the necessity of building *versatile* encoding/ modulation systems, capable of dealing with different channel conditions and at the same time with different user requirements.

A set of channel dependent constraints is, for example, the number of available transmitting and/or receiving antennas, the noise/interference level and its spectrum, and the time and or frequency selectivity of the channel. On the other side, a set of user dependent constraints could be the desired spectral efficiency, the maximum instantaneous or long term available power, the throughput, and the latency.

The joint optimization of the encoding and signaling part of the physical layer for all possible combinations of channel and user constraints requires an unacceptable complexity of the system on one side and a very low flexibility of the receiver on the other side. An efficient solution is then to decouple the signaling and encoding part into two separate subparts with sufficient degrees of freedom on both parts to cope with all possible channel and user conditions.

A typical structure of the encoding/modulation system includes the following parts

1. A strong and versatile *binary encoder* with variable rate and block sizes, to cope with different spectral efficiencies and latencies

2. A channel bit interleaver to maximize the diversity of each codeword

3. A modulation system made up of one OFDM modulator on each available transmitting antennas. The OFDM modulators have tunable power, constellations, and number of carriers. OFDM approach is particularly suited for frequency selective environment and also allows for flexible time/frequency allocation schemes (OFDMA).

A system like this, coupled with a well designed radio resource management, could virtually achieve the theoretical limits with all channel and user constraints.

Note that the presence of space-time encoder in the system, which is an encoding-modulation technique, breaks this decoupling assumption and requires specific design that depends on the channel scenario, i.e. the number of available transmitting or receiving antennas.

4.5.2 Receivers

At the receiver side the detector is that portion of the system that computes the LLR of the encoded bits starting from the received observed signals. Optimal detectors have complexity increasing with the number of waveforms while suboptimal detectors have complexity increasing only with the number of computed LLRs.

The decoder is in charge of computing the LLR of the information bits from the LLR of the coded bits, state of the art decoders nowadays are always based on a turbo-like encoders like LDPC, PCCC or SCCC.

A first receiver design architecture problem is to decide whether to perform iterations between the detecting part of the receiver and the decoding part or to proceed sequentially.

In literature, the first approach is often referred as the “pragmatic” receiver while the second is called iterative receiver.

When it is possible to transmit each encoded bit on an orthogonal resource (dimension) the pragmatic approach is always optimal as a-priori on other bits never affect the LLR of a given bit.

Several papers in literature deal with some important cases of the general problem just stated and for some of them we already know the answer. As an example in [CTB98], where BICM has been introduced, it is shown that pragmatic approach is almost as good as the iterative approach if the signaling part is a simple high cardinality constellation, provided that Gray mapping is used.

4.5.3 EXIT and GEXIT charts

A nice explanation of the trade-off between pragmatic and iterative receivers can be obtained with the EXIT chart analysis [tBK03b] and with more accuracy with the generalized EXIT (GEXIT) charts analysis introduced in [MMRU05].

With this approach detectors and decoders can be described through their EXIT charts, which relate the entropy of the input LLRs of a SISO system with the entropy of the corresponding output. We distinguish then from input-to-input EXIT charts (I-EXIT) and output-to-output EXIT (O-EXIT) charts distinguishing between input and output quantities of the transmitting block.

Since the encoding and signaling blocks are serially concatenated, the relevant EXIT charts in this case are the I-EXIT chart of the detector and the O-EXIT chart of the encoder. The parameter in this case is the capacity of the available channel.

For the pragmatic approach, as no iterations are performed between the two sub-blocks, optimization of the detector coincides with the minimization of the entropy of input bit messages with no a-priori on the other bits. This criterion leads to design techniques for the mapping of bits to waveforms that are extensions to the Gray mapping for the modulation schemes criterion derived in [CTB98]. The EXIT chart of the decoder on the other side should look as a step function and then coincide with that of a powerful capacity achieving binary encoder ([tBH02]).

Other relevant literature which covers several aspects of the problem can be found in [Ton00, SG01, tBH02, SPS02, HtB03, BGB03, NL00, Nar01, LWN02, HSMP03, tBKA04, tBK03b, tBK03a, TH02, SAL03, CBS02, BC02b].

A nice result found in [MMRU05] is that EXIT charts satisfy the area theorem when the input messages are obtained from a BEC channel and, more generally GEXIT charts always satisfy the area theorem for any kind of channel.

For optimal detectors, and applying the area theorem to the I-GEXIT chart, this means that the total area of the GEXIT chart equals the capacity of the channel constrained to the signaling system at hand and this happens independently from the particular mapping of bits to waveforms.

A consequence of this is that the mapping of bits into waveforms is not a crucial task for achieving the capacity for iterative detectors, provided that we have the capability of suitably designing the O-EXIT chart of the decoder to match the I-EXIT chart of the detector. This approach requires the availability of a binary encoder with tunable O-EXIT chart, a topic described in section 4.6.

On the other side, if the bit stream can be divided into a set of classes with associated different capacities the optimal decoder can be well approximated with the successive interference cancellation (SIC) decoder, which starts decoding the most protected bits and successively cancel the interference of most protected bits from the less protected channels before decoding the successive layers.

In summary, here is a set of interesting research areas to be further investigated

1. Compute expressions for the I-GEXIT chart for optimal and suboptimal detectors and investigate the trade-off between complexity and capacity loss, also considering the different impact on pragmatic and iterative receivers.

2. Find extended Gray-like mapping algorithm for mapping bits to a general set of waveforms optimizing the performance of pragmatic receivers.

3. Compare complexity, system and performance trade-offs between pragmatic, iterative and SIC receivers

A realistic system scenario for this study can be frequency and time selective OFDM-MIMO system.

4.6 Irregular Turbo Codes: Complexity Trade-Offs and Flexibility

Irregular LDPC codes, which are LDPC codes with variable right and/or left degree, were introduced by [RSU01] in order to reduce the convergence threshold of the iterative decoder.

The analysis through EXIT or GEXIT chart explains very well the underlying problem and translate into an equivalent "matching" condition between the EXIT charts of the constituent encoders of the concatenated encoder.

Several authors extended the concept of irregularity of LDPC codes to the more structured turbo-like encoders, which naturally maps to regular LDPC encoders. The most successful examples are the irregular repeat accumulate encoders (IRA) [JKM00] and irregular turbo codes [FM99]. The authors were able to achieve the capacity within fractions of dB ([BCV+02]).

In section 4.5 it is explained that binary decoders to be used in an iterative receiver have O-EXIT chart that must be matched to the I-EXIT chart of the corresponding detector. On the other side binary concatenated encoders, designed for achieving the capacity over a binary memoryless channel, have a total O-EXIT chart that is a step function. The use of binary encoders in iterative receivers then requires an additional flexibility that must be taken into account.

Previous work in this area includes for example [TFS07], which used irregular turbo codes in the effort of improving approaching the information capacity limit for severe ISI channels, [Saw04] used irregular turbo codes for optimizing the performance of an 8-PSK pragmatic modulator and [tBK03a, tBK03b, tBKA04] designed IRA and LDPC codes for iterative MIMO detection and decoding .

In this proposed research activity we would like to use the concept of irregularity to build flexible turbo-like encoders with tunable O-EXIT charts to be used in an iterative receiver. Several irregular design constructions will be compared under a unified framework, in terms of complexity, performance, and flexibility in conjunction with some OFDM-MIMO detectors.

4.7 Polar codes: Concept and Decoding Complexity

The concept of polar coding has already been described in detail in Section 4.7. In this section we will discuss complexity issues regarding polar codes and show that a polar code of block-length N can be encoded and decoded in complexity $\mathscr{O}(N\log N)$ irrespective of the code rate.

4.7.1 Encoding

Recall that a polar code is characterized by an information set $\mathscr{A}$ and a frozen vector. In this section, we will take the frozen vector as the all-zero vector since its choice does not affect the complexity of encoding and decoding algorithms. Thus, for the purposes of this section, an (N,K) polar code is a linear block code that maps a data vector $d_1^K \in \{0,1\}^K$ to a codeword $x_1^N \in \{0,1\}^N$ by

$$x_1^N = d_1^K G_N(\mathscr{A}) \tag{28}$$

where $\mathscr{A} \subset \{1,\dots,N\}$ is the information set, G_N is the full generator matrix for the code, and $G_N(\mathscr{A})$ denotes the submatrix of G_N consisting of rows with indices in $\mathscr{A}$.

It is shown in [Arı07] that the generator matrix G_N in polar coding is given by

$$G_N = F^{\otimes n}\tilde{\Pi}_N = \tilde{\Pi}_N F^{\otimes n} \tag{29}$$

where $F^{\otimes n}$ is the n-fold tensor product of $F \triangleq \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ with itself and $\tilde{\Pi}_N$ is a permutation matrix known as the *bit-reversal* operation. To describe the bit-reversal operation, we need to consider an alternative indexing for vectors. Given a vector a_1^N with length $N = 2^n$ for some $n \geq 0$, we may denote its ith element, a_i, $1 \leq i \leq N$, alternatively as $a_{b_n \cdots b_1}$ where $b_n \cdots b_1$ is the binary expansion of integer $i-1$, i.e., $i - 1 = \sum_{j=1}^{n} b_j 2^{j-1}$. Now, bit-reversal applied to a_1^N sends it to c_1^N such that $c_{b_n \cdots b_1} = a_{b_1 \cdots b_n}$. For example, $a_1^4 = (a_{00}, a_{01}, a_{10}, a_{11})$ is sent to $c_1^4 = (a_{00}, a_{10}, a_{01}, a_{11})$.

In Fig. 7 we show a circuit for realizing the transformation $F_2^{\otimes n}$ for $n = 3$. The input to the circuit is the vector u_1^8 with its elements arranged in the natural index order. At the output of the circuit, x_1^8 appears in the bit-reversed form $\tilde{x}_1^8 \triangleq u_1^8 F_2^{\otimes 3}$. We may recover x_1^8 through bit-reversal, $x_1^8 = \tilde{x}_1^8 \tilde{\Pi}_8$. Alternatively, if the bit-reversed vector $\tilde{u}_1^8 = u_1^8 \tilde{\Pi}_8$ is applied to the input of the circuit, then x_1^8 appears at the output directly.

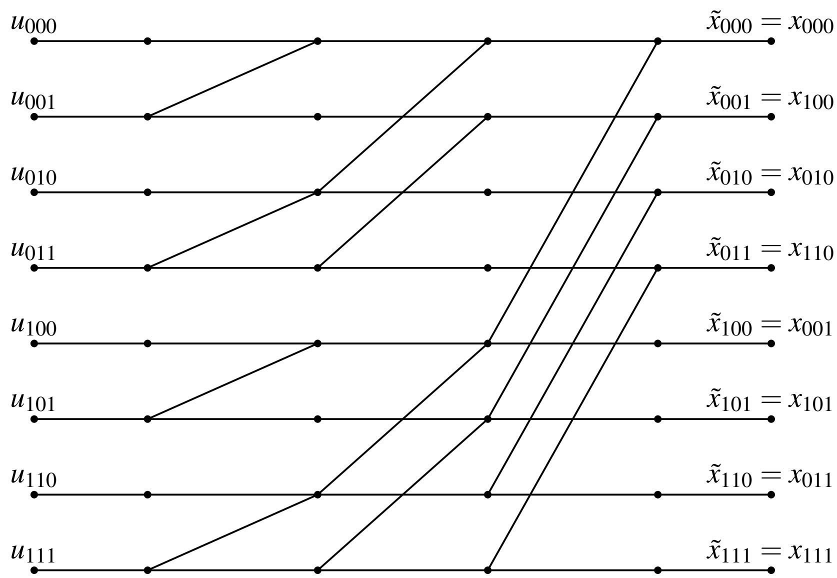

Figure 7: The transformation $F_2^{\otimes 3}$. Each edge carries a value 0 or 1 from left to right. At each node, all values arriving from the left are added modulo-2 and the result is forwarded on all outgoing edges to the right.

There are $N \log N$ nodes in a circuit of type Fig. 7 for a block-length N polar code. Each node carries out an operation of bounded complexity, independent of N. There is an additional bit-reversal operation that needs to be done to complete the encoding, which can also be carried out in circuit complexity $N \log N$. Thus, the overall the encoding complexity is $\mathcal{O}(N \log N)$.

We should note that the bit-reversal operation can be omitted in polar coding since the order in which the codeword elements are transmitted is immaterial, as long as the decoder keeps track of the correct order of transmission.

4.7.2 Connection with Reed-Muller codes

As the above discussion shows clearly, the recursive structure of polar codes is essentially congruent to that of the well-known Hadamard transform. This congruence establishes a link between polar coding and Reed-Muller (RM) coding [Mul54],[Ree54], since the latter, too, can be constructed on the basis of Hadamard transforms. In RM code constructions, one begins with a full-order generator matrix $G_{RM}(n,n)$ which may be taken as the nth order Hadamard transform $F^{\otimes n}$. The rth order RM code $\mathrm{RM}(r,n)$ is defined as the linear code with generator matrix $G_{RM}(r,n)$ which is obtained by expurgating rows of $F^{\otimes n}$ with

Hamming weights less than 2^{n-r}. This is the same type of construction followed in polar coding, except one begins with the matrix $G_N = F^{\otimes n}\tilde{\Pi}_N$ and obtains lower rate codes by expurgating rows using a channel-specific reliability-sensitive criterion as described in Section 4.7. These differences affect the relative performance of the two coding schemes, but do not prevent transferring ideas between them.

4.7.3 Decoding

The circuit in Fig. 7 (redrawn for the size of the polar code at hand) can be used for implementing a successive cancellation decoder for polar codes. Indeed, it is shown in [Arı07] that a successive cancellation decoder can be built as a message-passing decoder with a schedule of messages so that at most $\mathscr{O}(N \log N)$ messages (each computable within constant complexity) are exchanged.

Successive cancellation is a one-pass message passing algorithm; this makes it analytically tractable, but also suboptimal. Much better performance has been observed in experiments using a multi-pass belief-propagation algorithm. Belief propagation with any fixed number of iterations also has complexity $\mathscr{O}(N \log N)$.

4.7.4 Proposed research activities

Although encoding and decoding complexities of polar coding are well-understood, the complexity of code construction is an issue that needs to be studied further. The known methods of polar code construction have exponential complexity in code block length N in general. One exception is the binary erasure channel for which code construction complexity is $N \log N$. Heuristic methods of polynomial complexity exist for approximating the exact polar code construction algorithm for general channels. The development of better low-complexity heuristic algorithms is a subject for further research.

Another research direction is to study the complexity of implementing polar codes in systematic form. In many applications, systematic codes are preferable. Polar coding as described above does not generate systematic codes. A first research goal is to develop methods for systematic polar code construction that preserve the $N \log N$ encoding and decoding complexities. Once systematic polar codes are constructed, a subsequent research goal could be to construct polar turbo codes and study them with respect to complexity and performance.

5 SYNCHRONIZATION

5.1 Introduction

Synchronization, from the Greek *synchronos* (i.e., *syn* (together) + *chronos* (time)) denotes the function of making two systems or two signals running exactly together at the same pace. Specifically, the synchronization subsystems of a modem have the purpose of achieving the correct alignment of the incoming waveform with certain local references generated by the receiver [MMF97, MD97]. For instance, in digital bandpass transmission a coherent receiver needs *carrier, timing and frame synchronization*.

- Carrier synchronization means that the sinusoid for bandpass-to-baseband conversion at the receiver must be locked in phase and frequency to the incoming carrier.
- Timing synchronization means that the clock that determines the boundaries of each received symbol must be aligned with the symbol boundaries of the received signal.
- Frame synchronization means that the symbol count at the receiver must be the same as the symbol count at the transmitter.

Good signal synchronization often turns out to be the key to developing a good modem with fast signal acquisition and reliable steady-state operation.

In the following, we give an overview of the state of the art in synchronization, and point out some directions for further research within NEWCOM++.

5.2 Conventional synchronization

An optimum receiver would detect the transmitted data according to the *maximum a posteriori* (MAP) criterion, i.e. by selecting the data that yield the largest probability of occurrence, given the received signal. When the synchronization parameters (such as the symbol timing, the frame timing, the carrier phase and the carrier frequency) are known, the computation of the a posteriori probability (APP) of the data is feasible for all codes of practical interest. (For turbo and LDPC codes, an approximation of the APP is computed in an iterative decoding process.) However, in the presence of unknown synchronization parameters, the computation of the APP of the data is practically impossible, as it involves a very difficult averaging over the distribution of the synchronization parameters.

Because of these computational difficulties, a different detection strategy is traditionally used when unknown synchronization parameters are present: the receiver makes use of MAP detection assuming that the synchronization parameters are known, but that the detector is provided with estimates (rather than the true values) of the synchronization parameters. This strategy is referred to as *synchronized MAP* detection. This approach reduces the complexity of the detector (as compared to the true APP of the data), but requires the presence of a synchronization parameter estimator.

An acknowledged estimation algorithm is the MAP algorithm, which produces an estimate that maximizes the a posteriori probability density function of the synchronization parameters, conditioned on the received signal. For a uniform a priori probability density function, the MAP estimation rule reduces to the maximum likelihood (ML) estimation rule. As far as the estimation is concerned, the unknown data symbols are to be considered as nuisance parameters. Because the presence of nuisance parameters makes the computation of the a posteriori distribution (or the likelihood function) of the synchronization parameters in general very hard, true MAP estimation cannot be used in practice and one has to resort to approximation techniques. The various practical synchronization parameter estimation algorithms are the result of applying these techniques, which can either be systematic or ad hoc.

For communication systems without channel coding or with only a moderate coding gain the operating signal-to-noise ratio (SNR) is sufficiently high to achieve reliable synchronization parameter estimates by means of relatively simple algorithms. Basically these algorithms fall into one of the following categories: *pilot-based* (PB), *decision-directed* (DD), *non-data-aided* (NDA) [MMF97, MD97].

- In order to perform PB synchronization, some known symbols are multiplexed with the unknown data symbols at the transmitter. When ignoring the presence of the unknown data symbols, the receiver can easily compute the a posteriori distribution of the synchronization parameters, and perform MAP estimation (or an approximation thereof). The PB algorithm exploits only the signal energy contained in the pilot symbols, and is therefore suboptimum. Since pilot symbols reduce both spectral efficiency (i.e., the number of information bits per second that can be transmitted per Hertz of bandwidth) and power efficiency (i.e., the fraction of the available transmit power that is used for the transmission of information-bearing data symbols), it is desirable to limit the number of transmitted pilot symbols.

- In order to exploit for synchronization purposes also the energy contained in the unknown data symbols, one can resort to a DD algorithm. Basically, this involves using (for synchronization purposes) detected symbols as if they were pilot symbols. Typically, the receiver divides the data symbol sequence into blocks over which the synchronization parameters are considered constant; the parameter estimates obtained in the i-th block are used to detect the unknown data in the (i+1)-th block; then the data detected in the (i+1)-th block are used as pilot symbols for estimating the synchronization parameters the (i+1)-th block, and so on. This process is initialized by means of pilot symbols in the first block. The decisions that are conventionally used for synchronization purposes are hard symbol decisions that are made without taking into account the structure of the channel code.

- The use of a NDA synchronization algorithm is another way to exploit the energy contained in the unknown data symbols. The NDA algorithm results from an approximate computation of the a posteriori probability of the synchronization parameters. Conventional approximations in the derivation of NDA algorithms are the assumptions that the data symbols are independent and identically distributed and that the SNR is small. A typical example of a NDA algorithm is the N-th power carrier synchronizer, where the received signal samples are raised to the N-th power in order to remove the effect of the unknown data symbols. The NDA synchronizer does not make use of pilot symbols nor detected symbols for the purpose of synchronization.

5.3 Code-aided synchronization

The introduction of the Shannon capacity approaching turbo codes in the mid-90s marked the beginning of a lot of activity in the field of research, development, and standardization addressing the performance analysis, the design, and the application of *iterative* detectors in digital communications [GLL97, tBSY98, WP99]. The main advantage of such detectors is that they enable reliable communication at a very low SNR, while maintaining a reasonable computational complexity. Their iterative operation typically involves the exchange of soft (extrinsic) information between different soft-input/soft-output (SISO) stages. The performance of coded signals when applied to (wireless) communication terminals, however, implicitly assumes ideal synchronization of the received signal. So the adoption of powerful iteratively-decodable channel codes such as turbo and low-density parity-check (LDPC) codes [BGT93, Gal62, Mac99] engenders a main issue for the synchronization subsystems of a receiver: how to derive accurate synchronization parameter estimates from the received signal at those (extremely low) SNRs typical of such codes? Furthermore, how to do this with a reasonably short acquisition time, possibly using a short preamble of known pilot symbols, or even no preamble at all?

Earlier attempts of signal synchronization in the low-SNR regime focused on the traditional PB, DD and NDA algorithms, but it turns out that these algorithms perform poorly at low SNR.

- The PB algorithms require a lot of pilot symbols in order to achieve accurate synchronization, which in turn gives rise to a considerable loss in bandwidth and power efficiency

- The code-unaware hard decisions on the coded symbols are unreliable at low SNR, which deteriorates the DD synchronizer performance.

- As the derivation of the conventional NDA algorithms involves several approximations, they give rise to a noise enhancement that increases with decreasing SNR.

As a consequence, it was soon recognized [NSM05] that the only way to get good performance both in terms of acquisition time and steady-state accuracy is taking advantage of the coding gain not only for data detection, but for synchronization as well. This led to the notion of *code-aided* (CA) synchronization, i.e., explicitly using the channel code structure and properties to perform good synchronization at low SNR without requiring a very large number of pilot symbols. Although CA algorithms may appear as a natural solution to improve synchronization functions, its implementation is at times critical and has led to numerous contributions in the technical literature, see e.g., [HNL$^+$07] and references therein.

The iterative MAP detection of advanced codes (turbo, LDPC) involves the maximization of the APPs of the individual information bits. These APPs are obtained from a marginalization of the joint APP of all information bits and all coded symbols. The marginalization is accomplished by passing messages, computed by means of the sum-product (SP) algorithm, on a factor graph that represents a factorization of the joint APP. When the synchronization parameters are known, the factor graph has the information bits and the coded symbols as variable nodes, and the synchronization parameters appear as known parameters in some of the function nodes. As the factor graph contains cycles, the message-passing process becomes iterative, and the marginal APPs obtained after convergence are approximations of the true marginal APPs.

In the presence of unknown synchronization parameters, we can distinguish two strategies for the detection of the data that are protected by the advanced codes.

- In the case of synchronized detection, we use the same decoder as for known synchronization parameters. The decoder uses estimates (rather than the true values) of the synchronization parameters, which are updated after each decoding iteration. Hence, the synchronization process is also iterative. The joint iterative synchronization and detection process is initialized by conventionally obtained synchronization parameter estimates (e.g., from pilot symbols). The estimates are updated based on soft information provided by the decoder. This type of synchronization is often referred to as *turbo synchronization*.

- Alternatively, the receiver performs decoding based on the iterative computation of (an approximation of) the APPs of the data in the presence of the unknown synchronization parameters. This involves message passing in a factor graph that has the information bits, the coded symbols and the synchronization parameters as variable nodes. Strictly speaking, the receiver does not perform synchronization (because no explicit synchronization parameter estimates are used in the decoding process), but rather computes (as a by-product of the decoding process) the marginal a posteriori distributions of the synchronization parameters. Note that (unlike synchronized detection) this approach in general does not allow to use the decoder that assumes the synchronization parameters to be known.

In [OC01, LL04, ZB04, WX05], turbo synchronization is adopted. Although some of these techniques may be considered as ad hoc, they offer significant improvement over the conventional synchronization methods. Moreover, the additional complexity of the iterative turbo synchronization is usually quite modest when the synchronizer iterations and the decoder iterations are merged. In [OC01] a simple carrier phase recovery algorithm is proposed which exploits the information contained in the extrinsic values generated within the iterative MAP decoder. Based on the observation that the presence of a carrier phase offset results in a reduction of the magnitude of the decoder extrinsic values, the arithmetical average of the magnitude of the extrinsic values is considered as an objective function to be maximized by the phase estimate. This objective function is used to continuously track the (residual) carrier phase offset by means of a stochastic gradient-type update algorithm. Also an initial carrier phase acquisition algorithm is proposed, which provides a coarse initial phase offset estimate (a multiple of $\pi/2$, for QPSK) such that the initial estimation error falls within the pull-in range of the tracking algorithm. The initial carrier phase offset estimate is obtained through several additional decoder iterations with the input signal compensated with the candidate values of the initial phase offset estimate. The estimate is selected based upon the average magnitude of the extrinsic values generated by the second constituent decoder for these compensated signals. The initial carrier phase offset estimate is then utilized for all symbols in the code frame under decoding.

Another and refined low-complexity iterative CA carrier phase estimation algorithm for turbo-coded signals is introduced in [LL04]. The algorithm is based on a pseudo maximum-likelihood (ML) strategy and makes iterative use of soft decisions provided by the SISO constituent decoders at each iteration. The proposed scenario allows iterative decoding and carrier phase recovery in a so-called soft-decision-directed (SDD) mode. In other words, at each decoding iteration the soft decisions provided by the decoder are used to perform iterative soft-decision-aided estimation of the carrier phase. The logarithmic a posteriori probability ratios which are available at each iteration in the decoder are the soft values embedded in the computation of the phase estimate. This algorithm is labeled as iterative soft-decision-directed (ISDD) synchronization scheme. It is worth noting that the ISDD synchronizer needs to be preceeded by a conventional synchronization step, responsible for providing a sufficiently accurate intial carrier phase estimate. The ISDD algorithm itself is a blind recovery technique but it only converges to the true phase value after a small number of iterations and hence without appreciably degrading the turbo-decoder performance provided that the initial carrier phase estimation error is sufficiently small (e.g., up to 20 degrees for 16-QAM). With the same assumption and allowing for the number of iterations be extended up to 20, its performance is practically equal to the ideal phase recovery scenario.

A different concept of turbo carrier phase synchronization named a-priori-probability-aided (APPA) phase estimation is presented in [ZB04]. Here, similar to [LL04], the ML approach is applied to an iterative phase estimation process but (in contrast to [LL04]) it is realized with the aid of the extrinsic information obtained from the turbo decoder. The phase estimator and the turbo decoder operate simultaneously once per decoding iteration. The new phase estimate and extrinsic information are then employed in the next iteration. Applied iteratively, this technique allows successive refinement of the carrier phase estimate, until the joint decoder/synchronizer converges on the correct data and phase estimate. The key feature of the algorithm is that in the ML phase estimation the approximation of the log-likelihood function (LLF) is used in order to attain the estimate with low complexity. It is achieved by expanding the LLF as a Fourier series whose coefficients are precalculated and stored in lookup tables. This gives an advantage of not introducing excessive delay to the system. It was shown in [ZB04] that the APPA phase estimation achieves the performance of ideal synchronization without introducing the additional redundancy of a synchronization preamble (pilot symbols), when using N (e.g., N=4 for QPSK) identical joint phase recovery and decoding units that are (2π/N) apart.

Research on iterative CA synchronizers has also headed to the development of general *mathematical frameworks* for turbo synchronization. The common idea of these frameworks is to consider turbo synchronization as an iterative solution to the ML synchronization problem. In the literature have appeared frameworks based on the expectation maximization (EM) algorithm [NLD+05], the gradient method [Her06] and the sum-product algorithm (SPA) [HRV07]. These frameworks allowed to justify architectures and performance of earlier proposed turbo synchronization schemes. In particular, the approach introduced in [LL04] can be cast in the framework of the EM algorithm and the APPA phase estimation [ZB04] can be justified through the SP framework. The estimators proposed in [WX05] fall into gradient methods.

Finally, as far as decoding from the APPs in the presence of unknown synchronization parameters is concerned, we mention the use of the SP algorithm in [HRV07, DL04]. Approximating by Dirac functions the messages leaving the variable nodes that represent the synchronization parameters, the iterative decoder that assumes the synchronization parameters to be known can still be used. In this approach, the soft information to be provided by the decoder consists of the extrinsic probabilities of the coded symbols.

5.4 Extensions and low-complexity algorithms

Considering the notable performance of the above approaches to iterative CA carrier phase estimation, it seems justified and desirable to undertake further research in this area in some directions.

On the one hand, extending the performance results reported so far with these methods should be obtained. In particular, it concerns obtaining results of the mean square estimation error (MSEE) as a measure of the estimator accuracy, acquisition and/or operating range and the BER performance. In some cases it will require a non-trivial extension of the method. For instance, the APPA approach [ZB04] was developed for BPSK and QPSK signals only. A modification of the method for high-order two-dimensional modulations (e. g. 16-QAM) is not straightforward. Having obtained comprehensive numerical results, a comparison of performance and complexity of various approaches should be made.

On the other hand, research on modifications of the methods for more demanding scenarios is needed. Most of the approaches reported in the literature assume single carrier transmission and a carrier phase offset that is constant over one code block. These assumptions, however, are not always realistic ones, especially not in wireless communications. Therefore, modifications of the algorithms for multi-carrier systems and channels with a time-varying carrier phase should be a subject of further research. In general, the decoding/synchronization problem becomes much more complicated when the synchronization parameters are changing in time from one symbol to the next (e.g., when strong phase noise is present), because the number of unknown parameters drastically increases. In addition, for more complicated scenarios (e.g., when multiuser interference is present), it is more difficult to factorize the joint APP of the variables involved, which increases the computational complexity associated with the decoding.

Finally, derived from theoretical frameworks [NLD+05, Her06, HRV07], the development of new high-performance / low-complexity algorithms for other synchronization problems like a carrier frequency offset or timing delay will also be a direction for future research.

5.5 Time-varying synchronization parameters

As mentioned above, the investigation of the decoding/synchronization when the synchronization parameters are time-varying will be a subject of further research within NEWCOM++ WP4. Recently, in this context several turbo-synchronization-like techniques have appeared in literature. Similarly, the related problem of iterative joint channel estimation and decoding for channels with time-varying (flat) fading is a major focus of much current research.

A first approach to deal with a time-varying carrier phase consists in modifying the detection/decoding device in order to embed parameter tracking. In [AC01, CFR00] for example, combined iterative decoding and estimation is performed by forward-backward algorithms operating on a trellis and aided by some sort of (implicit or explicit) per-survivor parameter estimation technique. In [DL04, CBC05], the authors propose to construct the factor graph representing a factorization of the joint APP of the data symbols, the information bits and the phase trajectory, and to run the SPA to compute the marginal information bit APPs. To overcome the complexity problems due to the presence in the factor graph of continuous variables, several approximation techniques are proposed: in [CBC05], for example the authors advocate the method of canonical distributions, while in [DL04], numerical integration, gradient methods and particle filtering (also known as sequential Monte-Carlo (SMC) method) are applied. These approaches require a modification of the detector/decoder, as compared to synchronized detection. Such modification might not be desirable, as it precludes the use of standard "off-the-shelf" decoders.

Therefore, a second category of algorithms aims at extending the synchronized detection with turbo synchronization to the case of time-varying parameters. Recently, several turbo-synchronization-like phase tracking techniques have appeared in the literature; they are based on a combination of conventional and turbo techniques, such as feedback phase error control, Wiener filtering, Kalman filtering and the SMC method.

- Discrete-time phase error feedback control loops derive an estimate of the instantaneous phase error from the phase-corrected received noisy samples and feed this estimate back to the phase correction block. As a result, feedback structures are able to automatically track slow variations of the phase shift. Iterative CA feedback synchronizers for tracking a time-varying carrier phase have been presented in [OC01, LW98, LH00b, NDMB06]. These synchronizers are similar to the conventional DD feedback synchronizers [MMF97, MD97], with the important difference that after each decoding step information from the decoder is used to refine the phase estimation. While in [OC01, LW98, LH00b] the synchronizer exploits information from the decoder in a rather ad hoc way, in [NDMB06] the framework derived in [DL04] has been used to derive which information from the decoder is required by the synchronizer. The computational complexity of these synchronizers is lower than that of the algorithms from [DL04, AC01, CFR00, CBC05] which involve a modification of the decoder structure. Assuming a zero normalized frequency offset, simulations run for a first-order feedback loop in combination with an off-the-shelf BPSK turbo decoder already yielded very promising performance results. The conventional method to cope with nonzero frequency offsets is to use a second-order feedback loop. The investigation of the performance and the robustness of second-order feedback schemes needs to be further addressed. Similarly, the use of the proposed schemes in combination with higher-order signalling constellations remains a topic for future research.

- Wiener filtering, also known as linear minimum mean squared error (LMMSE) estimation [Kay93], is conventionally applied in the context of pilot-based estimation. There, it is a common technique, since, in the case of Gaussian noise on the channel observation, an LMMSE estimator yields the optimum result in terms of minimizing the mean squared estimation error. In [Cav91], Cavers successfully investigated the applicability of pilot-based LMMSE estimation for a bandlimited time-varying flat-fading channel. Later on, it was shown in [SSSM01] that LMMSE estimation can also be used in the context of PB phase noise estimation. However, LMMSE parameter estimation does not only yield accurate results in the context of PB estimation. Valenti demonstrated in [VW01] that in the context of the CA estimation of a time-varying flat fading channel, an LMMSE channel estimator can significantly improve the bit error rate performance of a turbo code. Valenti's work gave rise to several other publications in the field of CA channel estimation, cf. e.g. [YR03], [HR05]. Regarding phase noise compensation, it was shown in [GHP$^+$07] that CA LMMSE estimation can be adopted and yields accurate results, also for the case that only a few pilot symbols are present within a burst. However, as always when considering LMMSE estimation, we have to keep in mind that, depending on the chosen filter length, quite a significant amount of additional complexity at the receiver might be necessary. To be more specific, it needs to be carefully investigated whether the performance gain is worth this additional complexity. For instance in [GHP$^+$07], it is demonstrated that in certain scenarios conventional PB frequency estimation techniques approximate the phase noise processes rather well in certain scenarios.

- SMC methodologies provide a promising new paradigm for the design of low-complexity signal processing techniques for fast and reliable communication in a highly severe wireless environment. The SMC methodology, also called *particle filtering* [PS99], that has emerged in the field of statistics and engineering, has shown great promise to solve such problems. This technique can approximate the optimal solution directly without compromising the system model. Additionally, the decision made at time t does not depend on any decisions made previously, and thus, no error is propagated in their implementation. More importantly, the SMC methodology yields a fully blind algorithm and allows for both Gaussian and non-Gaussian ambient noise as well as high-speed parallel implementations. Furthermore, the tracking of time-varying parameters and the data detection are naturally integrated. The algorithm is self-adaptive and no training/pilot symbols or decision feedback are needed [LC98]. In [PCM05, PCMN06a, PCMN06b, PCMN06c, PCMN05], joint research between Kadir Has and Ghent University in the context of NEWCOM is described which highlights the potential use of the SMC technique for joint data detection and carrier phase tracking in single carrier and orthogonal frequency division multiplexing (OFDM) systems. In particular, in [PCM05, PCMN06a] tracking of phase noise is achieved by an extended Kalman filter (EKF).

The SMC method treats the transmitted symbols as "missing data" and draws samples sequentially of them based on the observed signal samples up to time t. This way, the Bayesian estimates of the phase noise and the incoming data are obtained through these samples, sequentially drawn, together with their importance weights. The proposed receiver structure is seen to be ideally suited for high-speed parallel implementation using very large scale integration technology. Although the EKF technique is the most widely used estimation algorithm for nonlinear systems, the long past experience has shown that it is difficult to implement, difficult to tune, and only reliable for systems that are almost linear on the time scale of the updates. Many of these difficulties are mainly due to the linearization process inherent in the EKF technique. To overcome this limitation, an *unscented filtering* (UF) technique was proposed in [PCMN06b] as an alternative for the EKF method. It was shown that the UF approach is more accurate, easier to implement and uses the same order of calculations as the EKF approach. The work presented in [PCM05, PCMN06a, PCMN06b] was extended later for estimating the residual phase noise, blindly, generated at the output the phase tracking loop employed in OFDM systems [PCMN06c, PCMN05]. Further investigations are needed to improve the estimation algorithm proposed in [PCMN06c, PCMN05].

Iterative algorithms for estimation and detection benefit from a factorization of the joint a posteriori probability of all unknown parameters when computing the marginal a posteriori probabilities. However, in several scenarios the joint a posteriori probability does not factorize into a product of simple functions, so that marginalization becomes more complicated. In such cases, marginalization can be simplified by approximating the true joint a posteriori probability with a more convenient function that is easily factorized. Such approximations can be obtained using variational optimization. In [Nis07], a variational Bayesian (VB) framework is deployed to estimate phase noise in the system. The scheme is based on the variational free energy. A formalization of the variational free energy methods called the variational free energy minimization (VFEM) framework was made in [Nis08]. A similar scheme was proposed by [LL07]. In [NP07], it was shown that in the presence of Rayleigh fading many soft-input soft-output equalizers can be viewed as instances of a variational optimization problem [NP07]. The VB framework is elaborated and extended in [Bea03] to the variational Bayesian expectation maximization (VBEM) algorithm by introducing the notion of the complete data set, known from the EM algorithm [DLR77]. The divergence minimization (DM) algorithm results from applying a VBEM-like framework [HLR+08] to situations where the number of mixture components is known. In the DM, the optimization is performed by minimizing the Kullback-Leibler (KL) divergence (or relative entropy). The KL divergence is similar to the variational free energy in the sense that it also measures the divergence between an auxiliary function and the true posterior. The DM method provides an approximation in the sense that only solutions that factorize in a given way are considered. The EM-algorithm has already shown good results for synchronization [HRV07]. It was shown in [NP07] that the EM-algorithm is a special case of the VBEM framework. Furthermore, Bayesian methods such as the VBEM have shown good results for joint channel estimation and data detection [CL06] in iterative receiver structures. In particular they were shown to outperform the EM/SAGE-based methods. The work in [Nis07] combined with the fact that the EM algorithm has already been applied to estimate synchronization parameters, like carrier phase and frequency offset, is a strong indicator that the application of the divergence minimization principle or free energy minimization to turbo-synchronization is a logical next step.

Considering the many different (possible) approaches for iterative decoding in the presence of time-varying parameter offsets that have recently been proposed, we will compare the various algorithms in terms of their mean square estimation error performance and bit error rate performance, their implementation complexity and their robustness with respect to the a priori information (such as the phase noise spectrum).

6 CONCLUSION

We have presented in four chapters the state of the art and outline of future research from today's viewpoint for WPR.4 of NEWCOM++. Regarding the theoretical framework for iterative receiver design, we have highlighted the information geometric approach and its interactions with convex optimization methods, and factor graph based design. We also delved into iterative processing for relay networks, with implications for network information theory and multi-user wireless communications. Regarding code design, we have presented plans to pursue research on the new and exciting topic of binary polar codes, as well as on the design of codes for applications with delay and bandwidth constraints, in particular non-binary codes. Regarding implementation, we have established plans for a theory of receiver design under quantization constraints, and considered the design of message passing with restricted message alphabets and low-complexity components. Finally, we established plans for collaborative research in the field of "turbo" synchronization, where iterative processing is used for synchronization as well as for detection and decoding.

Beyond the content of this deliverable, it is the collaborative manner in which the report was prepared that will contribute to the successful kick-off of the joint research aimed at within the Network of Excellence. Each chapter was prepared by a sub-constellation of participants:

Theoretical Framework involved Vienna University of Technology, L2S/CNRS, UCL, FTW and AAU

Code Design involved ENSEA/CNRS, PoliTorino and Bilkent.

Low Complexity and Implementation involved FTW, Bilkent, NOA/NKUA, PoliTorino and Kadir Has.

Synchronization involved University of Ghent, RWTH Aachen, Poznan University, Kadir Has, FTW and AAU.

In several cases, one partner's interests span more than one chapter, and so do the contributions to this deliverable, ensuring that there is cohesion and interaction between the research directions defined.

During the kick-off meeting in Munich, we established a map of mutual interests represented in Figure 8 that led to the assignment of topics and chapters for this deliverable. The map shows a simple arrow when one partner is interested in the research of another partner, and a thick double-arrow when the interest is mutual. Note that TUM and Technion are not currently linked within this map because they were unable to attend the meeting. They were however heavily involved in the preparation of the research program and we expect that they will remain active participants in the course of the life of the workpackage. Note also that Kadir Has has no incoming expressions of interests due to the fact that they could not present their work at the meeting as a result of visa restrictions. Their heavy involvement in the preparation of the deliverable shows their full integration in the research activities, and they would have several incoming arrows if the exercise was repeated with their presentation. Some focal regions of interests in this graph are clearly mapped in the chapter structure of the deliverable, e.g., the triangle PUT/RWTH/UGhent corresponds to the "synchronization" chapter. Some parallels between the graph and the research activities are more involved, e.g., the triangle FTW-ENSEA-Bilkent led to contributions both in the "code design" chapter and in the "implementation" chapter. Finally, the figure also shows arrows pointing to other workpackages, where one partner serves as a main interface for the common research interests that bridge the research in WPR.4 to other parts of NEWCOM++. Note that some of the common interests point to workpackages in the "physical layer" cluster (WPR.1, 3 and A)) that includes WPR.4, while some bridges are to workpackages in the other "networking cluster" (WPR.5, 6 and 7).

Both the map and the deliverable should be interpreted as reflecting the current state of interests within the workpackage, as captured and amplified to encourage collaborative research and foster interaction between partners. We expect that this map will change, as new research insights may lead to new joint initiatives. These changes in topics and interests will be closely monitored during subsequent workpackage meetings (we plan 2 physical meetings per year, plus possibly several virtual meetings) and reported upon in the next deliverable.

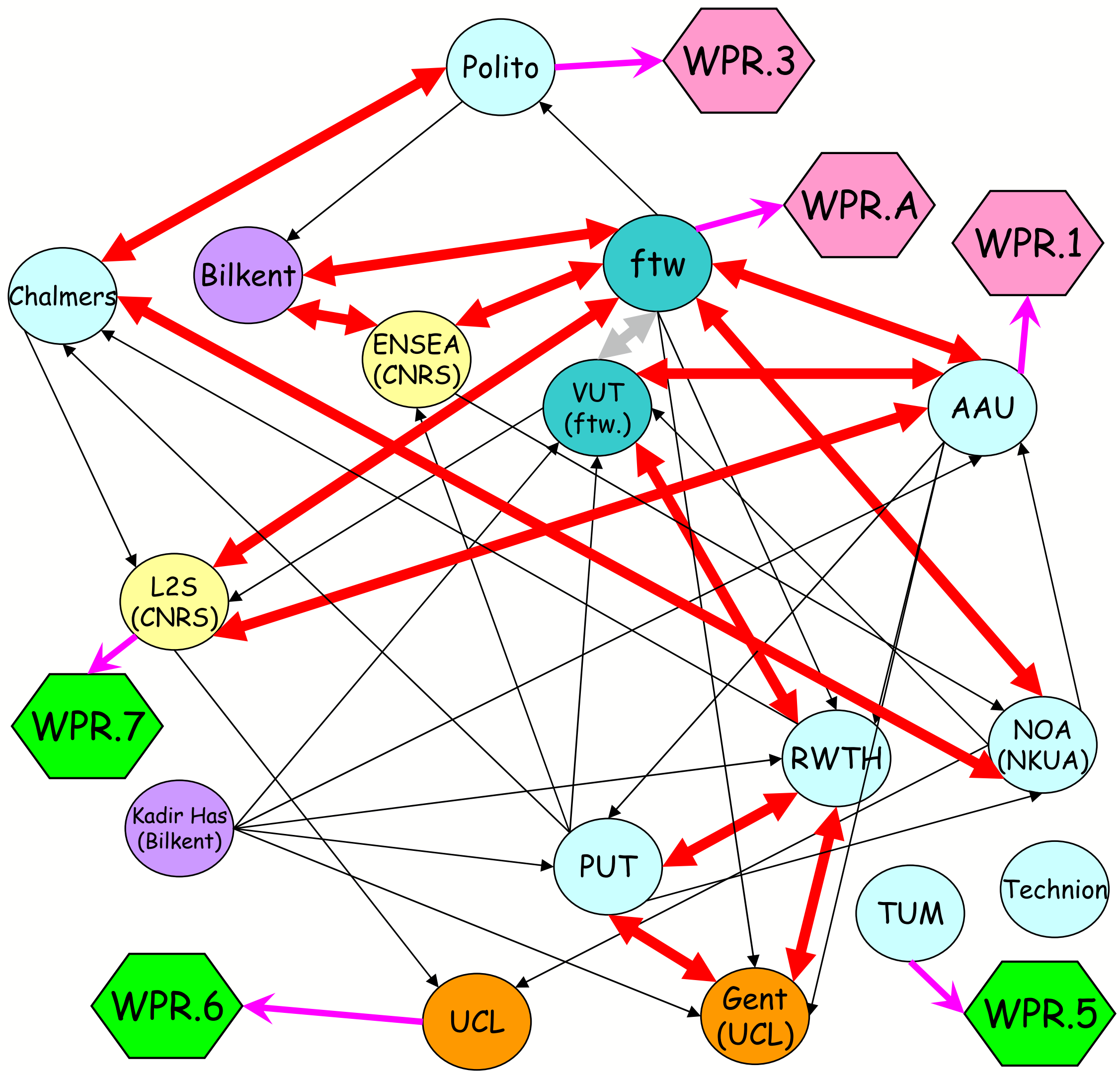

Figure 8: Map of mutual research interests of WPR.4 participants

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