216715 NEWCOM++
Deliverable Number: DR6.2
Intermediate report about distributed signal processing and distributed space-time codes

Contractual Date of Delivery to the CEC: T0+18
Actual Date of Delivery to the CEC: T0+18
Editor(s): L. Vandendorpe (UCL), L. Cottatellucci (CNRS/Eurecom), S. Lasaulce (CNRS/Supelec), G. M. Vitetta (CNIT/UNIMORE)
Participating institutions: Bilkent, CNIT/UNIBO, CNIT/UNIMORE, CTTC, CNRS/Eurecom, CNRS/LSS, CNRS/Supelec, IASA/NKUA, PUT, Technion, TUM, UCL/UGent
Contributors: G. Abraham (Technion), J. Alonso-Zárate (CTTC), E. V. Belmega (CNRS/LSS), H. Bogucka (PUT), C. Buratti (CNIT/BO), B. Krishna Chalise (UCL), L. Cottatellucci (CNRS/Eurecom), R. Couillet (CNRS/Supelec), B. Djeumou (CNRS/LSS), D. Duyck (UGent), P. Elia (CNRS/Eurecom), N. Fawaz (CNRS/Eurecom), S. Gezici (Bilkent), D. Gregoratti (CTTC), J. Hou (TUM), S. Lasaulce (CNRS/LSS), J. Louveaux (UCL), S. Medina (CNRS/LSS), M. Moeneclaey (UGent), O. Oguz (UCL), S. Sergi (CNIT/UNIMORE), H. Sneessens (UCL), S. Shamai (Technion), L. Vandendorpe (UCL), C. Verikoukis (CTTC), G. M. Vitetta (CNIT/UNIMORE), A. Zaidi (UCL), A. Zanella (CNIT/BO)
Internal Reviewer(s): S. Benedetto
Workpackage number: 6
Nature: R
Total Effort Spent:
Dissemination Level:
Version: 2

Abstract:

The purpose of this deliverable is to report on the results obtained in WPR6 about distributed signal processing and distributed space time codes after 18 months of research. The results originating from cooperation among partners will be emphasized.

Keyword list: cooperation, relays, distributed MIMO, distributed space-time-frequency codes

TABLE OF CONTENTS

1 INTRODUCTION

WP 6 addresses the potential of cooperative behavior in the view of obtaining significant capacity and multiplexing gain in wireless communications. This encompasses strategies and codes (including error correcting codes, linear precoders or both) which exploit relaying diversity in order to realize seamless communication and reduce complexity of routing in highly volatile/mobile networks.

WPR6 is organized for the time being in 6 tasks, described hereafter. Task 6.1 is entitled "Design of distributed Space-Time/Frequency codes for Multiuser OFDM systems with relay, relaying strategies and resource allocation". The goal of this task is to investigate and design Space-Time/Frequency codes for multiuser systems based on OFDM and relays. Assuming first CSI at the receiver side, codes will be optimized from the point of view of diversity gain. For CSI available at the transmitter side, the power allocation and the carrier allocation will be optimized for various criterions. These designs will be addressed for different relaying strategies (DF, AF, ...), and various levels of knowledge about the CSI. The optimization of MIMO relays will also be considered. The linear processing operated by the relay will be optimized for a number of criteria. Section 2 reports the achievements related to this task whose goal is in fact to present practical solutions to exploit the rate and/or diversity potential in relayed systems. In a first contribution, considering that several future air interfaces under consideration are based on OFDM, a setup has been investigated where point to point transmission operated by means of OFDM is helped by means of multiple relays implementing the DF protocol. Assuming perfect CSI is available, power allocation to the source and to the relays is optimized in order to maximize the rate of the system. This problem is investigated for a sum power constraint. The solution obtained shows that the power can be allocated according to the seminal waterfilling algorithm, how the water container has to be built and finally that a reallocation step has to be implemented. In next three contributions, communication systems enhanced by means of MIMO relaying is studied. A first scenario considers multiple single users link interfering with each other, that is to say an interference channel. Assuming perfect CSI, several MIMO relays are optimized to help the different links, according to different criteria. Computer simulations verify the improved performance of the jointly optimized MIMO relays when compared to zero-forcing (ZF) relays. A second scenario is considered where a multiantenna terminal communicates with a multiple antenna receiver. A MIMO relay equipped with a Zero forcing receiver and retransmitting the successfully decoded signals helps the systems. The outage probability of the setup is studied and obtained. The next contribution considers a similar setup. This time, CSI is used. The TX performs beamforming and matches his vectors either to the source-relay link or to the source-destination link. The outage probability is again investigated. Another way to benefit from diversity in systems with a relay is to use error correction coding, in a distributed arrangement. Two contributions deal with this technique. First LDPC codes are considered for relay channels in a slowly varying fading environment and with iterative decoding. Random LDPC codes cannot exhibit full-diversity for the maximum achievable coding rate yielding full-diversity. A new family of rate-compatible root-LDPC codes is introduced, which combines the rate-compatibility property with the full-diversity property for any coding rate fulfilling a certain condition. It is shown that the error rate performance of root-LDPC codes is close to the outage probability limit and this occurs for all block lengths (finite and infinite) and all rates fulfilling the condition. Its flexibility and high performance makes rate-compatible root-LDPC attractive for wireless cooperative communications scenarios with slowly varying fading. Next, LDPC codes are studied for the multiple access relay channel in a slowly varying fading environment under iterative decoding. LDPC codes must be carefully designed to achieve diversity on this channel and network coding must be applied to increase the achievable coding rate yielding full-diversity at a maximum rate Rcmax = 2/3. Combining network coding with full-diversity channel coding gave rise to a new family of full-diversity joint network-channel LDPC codes. Full-diversity is achieved for all rates. The results of this section originate from UCL and a close cooperation between Ghent University and Politecnico di Torino.

Task TR6.2 is "Linear Receiver Structures for Virtual MIMO Systems". Its goal is to develop linear receiver structures for virtual MIMO systems with various degrees of performance and complexity.

The first step is to design optimal linear algorithms according to the MMSE criterion, which facilitates optimal utilization of space and multipath diversity. For the cases in which the MMSE algorithm is computation intensive, multi-stage MMSE algorithms that provide complexity-performance trade-offs will be considered. Depending on the amount of sub-optimal combination of signal components from space and multipath domains, various degrees of computational complexity reduction can be obtained. In this deliverable no result is specifically reported for this task which will be mixed with others in the future.

Task TR6.3 is entitled "Randomized Distributed Space-Time Coding for Cooperative Networks and Adaptive Relay Schemes". In a first part of this task, distributed space-time coding schemes will be investigated for cooperative networks. One of the challenges in cooperative systems is that since there is no central control unit, often the cooperating nodes are not aware of each other. Randomized space-time coding schemes have been shown to attain diversity gains even when the number of cooperative nodes are unknown. However, as these coding schemes take advantage of diversity gain inherent in the channel only, one can provide further gains in spectral efficiency by designing space-time codes exploiting spatial multiplexing gains as well. Randomized distributed space-time codes will be investigated that can provide varying levels of diversity-multiplexing tradeoffs. Practical issues such as the decoding complexity and the asynchronism among the nodes will also be addressed. A second part of this task is to design "universal" estimate-and-forward protocols that are based on joint source-channel coding. Ideally the proposed scheme should be able to approximate any reference strategies such as amplify-and-forward and decode-and-forward in the regime where each of these strategies dominates the others. Additionally receiver design will be addressed. It has to be optimally adapted to the chosen relaying scheme. This issue appears in half duplex relay channel and can be critical for relays under bad reception conditions. Additionally these issues will be addressed both from the PHY- and cross-layer standpoints. In a first contribution reported in section (3), two relaying strategies have been considered: AF and EF. AF relaying with feedback has been considered. One of the weaknesses of AF is that it amplifies the reception noise at the relay, which is a critical issue when a cheap relay is used or/and a large amount of interference is received by the relay. To make the AF protocol more robust to this type of bad scenarios, two aspects have been studied: how to tune the amplification factor in order to maximize the information rate from the source to the destination and how to tune the clipping threshold in order to maximize the end-to-end distortion (mean square error on the channel input symbols). Then the interference relay channel has been studied. The contribution has been to analyze the case of the AF and EF protocols. Two EF relaying schemes have been proposed: a version based on a single compression level and a second one which is based on two compression levels. Still about the IRC channel resource (power) allocation is considered. For so called multiband IRC channels, the authors have tackled the existence issue of Nash equilibria (NE). Another important class of cooperative channels is the class of cooperative multiple access channels. So far as in intermediate and more simple case, multiple access channels have been studied. More precisely the network spectral efficiency of distributed vector multiple access channels (MACs) when the number of accessible dimensions per transmitter is strategically limited has been studied. The contributions of this section originate from cooperations between Laboratoire des Signaux et Systèmes (LSS, France), Poznan University of Technology (PUT, Poland), and Supélec (France).

Task TR6.4 is named "Oblivious cooperation protocols and scaling laws for relay networks". A first part addresses topological uncertainty, namely where the user is unaware whether a helper (relay) is present or not. Yet it benefits in cases such a helper exists and is not damaged if this is not the case. The research will be guided by information theoretic considerations, adhering to these practical assumptions. Within this procedure techniques will also be investigated that take advantage of side information present at the receiving end, as well as different broadcast and incremental Wyner-Ziv joint source-channel coding techniques. A second part is devoted to how the capacity scales with the use of relay nodes (virtual MIMO) in the case of full and partial cooperation between the nodes. Attention will in particular be focused on the system design and analysis for different level of channel knowledge at the transmitters (sources and/or relays) The framework will use asymptotic results of random matrix theory. Section (4) reports the results achieved after 18 months. A number of channel configurations have been investigated from the capacity viewpoint: the compound multiple access channel with a relay (cMACr) and a multirelay channel with non ergodic link-failures. Throughput scaling has been studied for wireless networks over channels with random connections. The multiple access relay channel with relay-source feedback is also studied and achievable rate regions are identified. For a system with multiple relays, operating with randomized Distributed Space-Time Coding, an asymptotic analysis is conducted for the Outage Probability and the Diversity. Results are provided for AF and DF relays. Then a cooperative ad hoc network is considered. Nodes are grouped in three types of cooperative clusters. This system forms a virtual MIMO multi-hop relay network. Using tools from random matrix and free probability theories, the asymptotic instantaneous end-to-end mutual information between the source input and the destination output is obtained, as the number of nodes in all clusters grows large. Finally a relay assisted CDMA network with a large number of sources and half duplex relays and a unique destination is considered. Direct relaying (DR) and full relaying (FR) are proposed. An analytical framework for the analysis of the achievable rates in such a network, as the number of nodes and relays becomes asymptotically large, is proposed. The results of this task originate from Technion, CNIT (CRS), CTTC, and CNRS (Eurecom, Supelec), and a starting cooperation between UCL and TUM.

Task TR6.5 is named "Cooperation in mobile ad-hoc networks (Virtual Antenna Arrays/VAAs)". This task intends to investigate the following issues: which neighbors should assist the transmission, and with which processing, and how to perform the cooperative relaying so the battery life of a mobile relay node is kept at a reasonable level. This task will target the selection of the appropriate network layer protocol and cross-layer optimisation technique. The examination and the choice of the network layer protocol and optimisation across multiple OSI layers is essential so that additional routing information can be used for the purposes of organising cooperative relaying at the link layer. A possible selection is the well-known Optimised Link State Routing protocol (OLSR) which features some advanced neighbour discovery and pre-selection mechanisms that seem also applicable for selecting the mobile relays forming VAAs. Section (5) reports the results associated with this task. For cooperative ad hoc networks strategies are first discussed for the selection of wireless nodes which should assist data communications in order to maximize network performance. In order to exploit cooperation at the receive side, linear combining schemes are then presented for Virtual MIMO Systems. Finally, the design and analysis of a MAC protocol that allows executing a Cooperative Automatic Retransmission reQuest (C-ARQ) scheme in wireless networks is presented and analysed. Its principle is that whenever a destination station receives a data packet with errors, it requests retransmissions from any of the stations which overheard the original transmission and can act as spontaneous relays or helpers. This work originates from Bilkent, Technion and CNIT/MORE, and from a cooperation between CNIT/MORE and UCL, and from CTTC.

Task TR6.6 is entitled "Distributed inference". Distributed estimation problems rely on inexpensive pervasive nodes that individually could be unreliable but the exploitation of their interconnections makes them a powerful tool capable of achieving complex network-wide goals. Distributed signal processing is based on nodes with communication, processing and possibly sensing capabilities that self-organise and adapt themselves to the variations of the network as well as of the topology conditions. The nodes process signals in a distributed manner by taking advantage of their density and redundancy. Distributed estimation and decision making are part of the self-learning approach of cognitive networks. The goal of this task is to study distributed synchronization, distributed localization and distributed interference mitigation. No specific results are reported here but rather in WPR.B.

2 TASK TR6.1: DESIGN OF DISTRIBUTED SPACE-TIME-FREQUENCY CODES FOR MULTIUSER OFDM SYSTEMS WITH RELAYS, RELAYING STRATEGIES AND RESOURCE ALLOCATION

2.1 Power loading for point to point OFDM transmission enhanced by multiple DF relays

In previous contributions we have considered OFDM (orthogonal frequency division multiplexing) transmission schemes improved by means of a single relay operating in the decode and forward (DF) or regenerative mode. Two protocols have been considered, differing by the behavior of the source during the second time slot. In a first case, the source is always idle during the second time slot even when the relay is non assisting. In an improved protocol, the source sends during the second time slot a new symbol on each carrier for which the relay is non assisting. The power allocation problem has been solved for an objective function which is the rate of the system. The problem has been solved with a proper handling of the decodability constraint at the relay, and for both types of constraints on the power: a sum power constraint or individual power constraints at the source and at the relay. These results have been reported in [VDLZ08, LTV08, VLOZ08].

In the current section we consider a similar setup where now the transmission is helped by means of multiple DF/regenerative relays. The goal of this contribution is to solve the power allocation problem again for a constraint on the sum of the powers at the source and at the relays. This appears as a natural step to solve the problem for individual power constraints, which will be reported in a future contribution.

Setups with multiples relays have also been considered in the literature. In [GC07], the authors have considered OFDM with multiple decode and forward relays. The objective of their work is to minimize the total transmission power by allocating bits and power to the individual subchannels. A selective relaying strategy is chosen. In the current section, the objective function chosen is the maximization of the rate. Besides that, the truly optimum allocation is obtained and shown to use several relays. It is compared with a suboptimum selective relaying strategy. Li et al. in [LL06] investigate the resource allocation problem for a scenario with multiple source nodes, multiple relays and a single destination. During each slot there is either a single source or a single relay transmitting at a time. They optimize the rate of the system under fairness constraints. They solve their problem by means of a graph theoretical approach. In the current section, we consider a single source. We actually show that it is optimum to have multiple relays sending at the same time and not a single relay, even if we make a comparison with a relay selection mode. Besides that we handle properly the constraints on the decode and forward capability of each node. Moreover an explicit solution is obtained by means of a waterfilling formulation.

2.1.1 System description

Assuming the cyclic prefix technique works properly everywhere, the system can be described by looking at each individual carrier. The block diagram associated with the system for one particular carrier is depicted in figure 1.

During the first signaling period, a symbol is sent by the source on each carrier. The relays then decode and possibly relay some of the symbols during the second time slot. When assisting, the relays are constrained to use the same carrier as that used by the source. Based on the two signaling intervals, the destination implements maximum ratio combining for the carriers with relaying.

Let us denote by $\sqrt{P_s(k)}$ (resp. $\sqrt{P_{r,i}(k)}$) the amplitude of the symbol at the source (resp. ith relay) for carrier k, and by $\lambda_{sd}(k)$ (resp. $\lambda_{r_id}(k)$) the complex channel gain for tone k between source (resp. relay i) and destination. The noise sample observed by the destination at tone k during the first period is $n_1(k)$, and $n_2(k)$ during the second period. These two noise samples are zero-mean circular Gaussian, white and uncorrelated with the same variance σ_n^2. Denoting by $s(k)$ the unit energy symbol transmitted over tone k, the destination gets at the end of the first time slot,

$$y_{sd}(k) = \sqrt{P_s(k)}\,\lambda_{sd}(k)\,s(k) + n_1(k). \tag{1}$$

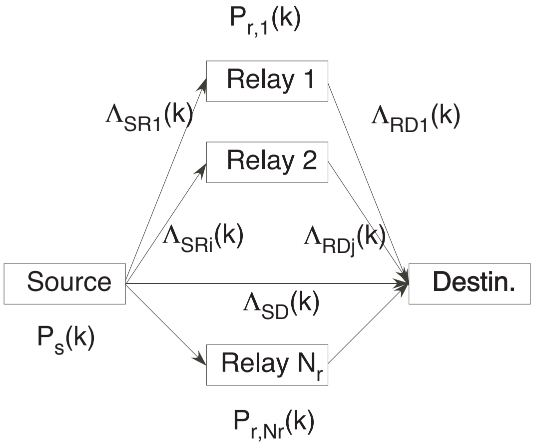

Figure 1: Structure of the system for carrier k.

During the second time slot, for coherence issues at the receiver side, each assisting relay $1 \leq i \leq N_r$ (N_r is the number of relays) sends $\sqrt{P_{r,i}(k)} \exp^{-j \arg \lambda_{r_i d}(k)} s(k)$ which requires the relays to know their respective gains $\lambda_{r_i d}(k)$. Hence the destination receives during the second time slot

$$y_{rd}(k) = \sum_i \sqrt{P_{r,i}(k)} |\lambda_{r_i d}(k)| s(k) + n_2(k). \tag{2}$$

For a carrier with relaying, after proper maximum ratio combining over the two time slots at the destination, the decision variable $r(k)$ obtained at the k-th output of the N_t-FFT (Fast Fourier transform of size N_t, N_t being the number of carriers) and the related signal to noise ratio $\gamma_r(k)$ are given by

$$\begin{aligned} r(k) &= \sqrt{P_s(k)}\, \lambda_{sd}^*(k)\, y_{sd}(k) \\ &+ \Big(\sum_i \sqrt{P_{r,i}(k)} |\lambda_{r_i d}(k)|\Big)^* y_{rd}(k) \qquad (3) \\ \gamma_r(k) &= \frac{P_s(k)\, |\lambda_{sd}(k)|^2 + (\sum_i \sqrt{P_{r,i}(k)} |\lambda_{r_i d}(k)|)^2}{\sigma_n^2}. \qquad (4) \end{aligned}$$

For a not relayed carrier, the signal to noise ratio $\gamma_s(k)$ is given by

$$\gamma_s(k) = \frac{P_s(k)\, |\lambda_{sd}(k)|^2}{\sigma_n^2}. \tag{5}$$

2.1.2 Rate optimization for a sum power constraint

The rate achievable by the system for a duration of 2 OFDM symbols is defined by [LTW04c]:

$$R = 2 \sum_{k \in S_s} \log\left(1 + \gamma_s(k)\right) + \sum_{k \in S_r} \log\left(1 + \gamma_r(k)\right) \tag{6}$$

where S_s is the set of carriers (or tones) receiving power at the source only, and S_r the complementary set, that is, carriers receiving power at the source and at one relay at least. At this point the sets S_s and S_r are not known. The optimization procedure should tell us in which set to allocate each carrier. For a relayed carrier, one has to remember the assumption on the decode and forward operating mode of the assisting relays. For any relay j assisting in the relaying phase for carrier k, one must have that

$$\frac{P_s(k)\, |\lambda_{sr_j}(k)|^2}{\sigma_n^2} \geq \gamma_r(k) \tag{7}$$

where $\lambda_{sr_j}(k)$ is the channel gain between the source and relay j for carrier k. This constraint means that for each relay which is assisting for carrier k, the global signal to noise ratio at the destination has to be lower than or equal to the signal to noise ratio at each one of the contributing relays. As a matter of fact, the signal to noise ratios translate into rates. Hence the global rate of the system between source and destination cannot be above the rate achievable on any of the links between the source and the assisting relays, otherwise some relays would not be able to decode which is in contradiction with our assumptions. Note that for each relayed carrier, the set of relays assisting for that carrier has to be determined.

We want to maximize the rate under a constraint on the sum power, given by

$$\left[\sum_{k\in S_s} 2P_s(k) + \sum_{k\in S_r} [P_s(k) + \sum_i P_{r,i}(k)]\right] \le P_t \tag{8}$$

where P_t is the total power budget. The objective function together with the constraints lead to the following Lagrangian:

$$\begin{aligned}
\mathscr{L}_1 &= 2\sum_{k\in S_s} \log\left(1+\frac{P_s(k)|\lambda_{sd}(k)|^2}{\sigma_n^2}\right) \\
&+ \sum_{k\in S_r} \log\left(1+\frac{P_s(k)\,|\lambda_{sd}(k)|^2}{\sigma_n^2}\right. \\
&+ \left.\frac{(\sum_i \sqrt{P_{r,i}(k)}|\lambda_{r_id}(k)|)^2}{\sigma_n^2}\right) \\
&- \mu\left[\sum_{k\in S_s} 2P_s(k) + \sum_{k\in S_r} [P_s(k) + \sum_i P_{r,i}(k)] - P_t\right] \\
&- \sum_{k\in S_r}\sum_j \rho_{kj}\left[P_s(k)\,|\lambda_{sd}(k)|^2\right. \\
&+ \left.(\sum_i \sqrt{P_{r,i}(k)}|\lambda_{r_id}(k)|)^2 - P_s(k)\,|\lambda_{sr_j}(k)|^2\right].
\end{aligned} \tag{9}$$

From constraint (7) a first conclusion can be drawn about the allocation of a carrier to set S_s or set S_r. If for carrier k and relay j, $|\lambda_{sr_j}(k)|^2 < |\lambda_{sd}(k)|^2$, obviously $P_s(k)|\lambda_{sr_j}(k)|^2 < P_s(k)|\lambda_{sd}(k)|^2$. Because of the DF constraint (7) the rate of the carrier if relayed by relay j would be limited by $P_s(k)|\lambda_{sr_j}(k)|^2$. Therefore the power is better used by not using relay j on that carrier.

On the contrary, if relay j satisfies $|\lambda_{sr_j}(k)|^2 > |\lambda_{sd}(k)|^2$, then it might be beneficial to use this relay on the considered carrier and it should be further investigated in the optimization procedure. Let us now consider the relayed carriers i. e. for which we have at least one j such that $|\lambda_{sr_j}(k)|^2 > |\lambda_{sd}(k)|^2$.

It is known that when a constraint is inactive, its associated Lagrange multiplier is 0. On the contrary when a constraint is active or saturated its multiplier may be different from zero. Assuming that none of the decodability constraints is active (all multipliers are 0), let us take the derivative of Lagrangian (9) with respect to the source power and to the relay power. Forcing them to zero leads to

$$\begin{aligned}
\frac{\partial R}{\partial P_s(q)} &= \left(1+\frac{P_s(q)\,|\lambda_{sd}(q)|^2}{\sigma_n^2}\right.\\
&+ \left.\frac{(\sum_i \sqrt{P_{r,i}(q)}|\lambda_{r_id}(q)|)^2}{\sigma_n^2}\right)^{-1}\frac{|\lambda_{sd}(q)|^2}{\sigma_n^2}\\
&= \mu \qquad (10)\\
\frac{\partial R}{\partial P_{r,j}(q)} &= \left(1+\frac{P_s(q)\,|\lambda_{sd}(q)|^2}{\sigma_n^2}\right.\\
&+ \left.\frac{(\sum_i \sqrt{P_{r,i}(q)}|\lambda_{r_id}(q)|)^2}{\sigma_n^2}\right)^{-1}\\
&\times \frac{\sum_i \sqrt{P_{r,i}(q)}|\lambda_{r_id}(q)|}{\sigma_n^2}\frac{|\lambda_{r_jd}(q)|}{\sqrt{P_{r,j}(q)}}\\
&= \mu. \qquad (11)
\end{aligned}$$

From the last equation we must have that, for any j and j' belonging to the set of relays assisting for carrier q,

$$\frac{|\lambda_{r_jd}(q)|}{\sqrt{P_{r,j}(q)}} = \frac{|\lambda_{r_{j'}d}(q)|}{\sqrt{P_{r,j'}(q)}}. \qquad (12)$$

Considering also that we must have

$$\frac{\sum_i \sqrt{P_{r,i}(q)}|\lambda_{r_id}(q)|}{\sigma_n^2}\frac{|\lambda_{r_jd}(q)|}{\sqrt{P_{r,j}(q)}} = \frac{|\lambda_{sd}(q)|^2}{\sigma_n^2} \qquad (13)$$

and using result (12) this is only possible if

$$\sum_i |\lambda_{r_id}(q)|^2 = |\lambda_{sd}(q)|^2. \qquad (14)$$

When this is not fulfilled, what we assume here, not all constraints can be inactive at the same time.

On the other hand, if two relays j and j' assisting for carrier k have the constraint saturated, it means that

$$P_s(k)\,|\lambda_{sr_j}(k)|^2 = P_s(k)\,|\lambda_{sr_{j'}}(k)|^2. \qquad (15)$$

This is only possible when all the $|\lambda_{sr_j}(k)|^2$ are equal for the active constraints. As this is generally not the case (two channel gains statistically never have the exact same amplitude), we can only have one relay at a time per carrier for which the constraint is saturated. In conclusion, for each relayed carrier, there must be exactly one (and no more) relay with a saturated constraint. At this point the question which is still open is to know whether we might have for carrier k one relay with the saturated constraint and one or more additional ones which are assisting but with the nonactive (or nonsaturated) constraint. Assuming this is the case, we have the following Lagrangian:

$$\begin{aligned}
\mathscr{L}_2 &= 2\sum_{k\in S_s} \log\left(1+\frac{P_s(k)|\lambda_{sd}(k)|^2}{\sigma_n^2}\right) \\
&+ \sum_{k\in S_r} \log\left(1+\frac{P_s(k)\,|\lambda_{sd}(k)|^2}{\sigma_n^2}\right. \\
&+ \left.\frac{(\sum_i \sqrt{P_{r,i}(k)}|\lambda_{r_id}(k)|)^2}{\sigma_n^2}\right) \\
&- \mu\left[\sum_{k\in S_s} 2P_s(k) + \sum_{k\in S_r}[P_s(k)+\sum_i P_{r,i}(k)] - P_t\right] \\
&- \sum_{k\in S_r} \rho_{kj_k}\left[P_s(k)\,|\lambda_{sd}(k)|^2\right. \\
&+ \left.(\sum_i \sqrt{P_{r,i}(k)}|\lambda_{r_id}(k)|)^2 - P_s(k)\,|\lambda_{sr_{j_k}}(k)|^2\right] \qquad (16)
\end{aligned}$$

where j_k is the index of the only relay with the active (saturated) constraint for carrier k. The derivatives are given by

$$\begin{aligned}
\frac{\partial R}{\partial P_s(q)} &= \left(1+\frac{P_s(q)\,|\lambda_{sd}(q)|^2}{\sigma_n^2}\right. \\
&+ \left.\frac{(\sum_i \sqrt{P_{r,i}(q)}|\lambda_{r_id}(q)|)^2}{\sigma_n^2}\right)^{-1} \frac{|\lambda_{sd}(q)|^2}{\sigma_n^2} \\
&= \mu+\rho_{qj_q}\left[|\lambda_{sd}(q)|^2 - |\lambda_{sr_{j_q}}(q)|^2\right]. \qquad (17) \\
\frac{\partial R}{\partial P_{r,j}(q)} &= \left(1+\frac{P_s(q)\,|\lambda_{sd}(q)|^2}{\sigma_n^2}\right. \\
&+ \left.\frac{(\sum_i \sqrt{P_{r,i}(q)}|\lambda_{r_id}(q)|)^2}{\sigma_n^2}\right)^{-1} \\
&\times \frac{\sum_i \sqrt{P_{r,i}(q)}|\lambda_{r_id}(q)|}{\sigma_n^2}\,\frac{|\lambda_{r_jd}(q)|}{\sqrt{P_{r,j}(q)}} \\
&= \mu+\rho_{qj_q}\left[\sum_i \sqrt{P_{r,i}(q)}|\lambda_{r_id}(q)|\right] \\
&\times \frac{|\lambda_{r_jd}(q)|}{\sqrt{P_{r,j}(q)}}. \qquad (18)
\end{aligned}$$

From the last equation we must again have for any two assisting relays j and j' that

$$\frac{|\lambda_{r_jd}(q)|}{\sqrt{P_{r,j}(q)}} = \frac{|\lambda_{r_{j'}d}(q)|}{\sqrt{P_{r,j'}(q)}}. \qquad (19)$$

Because of the saturation or active constraint for relay j_q we also have

$$\begin{aligned}
P_s(q)\,|\lambda_{sd}(q)|^2 &+ (\sum_i \sqrt{P_{r,i}(q)}|\lambda_{r_id}(q)|)^2 \\
&= P_s(q)\,|\lambda_{sr_{j_q}}(q)|^2. \qquad (20)
\end{aligned}$$

Using (19), the active constraint leads to

$$
\begin{aligned}
(\sum_i \sqrt{P_{r,i}(q)}|\lambda_{r_i d}(q)|)^2 &= P_s(q)\left[|\lambda_{sr_{j_q}}(q)|^2 - |\lambda_{sd}(q)|^2\right] \\
&= P_{r,1}(q)\left(\sum_i \frac{|\lambda_{r_i d}(q)|^2}{|\lambda_{r_1 d}(q)|}\right)^2. \qquad (21)
\end{aligned}
$$

This leads to the following values for the relay powers:

$$
\begin{aligned}
P_{r,j}(q) &= P_s(q)\left[|\lambda_{sr_{j_q}}(q)|^2 - |\lambda_{sd}(q)|^2\right] \\
&\times \frac{|\lambda_{r_j d}(q)|^2}{(\sum_i |\lambda_{r_i d}(q)|^2)^2}. \qquad (22)
\end{aligned}
$$

The power allocated to carrier q is given by $P(q)$ which is obtained by

$$
\begin{aligned}
P(q) &= P_s(q) + \sum_i P_{r,i}(q) \\
&= P_s(q)\left[1 + \frac{|\lambda_{sr_{j_q}}(q)|^2 - |\lambda_{sd}(q)|^2}{\sum_i |\lambda_{r_i d}(q)|^2}\right] \qquad (23)
\end{aligned}
$$

which shows the link between $P(q)$ and $P_s(q)$. In view of all these, we can conclude that for a carrier with one assisting relay fulfilling the constraint, and several other assisting relays with non-saturated constraint, the power $P(q)$ leads to a contribution to the rate given by

$$
\begin{aligned}
R_r(q) &= \log\left(1 + \frac{P_s(q)\,|\lambda_{sr_{j_q}}(q)|^2}{\sigma_n^2}\right) \\
&= \log\left(1 + \frac{P(q)\,|\lambda_{sr_{j_q}}(q)|^2\,\beta(q)}{\sigma_n^2}\right) \qquad (24)
\end{aligned}
$$

with

$$
\beta(q) = \frac{\sum_i |\lambda_{r_i d}(q)|^2}{|\lambda_{sr_{j_q}}(q)|^2 - |\lambda_{sd}(q)|^2 + \sum_i |\lambda_{r_i d}(q)|^2}. \qquad (25)
$$

It appears that the impact of $P(q)$ on the rate is increased with the factor $|\lambda_{sr_{j_q}}(q)|^2\,\beta(q)$. As $|\lambda_{sr_{j_q}}(q)|^2 \geq |\lambda_{sd}(q)|^2$, $\beta(q)$ increases with the value of $\sum_i |\lambda_{r_i d}(q)|^2$. Therefore, for a given choice of the saturated relay j_q, all possible additional values of $|\lambda_{r_i d}(q)|^2$ should be retained. In other words, all relays i for which decoding is possible, which means $|\lambda_{sr_i}(q)|^2 > |\lambda_{sr_{j_q}}(q)|^2$, should be used. The choice of the saturated relay j_q is a compromise between a high value of $|\lambda_{sr_{j_q}}(q)|^2$ that is directly beneficial to the rate, and a lower value that allows to select more relays $|\lambda_{r_i d}(q)|^2$ fulfilling the decoding constraint. Hence all the values $|\lambda_{sr_j}(q)|^2 > |\lambda_{sd}(q)|^2$ should be ranked in increasing order. For each one considered as candidate for the relay with saturated constraint j_q, the value of $|\lambda_{sr_{j_q}}(q)|^2\,\beta(q)$ should be computed. The maximum is then kept.

About a non relayed carrier, we have by taking the derivative of the Lagrangian (9) that

$$
\begin{aligned}
\frac{\partial R}{\partial P_s(q)} &= 2\left(1 + \frac{P_s(q)\,|\lambda_{sd}(q)|^2}{\sigma_n^2}\right)^{-1} \frac{|\lambda_{sd}(q)|^2}{\sigma_n^2} \\
&= 2\mu. \qquad (26)
\end{aligned}
$$

For a total power $P(q)$ allocated to carrier q (over the two instants), the rate evolves as

$$
R_s(q) = \log\left[\left(1 + \frac{P(q)\,|\lambda_{sd}(q)|^2}{2\,\sigma_n^2}\right)^2\right]. \qquad (27)
$$

Let us denote by $|\lambda_\beta(q)|^2$ the maximum value that can be found for $|\lambda_{sr_{j_q}}(q)|^2\beta(q)$. When $|\lambda_{sd}(q)|^2 > |\lambda_\beta(q)|^2$ we have for any value of P that $R_s(q) > R_r(q)$. On the contrary, when $|\lambda_{sd}(q)|^2 < |\lambda_\beta(q)|^2$ we have $R_s(q) < R_r(q)$. However this is valid only for $P(q) \le \lambda_t(q)$ where

$$\lambda_t(q) = 4\,\sigma_n^2\,\frac{|\lambda_\beta(q)|^2 - |\lambda_{sd}(q)|^2}{|\lambda_{sd}(q)|^4}. \tag{28}$$

Based on this a new form can obtained for a Lagrangian:

$$\begin{aligned}\mathscr{L}_3 &= 2\sum_{k\in S_s}\log\left(1+\frac{P(k)\,|\lambda_{sd}(k)|^2}{2\,\sigma_n^2}\right)\\ &+ \sum_{k\in S_r}\log\left(1+\frac{P(k)\,|\lambda_\beta(k)|^2}{\sigma_n^2}\right)\\ &- \mu\left[\sum_{k\in S_s}P(k)-P_t\right]\end{aligned} \tag{29}$$

with $P(k) = P_s(k) + \sum_i P_{ri}(k)$ for a relayed carrier where the relay powers can be computed from (22). For a non relayed carrier, $P(k) = 2P_s(k)$.

Equating to 0 the derivatives of this Lagrangian with respect to the power, we get for $k \in S_s$,

$$P(k) = 2\left[\frac{1}{\mu} - \frac{\sigma_n^2}{|\lambda_{sd}(k)|^2}\right]_+ \tag{30}$$

where $[.]_+$ stands for $\max[0,.]$. Similarly, for $k \in S_r$,

$$P(k) = \left[\frac{1}{\mu} - \frac{\sigma_n^2}{|\lambda_\beta(k)|^2}\right]_+. \tag{31}$$

All these derivations basically show that our constrained optimization problem can actually be solved using the seminal waterfilling algorithm, applied to a water container built either from $\frac{\sigma_n^2}{|\lambda_{sd}(k)|^2}$ or from $\frac{\sigma_n^2}{|\lambda_\beta(k)|^2}$. The latter values actually show that the constraint related to the use of the relay leads to particular values to be used for the container. More specifically, for the set S_r, these values are modified values with respect to the $|\lambda_{sr_{j_k}}(k)|^2$. It is also important to note that for the non relayed carriers two identical values have to be used for the water container, corresponding to the two protocol instants.

At the end of the waterfilling one checks if any of the relayed carriers receives an amount of power above the threshold given by (28). If this happens, the relayed carrier fulfilling this condition and for which the rate increase is the largest one, is moved from the set S_r to the set S_s. The waterfilling is applied again. This procedure is iterated till none of the relayed carrier receives an amount of power larger than its associated threshold. This procedure is referred below as the reallocation step.

2.1.3 A simplified approach based on relay selection

In the previous section we have obtained the optimum allocation rule for each carrier, to the set S_s or to the set S_r. For a carrier in this second set, one further has to decide about the relay which saturates the constraint on decodability and that is helped by nonsaturated relays.

In this section we investigate a simplified approach for a relayed carrier, based on the selection of a single relay for which the constraint will be saturated. The main advantage of this simplified approach is that it removes the need for channel knowledge at the relays, which was assumed in (2), and is essentially necessary to benefit from multiple relays simultaneously. In addition, the process of selecting one relay to use is also simpler than the selection of the saturated relay described above.

Actually the results derived in the previous section can be used to decide about which relay to use. If a single and saturated relay is used for carrier q, the value $|\lambda_\beta(q)|^2$ will be given by

$$|\lambda_\beta(q)|^2 = \frac{|\lambda_{sr_{j_q}}(q)|^2 |\lambda_{r_{j_q}d}(q)|^2}{|\lambda_{sr_{j_q}}(q)|^2 - |\lambda_{sd}(q)|^2 + |\lambda_{r_{j_q}d}(q)|^2}. \quad (32)$$

The relay selection amounts to choosing the relay with the largest $|\lambda_\beta(q)|^2$ out of all relays j having $|\lambda_{sr_j}(q)|^2 \geq |\lambda_{sd}(q)|^2$. If at least one of the $|\lambda_\beta(q)|^2$ is large enough such that $|\lambda_\beta(q)|^2 \geq |\lambda_{sd}(q)|^2$ then the carrier is allocated to set S_r. Otherwise the carrier is allocated to set S_s.

Once the selection has been achieved for each carrier, we are again faced with a waterfilling problem where the container is made of the $\frac{\sigma_n^2}{|\lambda_{sd}(k)|^2}$ or from $\frac{\sigma_n^2}{|\lambda_\beta(k)|^2}$. The difference with the optimized approach is in the way the $|\lambda_\beta(k)|^2$ are obtained. The reallocation step to move carriers from set S_r to set S_s when relevant is maintained.

2.1.4 Results

In order to illustrate the theoretical analysis, numerical results are provided and discussed. The number of carriers is set to $N_t = 128$. Channel impulse responses (CIR) of length 32 taps are generated. The taps are i. i. d. zero-mean circular complex random variables. They have a unit variance for all links except the links λ_{sr_i} which are computed from CIRs with taps of variance 10. From these CIRs, FFTs are computed to provide the corresponding λ_{xy}. We set $\sigma_n^2 = 1$.

For the purpose of illustration, a situation with $N_r = 3$ relays is first considered. The sum power constraint is chosen to be $P_t = 300$. Figure 2 provides the gains $|\lambda_{sr_i}(k)|^2$ (solid curve), $|\lambda_{sd}(k)|^2$ (dash-dotted), $|\lambda_{r_id}(k)|^2$ (dashed) in dB for the realization under consideration. Figure 3 shows, for the sum power constraint, the result about the power allocation ($\circ$) obtained for the optimized approach including the reallocation step. The possible further split where appropriate among source power (solid line) and relay power (dashed) is shown. The $\times$s indicate that at least one relay is assisting ($\times$ at the top of the figure) or none is helping ($\times$ in 0). When no relay is assisting it has to be remembered that the corresponding amount of power ($\circ$) has to be shared over two successive time slots. Figure 4 shows for each carrier the associated rate over two time slots, after the reallocation step. The total rate here is 488 bits per 2 OFDM symbols duration before the reallocation step (not shown here for the sake of concision) and 525 after the reallocation step. It turns out that for the parameters of the setup used here the reallocation step brings an additional rate increase of 7.5%.

For the suboptimum relay selection, similar results have been produced but are not shown here. Before the reallocation step the bit rate achieved here is 482 bits per 2 OFDM symbols duration, and 512 after this step. It appears that the penalty associated with the relay selection rather than the correct implementation is around 3% for the parameters used here.

Systematic results have also been produced for the optimum and the selection methods. Situations with $N_r = 3$, 6, 12 and 24 relays have been considered. The rates (bits per duration of 2 OFDM symbols) obtained with the two approaches are reported versus $10\log(P_t/\sigma_n^2)$. Figure 5 reports the rates obtained when the CIRs associated with the $\lambda_{r_id}(k)$ and $\lambda_{sd}(k)$ all have unit variance, and those associated with the $\lambda_{sr_i}(k)$ have a variance 10. Figure 6 reports the bit rate gain (in %) for the optimum approach compared to the relay selection approach, for the same variances of the λ's. For each value of the power P_t, the results are obtained by averaging over 200 channel realizations. As mentioned above the 32 taps are i. i. d. zero-mean complex gaussian. The difference is rather important up to a value of $10\log(P_t/\sigma_n^2)$ where the power is high enough so that all carriers are mainly allocated to the set S_s. In this case, the optimum method and the relay selection method are the same, because relaying is not used anymore.

2.1.5 Conclusions

This section is devoted to the optimization of the power allocation for the rate of an OFDM transmission between a source and a destination, helped by several relays. The relays are of the decode and forward

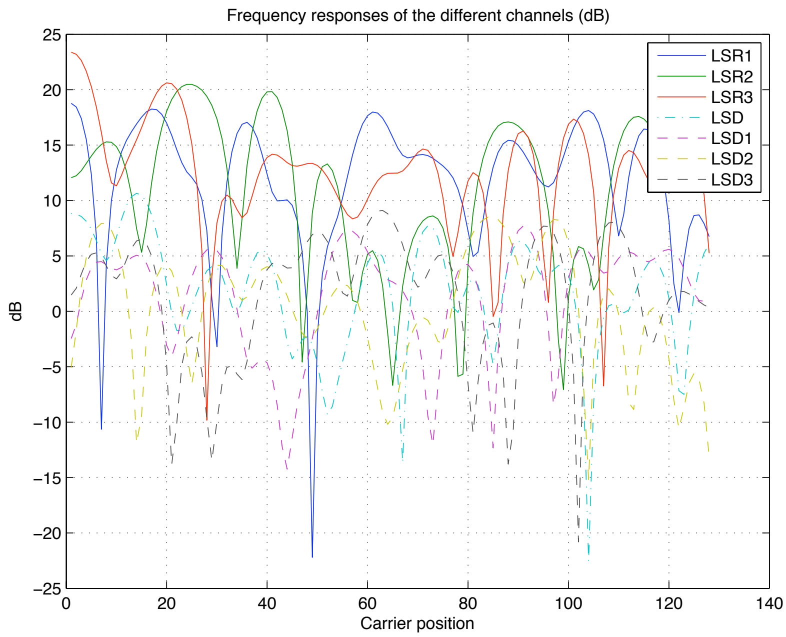

Figure 2: Gains $|\lambda_{sr}(k)|^2, |\lambda_{sd}(k)|^2, |\lambda_{rd}(k)|^2$ in dB.

or regenerative type. The optimization problem is solved for a sum power constraint and by handling properly the decode and forward constraints. The protocol under consideration enables the source to send a new symbol during the second time slot when the relays are all non assisting. A major conclusion is that for a carrier assisted by relaying, a single relay has the saturated constraint (associated with decode and forward). It is shown how to decide about the number of other assisting relays. Further to this optimized approach a suboptimum solution based on relay selection is proposed. The theoretical results are illustrated by numerical simulations. The two approaches are compared.

As shown by previous contributions the solution for a sum power constraint is a natural path to the case of individual power constraints that will be reported in a future contribution.

2.2 Joint Optimization of Multiple MIMO relays

In this work, we propose algorithms to jointly optimize the multiple multi-antenna relays which assist the multi-point to multi-point communication in wireless networks. Assuming that perfect channel state information (CSI) is available for all channels, we design the multiple-input-multiple-output (MIMO) relays with three different methods; 1) maximize the minimum of the signal-to-interference-plus-noise ratios (SINRs) of all destinations satisfying the transmit power constraint of each MIMO relay, 2) minimize the sum of the powers of the relays while fulfilling the SINR requirements for all destinations (also quality of service (QoS) approach), and 3) minimize the total interference and noise power received at each destination under the conditions that the desired signal is preserved and the transmit power of each relay is kept below its power budget. It is shown that the former two problems are non-convex but they can be solved accurately and efficiently using the standard semidefinite relaxation and randomization techniques. However, the last problem can be reformulated into a second-order cone programming (SOCP) problem. Computer simulations verify the improved performance of the jointly optimized MIMO relays when compared to zero-forcing (ZF) relays.

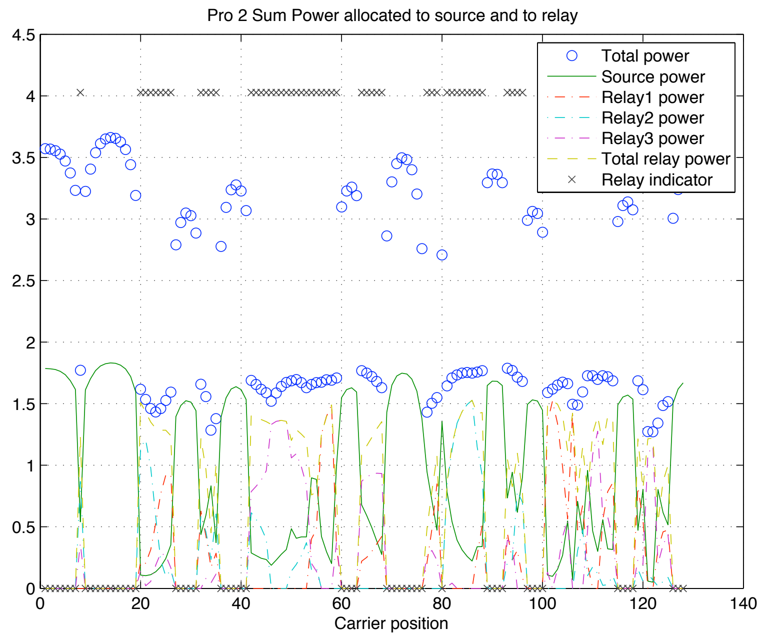

Figure 3: Final power allocation to source and relays in the case of a sum power constraint after the reallocation step for the optimum approach.

2.2.1 System model and description

We consider the scenario where the signal transmissions from K single-antenna sources to K single-antenna destinations are supported by R MIMO relays. The block diagram is shown in Fig. 7. Let $N_{\mathrm{r},i}$ be the number of antennas of ith MIMO relay. It is assumed that each destination is targeted by a single source. More, specifically, kth destination sees only the signal from kth source as a desired signal. The MIMO relays linearly process the signals received from all sources and forward them to the destinations. Note that the direct links between the sources and destinations are not considered since we assume that these direct links undergo relatively larger path attenuations compared to the links via MIMO relays. Furthermore, we assume that the MIMO relays have complete CSI which in our case is the knowledge of all instantaneous channels. It is also important to emphasize here, that the methods to be presented in the sequel can be easily applied to the cases where only the second-order statistics of the channels are available. We consider flat fading channels between sources and the relays as well as between the relays and destinations.

The instantaneous SINR for kth destination can be expressed as [CV09a]

$$\gamma_k = \frac{P_k \mathbf{z}_{\mathrm{L}}^H \mathbf{a}_{k,k} \mathbf{a}_{k,k}^H \mathbf{z}_{\mathrm{L}}}{\sum_{j=1,j\neq k}^{K} \mathbf{z}_{\mathrm{L}}^H \left[P_j \mathbf{a}_{k,j} \mathbf{a}_{k,j}^H + \mathbf{D}_k \right] \mathbf{z}_{\mathrm{L}} + \sigma_{\mathrm{d},k}^2}, \ \forall k. \tag{33}$$

where

$$\mathbf{a}_{k,j} = \begin{bmatrix} \mathrm{vec}(\mathbf{g}_{1,k}\mathbf{h}_{j,1}^H) \\ \vdots \\ \mathrm{vec}(\mathbf{g}_{R,k}\mathbf{h}_{j,R}^H) \end{bmatrix}, \ \mathbf{z}_{\mathrm{L}} = \begin{bmatrix} \mathbf{z}_{\mathrm{L},1} \\ \vdots \\ \mathbf{z}_{\mathrm{L},R} \end{bmatrix} \in \mathscr{C}^{\sum_{i=1}^{R} N_{\mathrm{r},i}^2 \times 1}, \ \ \mathbf{z}_{\mathrm{L},i} = \mathrm{vec}(\mathbf{Z}_i) \in \mathscr{C}^{N_{\mathrm{r},i}^2 \times 1} \tag{34}$$

$$\mathbf{D}_k = \sum_{i=1}^{R} \sigma_{\mathrm{r},i}^2 \mathbf{A}_i^H (\mathbf{I}_{N_{\mathrm{r},i}} \otimes \mathbf{g}_{i,k}\mathbf{g}_{i,k}^H)\mathbf{A}_i, \rightarrow \mathbf{A}_i = \left[\tilde{\mathbf{A}}_1, \cdots, \tilde{\mathbf{A}}_R\right] \in \mathscr{R}^{N_{\mathrm{r},i}^2 \times \sum_{i=1}^{R} N_{\mathrm{r},i}^2}, \tag{35}$$

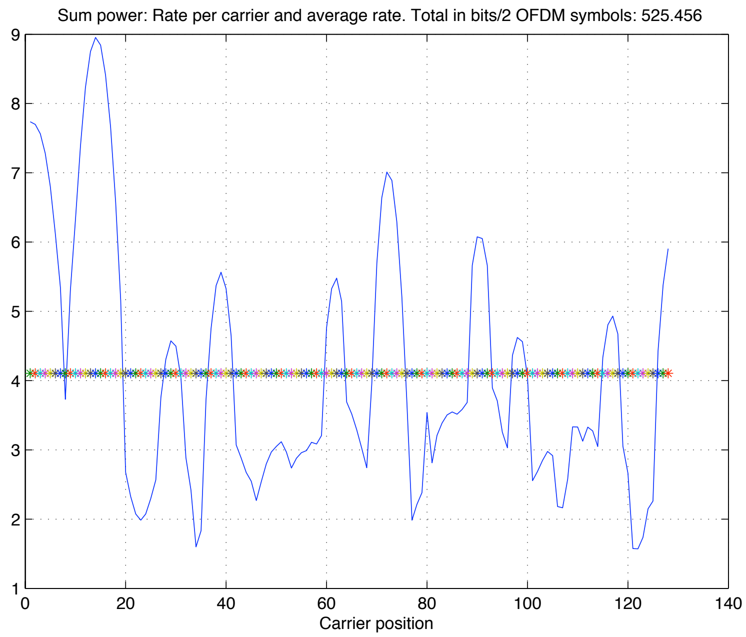

Figure 4: Rate for each carrier and average rate in the case of a sum power constraint after the reallocation step for the optimum approach.

$\{P_k\}_{k=1}^K$ are the powers of source-signals, and $\sigma^2_{\mathrm{d},k}$ is the noise power at kth destination. Note that in (34), $\mathbf{h}_{k,i} \in \mathscr{C}^{N_{\mathrm{r},i}\times 1}$ and $\mathbf{g}_{i,k} \in \mathscr{C}^{N_{\mathrm{r},i}\times 1}$ represent the vector channels between kth source and ith MIMO relay, and ith relay and kth destination, respectively. In (35), $\tilde{\mathbf{A}}_i$ is $N_{\mathrm{r},i} \times N_{\mathrm{r},i}$ identity matrix and all $\{\tilde{\mathbf{A}}_j\}_{j=1,j\neq i}^R$ are all-zero matrices of sizes $N_{\mathrm{r},i} \times N_{\mathrm{r},i}$, $\sigma^2_{\mathrm{r},i}$ is the noise power at ith relay and the symbol $\otimes$ stands for Kronecker product. The transmit power of the relay in terms of $\mathbf{z}_{\mathrm{L}}$

$$P_{\mathrm{r},i} = \sum_{k=1}^K P_k \mathbf{z}_{\mathrm{L}}^H \mathbf{C}_{i,k} \mathbf{z}_{\mathrm{L}} + \sigma^2_{\mathrm{r},i} \mathbf{z}_{\mathrm{L}}^H \mathbf{A}_i^H \mathbf{A}_i \mathbf{z}_{\mathrm{L}}, \text{ where } \mathbf{C}_{i,k} = \mathbf{A}_i^H (\tilde{\mathbf{H}}_{k,i}^T \otimes \mathbf{I}_{N_{\mathrm{r},i}}) \mathbf{A}_i, \text{ and } \tilde{\mathbf{H}}_{k,i} = \mathbf{h}_{k,i}\mathbf{h}_{k,i}^H. \tag{36}$$

2.2.2 Optimization methods

Max-min Approach: The objective of Max-min approach is to maximize the minimum of the SINRs $\{\gamma_k\}_{k=1}^K$ under the condition that the power of each MIMO relay does not exceed its predetermined power budget. This optimization problem can be expressed as

$$\max_{\mathbf{z}_{\mathrm{L}}} \min_{\{k\}_{k=1}^K} \gamma_k \text{ s. t.}$$
$$P_{\mathrm{r},i} \leq P_i^{\mathrm{R}},\ i \in \{1,\cdots,R\} \tag{37}$$

where $\{P_i^{\mathrm{R}}\}_{r=1}^R$ are the powers available for all relays. Using the semidefinite relaxation (SDR) technique, the bisection algorithm, and the randomization method, we solve this problem in terms of $\tilde{\mathbf{Z}} = \mathbf{z}_{\mathrm{L}}\mathbf{z}_{\mathrm{L}}^H$ [CV09a].

QoS Approach: In this method, we minimize the sum of the powers of MIMO relays such that the SINRs for all destinations are maintained above a predetermined threshold value γ_{th}. This problem can be formulated as

$$\min_{\mathbf{z}_{\mathrm{L}}} \sum_{i=1}^R P_{\mathrm{r},i} \text{ s. t.}$$
$$\gamma_k \geq \gamma_{\mathrm{th}},\ k \in \{1,\cdots,K\} \tag{38}$$

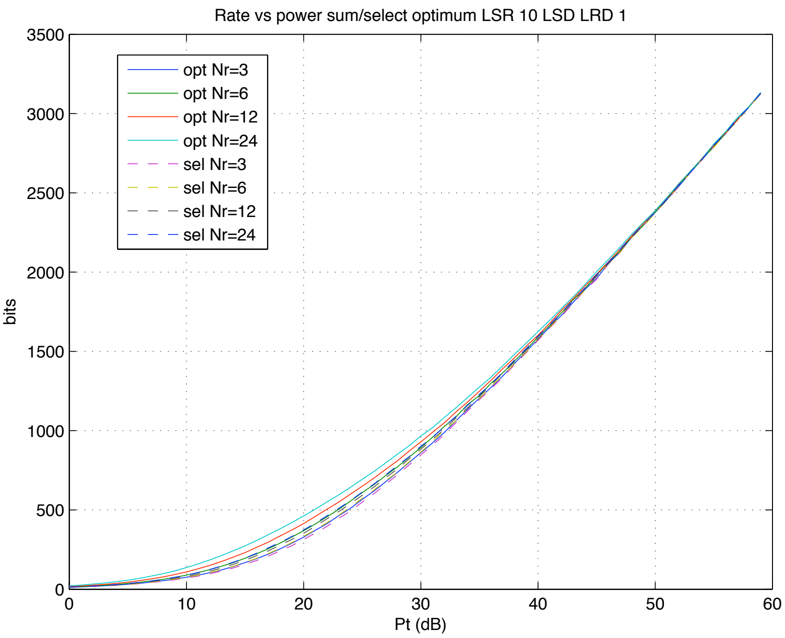

Figure 5: Rate for the optimum method and the suboptimum selection based approach vs $10\log(P_t/\sigma_n^2)$. The number of relays is $N_r = 3,6,12,24$. Tap variances for $\lambda_{r_id}(k)$ and $\lambda_{sd}(k)$: $= 1$. Tap variances for $\lambda_{sr_i}(k)$: 10

Applying the SDR and randomization techniques, the above QoS problem is solved in terms of $\tilde{\mathbf{Z}}$.

Minimize interference plus noise power: We minimize the interference and noise power received at each destination while preserving its desired signal response and maintaining the power constraint of each MIMO relay. This optimization problem can be expressed as

$$\begin{aligned}
&\min_{\mathbf{z}_\mathrm{L},\{t_k\}_{k=1}^K} \sum_{k=1}^K t_k \;\; \text{s. t.} \\
&\mathbf{z}_\mathrm{L}^H \mathbf{Q}_k \mathbf{z}_\mathrm{L} \le t_k, \;\; k \in \{1,\cdots,K\} \\
&\mathbf{z}_\mathrm{L}^H \mathbf{E}_i \mathbf{z}_\mathrm{L} \le P_i^\mathrm{R}, \;\; i \in \{1,\cdots,R\} \\
&\sqrt{P_k}\mathbf{a}_{k,k}^H \mathbf{z}_\mathrm{L} = f_k, \;\; k \in \{1,\cdots,K\}
\end{aligned} \tag{39}$$

where $\mathbf{Q}_k = \sum_{j=1,j\neq k} P_j \mathbf{a}_{k,j}\mathbf{a}_{k,j}^H + \mathbf{D}_k$, $\mathbf{E}_i = \sum_{k=1}^K P_k \mathbf{C}_{i,k} + \sigma_{\mathrm{r},i}^2 \mathbf{A}_i^H \mathbf{A}_i$ and f_k is the scalar (assumed to be known) used for keeping the desired signal response to the kth destination constant. This problem is solved using the SOCP method. Note that if only the desired signal power $|f_k|^2$ is known instead of f_k, then the last constraints in (39) will be $P_k|\mathbf{a}_{k,k}^H\mathbf{z}_\mathrm{L}|^2 = |f_k|^2, \forall k$ which are non-convex. In this case, the optimization problem can be only approximately solved by changing the latter constraints into $\mathscr{Re}\left\{\sqrt{P}_k \mathbf{a}_{k,k}^H \mathbf{z}_\mathrm{L}\right\} = |f_k|, \forall k$.

2.2.3 Numerical results

For numerical simulations, we take $K = 3$, $R = 2$, $\{N_{\mathrm{r,i}}\}_{i=1}^R$=3, $\left\{\sigma_{\mathrm{r,i}}^2\right\}_{i=1}^R = \sigma_\mathrm{r}^2$ and $\left\{\sigma_{\mathrm{d,k}}^2\right\}_{k=1}^K = \sigma_\mathrm{d}^2$, and $\{P_k\}_{k=1}^K = 1$. All channel coefficients are zero-mean complex Gaussian distributed with the unit variance. As a benchmark, we also include the performance of ZF-MIMO relays that are given by $\mathbf{Z}_i = \alpha_i \mathbf{G}_i[\mathbf{G}_i^H\mathbf{G}_i]^{-1}[\mathbf{H}_i^H\mathbf{H}_i]^{-1}\mathbf{H}_i^H, \forall i$, where $\mathbf{G}_i = [\mathbf{g}_{i,1},\cdots \mathbf{g}_{i,K}]$ and $\mathbf{H}_i = [\mathbf{h}_{1,i},\cdots \mathbf{h}_{K,i}]$ are $N_{\mathrm{r},i} \times K$ matrices and α_i is the scaling factor that needs to be properly selected for satisfying the power constraint of ith MIMO

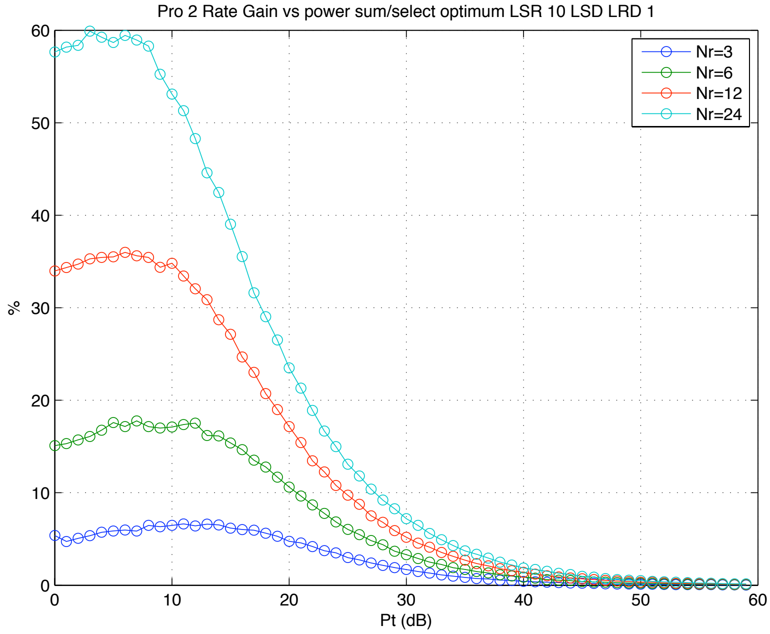

Figure 6: Bit rate gain (in %) of the optimum method compared to the relay selection method vs $10\log(P_t/\sigma_n^2)$. The number of relays is $N_r = 3, 6, 12, 24$. Tap variances for $\lambda_{r_id}(k)$ and $\lambda_{sd}(k)$: $= 1$. Tap variances for $\lambda_{sr_i}(k)$: 10

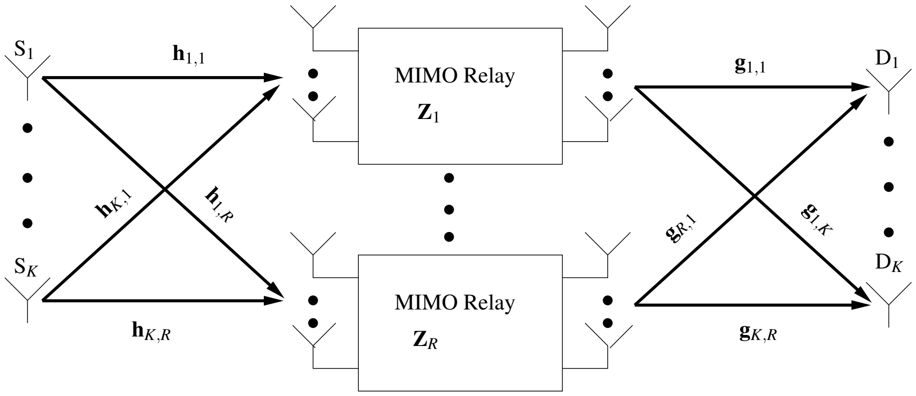

Figure 7: MIMO relays for multiuser cooperative communications (S_k sources and D_k destinations, $\forall k$).

relay. All simulation results are obtained by averaging over 1000 independent channel realizations. It can be observed from Fig. 8 that the proposed methods perform significantly better than that of the ZF-MIMO relays while the Max-min approach outperforms (39) for smaller σ_r^2. The average powers of all MIMO relays and their sum are shown in Fig. 9 for different γ_{th} using the proposed QoS approach. We take $\sigma_{\text{r}}^2 = -20$ dBw. Here, no randomization technique is required since the optimal $\tilde{\mathbf{Z}}$ are found to be always rank-one. It can be seen from Fig. 9 that the relays require more power when the QoS level and the destination noise power increase.

2.3 Performance analysis of linear receivers in MIMO relay channels

We investigate the performance of linear receivers in a cooperative system which consists of a source, a decode-and-forward (DF) relay and a destination that are all multi-antenna nodes. The relay uses zero-forcing (ZF) equalization whereas the destination employs maximum ratio combining (MRC) as well as

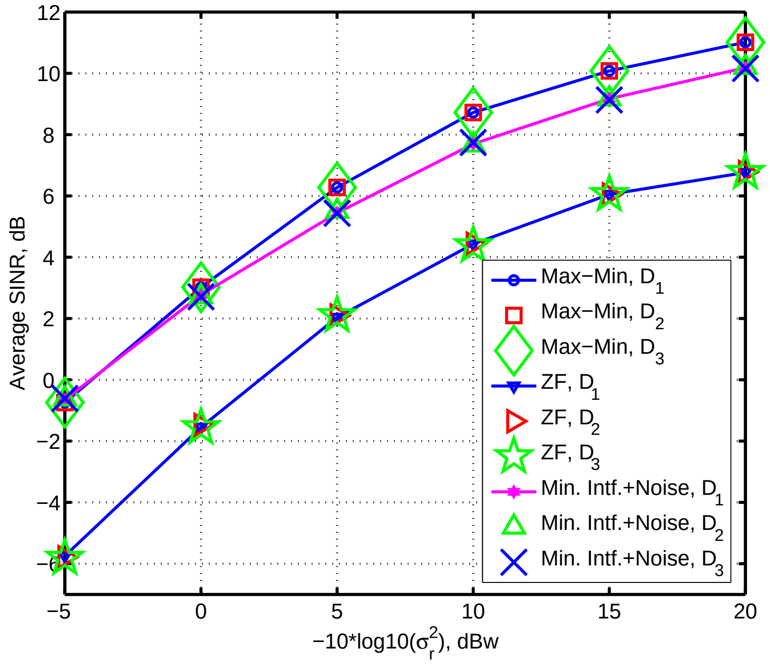

Figure 8: SINR for all destinations versus relay noise power ($\sigma_{\mathrm{d}}^2 = -5$ dBW).

ZF techniques. Considering that perfect channel state information is available at the relay and destination, and the fading is Rayleigh, we derive a closed-form approximate expression for the outage probability of the post-receiver signal-to-noise ratio (SNR) of each data stream at the destination and analyze the diversity order. The validity of the outage probability expression is confirmed with the numerical results.

2.3.1 System description

We study the decode-and-forward (DF) cooperative MIMO relay system as shown in Fig. 10. The source, relay and destination have n_s, n_{r} and n_d antennas, respectively. We consider flat fading spatially uncorrelated Rayleigh MIMO channels where $\mathbf{H}_0 \in \mathscr{C}^{n_d \times n_s}$, $\mathbf{H}_1 \in \mathscr{C}^{n_d \times n_r}$, and $\mathbf{H}_2 \in \mathscr{C}^{n_r \times n_s}$ represent the source-destination, relay-destination, and source-relay channels, respectively. The entries of $\mathbf{H}_0$, $\mathbf{H}_1$ and $\mathbf{H}_2$ are assumed to be independent, zero-mean circularly symmetric complex Gaussian (ZMCSCG) random variables with unit variance. We also assume that the shadowing is negligible by considering the scenario where there are no big obstacles in the vicinity of the source, relay and destination. It is considered that the communication from source to destination, source to relay and relay to destination takes place in two time-slots. In the first time-slot, the source sends its spatially multiplexed (SM) signal to the relay and the destination. During the same time-slot, the relay uses ZF equalizer and decodes the signal from the source. It is assumed that the relay transmits the decoded signal in the second time-

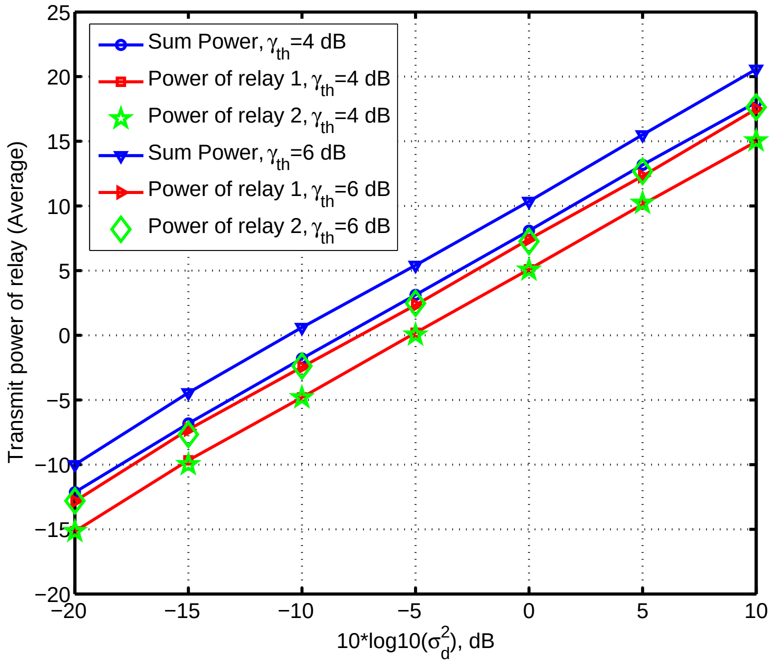

Figure 9: Powers of the relays versus σ_d^2 for the QoS approach.

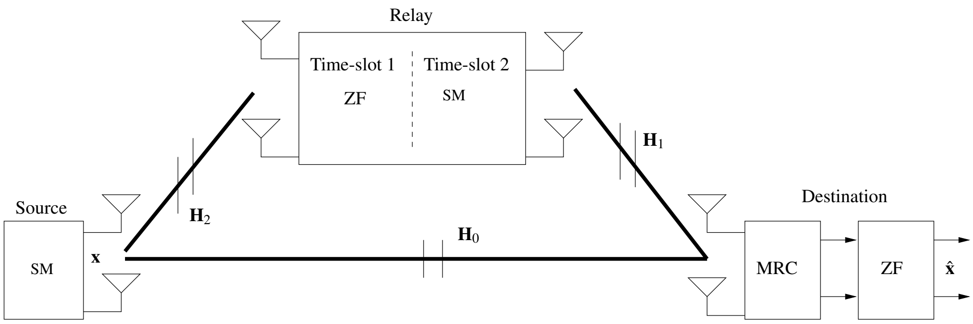

Figure 10: MIMO relay cooperative system with linear receivers.

slot if the relay can correctly decode all source symbols otherwise the relay remains idle. Hence, in our protocol, the instantaneous SNR for each source symbol at the relay is compared with a certain predefined threshold value γ_0 and the random binary integer B_R is assigned to the relay for determining whether the relay transmits the source symbols or remains idle.

2.3.2 *Performance analysis*

The SNR for ith symbol at the destination when the relay is active ($B_{\mathrm{R}} = 1$) is given by [CV09b]

$$\gamma_i^{\mathrm{D}}|_{B_{\mathrm{R}}=1} = \frac{P_{\mathrm{T}}}{2n_s\sigma_{n_{\mathrm{d}}}^2}\frac{1}{[(c_0\mathbf{H}_0^H\mathbf{H}_0 + c_1\mathbf{H}_1^H\mathbf{H}_1)^{-1}]_{i,i}}. \tag{40}$$

where P_{T} is the total power available for the network, n_s is the number of antennas at source node, $\sigma_{n_{\mathrm{d}}}^2$ is the destination noise power, and c_0, and c_1 are the distance-dependent power transfer factors for source-destination and relay-destination channels, respectively. Similarly, the SNR for ith symbol at the destination when the relay is idle ($B_{\mathrm{R}} = 0$) is given by [CV09b]

$$\gamma_i^{\mathrm{D}}|_{B_{\mathrm{R}}=0} = \frac{P_{\mathrm{T}}}{2n_s\sigma_{n_{\mathrm{d}}}^2}\frac{1}{[(c_0\mathbf{H}_0^H\mathbf{H}_0)^{-1}]_{i,i}}. \tag{41}$$

The performance measure such as the outage probability of ith source-stream is thus given by [CV09b]

$$\Pr\left\{\gamma_i^{\mathrm{D}} \le \gamma_0\right\} = \Pr\left\{\gamma_i^{\mathrm{D}}|_{B_{\mathrm{R}}=0} \le \gamma_0\right\}\Pr\{B_{\mathrm{R}}=0\} + \Pr\left\{\gamma_i^{\mathrm{D}}|_{B_{\mathrm{R}}=1} \le \gamma_0\right\}\Pr\{B_{\mathrm{R}}=1\} \tag{42}$$

In (42), we have

$$\Pr\{B_{\mathrm{R}}=1\} = \Pr\left\{\left\{\gamma_i^{\mathrm{R}}\right\}_{i=1}^{n_s} \ge \gamma_0\right\} = \Pr\left\{\frac{P_{\mathrm{T}}}{2\sigma_{n_{\mathrm{r}}}^2 n_s}\frac{1}{[(c_2\mathbf{H}_2^H\mathbf{H}_2)^{-1}]_{i,i}} \ge \gamma_0, \forall i \in \{1, n_s\}\right\} \tag{43}$$

where c_2 is the distance dependent power transfer factor of source-relay channel. It has been shown in [CV09b], that the outage probability for ith source signal can be approximated as

$$\begin{aligned}\Pr\left\{\gamma_i^{\mathrm{D}} \le \gamma_0\right\} &= \frac{\gamma\left(n_d - n_s + 1, \frac{\gamma_0}{\rho_2}\right)}{\Gamma(n_d - n_s + 1)}\left(1 - \mathrm{e}^{-\frac{n_s\gamma_0}{\rho_1}}\right) + \frac{\gamma\left(\tilde{n} - n_s + 1, \frac{\gamma_0 b}{\rho_3}\right)}{\Gamma(\tilde{n} - n_s + 1)}\mathrm{e}^{-\frac{n_s\gamma_0}{\rho_1}}, \text{where} \\ \tilde{n} &= \left[1 + \frac{2c_0c_1}{c_0^2 + c_1^2}\right]n_d,\ b = \frac{c_0^2 + c_1^2}{c_0 + c_1},\ \rho_1 \triangleq \frac{P_{\mathrm{T}}c_2}{2n_s\sigma_{n_{\mathrm{r}}}^2},\ \rho_2 \triangleq \frac{P_{\mathrm{T}}c_0}{2n_s\sigma_{n_{\mathrm{d}}}^2}, \rho_3 \triangleq \frac{P_{\mathrm{T}}}{2n_s\sigma_{n_{\mathrm{d}}}^2}.\end{aligned} \tag{44}$$

where $\gamma(a,x)$ is the lower-incomplete Gamma function and $\Gamma(\cdot)$ is the Gamma function. In this case, the asymptotic analysis (assuming $\sigma_{n_{\mathrm{d}}}^2 = \sigma_{n_{\mathrm{r}}}^2 = \sigma_n^2$) is given by [CV09b]

$$\begin{aligned}&\frac{n_s\gamma_0^{n_d - n_s + 1 + 1}\rho_0^{-[(n_d - n_s + 1)+1]}}{c_0^{n_d - n_s + 1}c_2\Gamma(n_d - n_s + 1)(n_d - n_s + 1)} + \frac{(\gamma_0 b)^{\tilde{n} - n_s + 1}}{\Gamma(\tilde{n} - n_s + 1)}\frac{\rho_0^{-(\tilde{n} - n_s + 1)}}{(\tilde{n} - n_s + 1)} - \\ &\frac{n_s\gamma_0^{\tilde{n} - n_s + 1 + 1}b^{\tilde{n} - n_s + 1}\rho_0^{-[(\tilde{n} - n_s + 1)+1]}}{c_2(\tilde{n} - n_s + 1)\Gamma(\tilde{n} - n_s + 1)}\end{aligned} \tag{45}$$

where $\rho_1 \triangleq \rho_0 c_2$, $\rho_2 \triangleq \rho_0 c_0$ and $\rho_3 \triangleq \rho_0 = \frac{P_{\mathrm{T}}}{2n_s\sigma_n^2}$. The outage probability at high SNR (ρ_0) is dominated by the first two terms. Therefore, the diversity order is given by $\min(n_d + 1, \tilde{n}) - n_s + 1$.

2.3.3 *Numerical results*

Monte-Carlo simulations are used to assess the accuracy of the closed-form expression (44). We take $n_s = n_r = 2$, $c_0 = 0.8$, $c_1 = 0.9$, $c_2 = 0.95$, $\gamma_0 = 5$ dB and $P_T = 3$ dBw. Fig. 11 compares the outage probability obtained via simulations and theoretical evaluation versus ρ_0 for cooperative scheme with different numbers of antennas at the destination. As a benchmark, we also show the outage probability for non-cooperative scheme where the destination receives the signal only from the source. It can also be observed from this figure that the cooperative scheme outperforms the non-cooperative scheme in terms of diversity and the performance of the former method improves with increasing n_d. Furthermore, Fig. 11 demonstrates a fine agreement between the theoretical and numerical results, and verifies the validity of outage probability analysis.

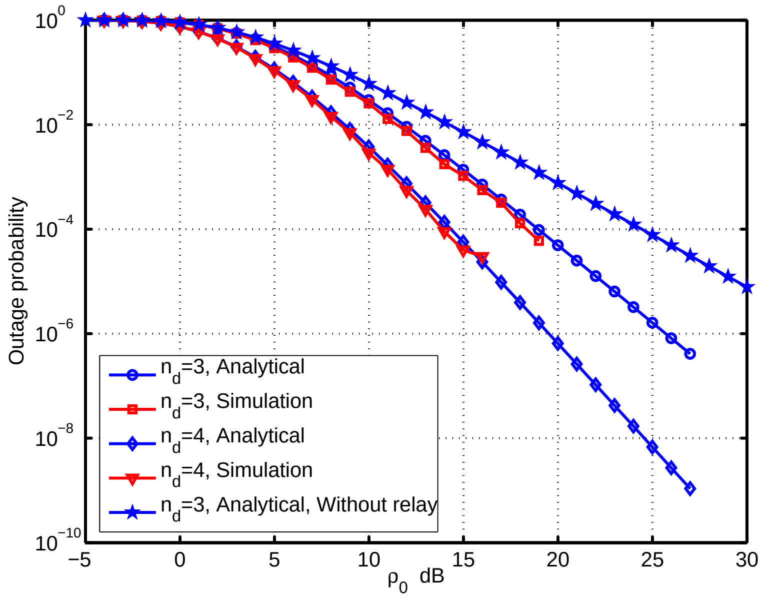

Figure 11: Comparison of analytical and simulated outage probabilities.

2.4 Selective source beamforming

We investigate the performance of selection diversity for a relay system that consists of a source, an amplify-and-forward (AF) relay and a destination, which are all multi-antenna nodes. The beamforming technique is applied at the source and destination and the linear processing operator of AF-MIMO is optimized. The source matches its beamforming vector either to the source-relay channel or the source-destination channel depending on the channel conditions of direct link and the link via relay. Considering that perfect channel state information is available at the source, relay and destination, and the fading is Rayleigh, we derive a closed-form approximate expression for the outage probability of the post-receiver signal-to-noise ratio (SNR) at the destination and analyze the diversity order. The validity of the outage probability expression is confirmed with the numerical results.

2.4.1 System model and protocol description

We study the cooperative MIMO relay system as shown in Fig. 12. In the first-time slot, if the source finds that the source-destination link is stronger than the link via relay, the source matches its beamforming vector $\mathbf{w}_s \in \mathscr{C}^{n_s \times 1}$ to the source-destination channel and also asks the relay to be silent. However, if the source observes that the link via relay is stronger than the source-destination link, the source matches its beamformer $\mathbf{w}_s$ to the source-relay link while informing the destination to be silent during the first-time slot. The relay processes its received signal and transmits the resulting signal to the destination in the second-time slot. In this way, the destination receives signals either from the direct link or the link via relay depending on the corresponding channel conditions. Thus, at any instant of signal transmission from source to destination either the relay is active or idle. Let B_R denotes the state of the relay which is

1 if the relay is active and 0 if the relay is idle.

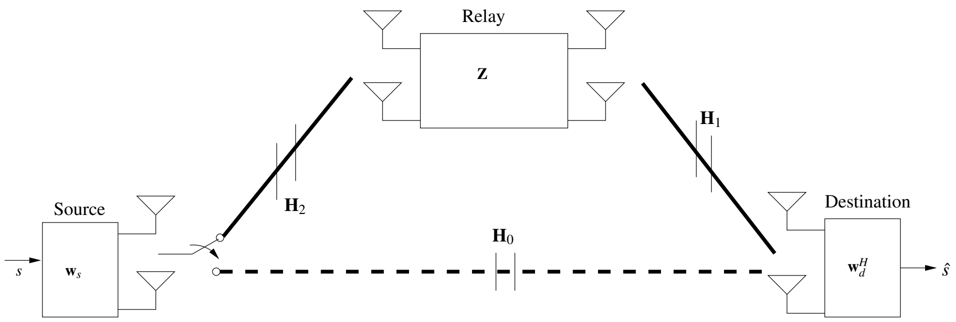

Figure 12: Selective source beamforming for AF-MIMO relay channel.

2.4.2 *Performance analysis*

It can be easily shown that for the optimized relay $\mathbf{Z} \in \mathscr{C}^{n_r \times n_r}$, and the optimal source and destination beamformers $\mathbf{w}_s$ and $\mathbf{w}_d$, respectively, the destination SNR for the case $B_{\mathrm{R}} = 1$ can be expressed as [CV09c]

$$\gamma^o|_{B_{\mathrm{R}}=1} = \frac{P_{\mathrm{S}} P_{\mathrm{R}} \alpha_1 \alpha_2}{P_{\mathrm{R}} \sigma_{n_{\mathrm{r}}}^2 \alpha_1 + P_{\mathrm{S}} \sigma_{n_{\mathrm{d}}}^2 \alpha_2 + \sigma_{n_{\mathrm{d}}}^2 \sigma_{n_{\mathrm{r}}}^2} \triangleq \frac{c\alpha_1\alpha_2}{e\alpha_1 + f\alpha_2 + 1} \tag{46}$$

where $\alpha_1 = |\sigma_{h_1}^1|^2$ and $\alpha_2 = |\sigma_{h_2}^1|^2$ are the largest eigenvalues of $\mathbf{H}_1^H \mathbf{H}_1$ and $\mathbf{H}_2 \mathbf{H}_2^H$, respectively, $c \triangleq \frac{P_{\mathrm{S}}}{\sigma_{n_{\mathrm{r}}}^2} \frac{P_{\mathrm{R}}}{\sigma_{n_{\mathrm{d}}}^2}$, $e = \frac{P_{\mathrm{R}}}{\sigma_{n_{\mathrm{d}}}^2}$ and $f = \frac{P_{\mathrm{S}}}{\sigma_{n_{\mathrm{r}}}^2}$. Note that, here, P_{R} and P_{S} are the powers allocated to the source and relay, respectively. Similarly, the optimal destination SNR when $B_{\mathrm{R}} = 0$ is given by [CV09c]

$$\gamma^o|_{B_{\mathrm{R}}=0} = \frac{P_{\mathrm{s}} \alpha_0}{\sigma_{n_{\mathrm{d}}}^2} \tag{47}$$

where $\alpha_0 = |\sigma_{h_0}^1|^2$ is the largest eigenvalue of $\mathbf{H}_0^H \mathbf{H}_0$. The outage probability of destination SNR is given by [CV09c]

$$\Pr\{\gamma^o \leq \gamma_0\} \;=\; \Pr\left\{\gamma^o|_{\mathbf{w}_s=\mathbf{v}_{h_0}^1} \leq \gamma_0\right\} \Pr\left\{\mathbf{w}_s = \mathbf{v}_{h_0}^1\right\} + \Pr\left\{\gamma^o|_{\mathbf{w}_s=\mathbf{v}_{h_2}^1} \leq \gamma_0\right\} \Pr\left\{\mathbf{w}_s = \mathbf{v}_{h_2}^1\right\} \tag{48}$$

where $\mathbf{v}_{h_0}^1$ and $\mathbf{v}_{h_2}^1$ are the right singular vectors corresponding to the largest singular values of the singular value decompositions (SVDs) of $\mathbf{H}_0$ and $\mathbf{H}_2$, respectively, and the conditions $\mathbf{w}_s = \mathbf{v}_{h_2}^1$ and $\mathbf{w}_s = \mathbf{v}_{h_0}^1$ are equivalent to $B_{\mathrm{R}} = 1$ and $B_{\mathrm{R}} = 0$, respectively. We use the following criteria for evaluating $\Pr\left\{\mathbf{w}_s = \mathbf{v}_{h_2}^1\right\}$:

- $\Pr\{\alpha_2 \geq \alpha_0\}$
- $\Pr\{\alpha_2\alpha_1 \geq \alpha_0\}$
- $\Pr\left\{\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2} \geq \alpha_0\right\}$
- $\Pr\left\{\frac{\alpha_1+\alpha_2}{2} \geq \alpha_0\right\}$

The first criterion does not need the knowledge of relay-destination channel at the source whereas the last three criteria require that knowledge. We obtain closed-form solutions for the above mentioned criteria as well as the outage probabilities for (46) and (47) to evaluate the overall outage probability (48). The diversity order has been analyzed, and it has been found that the diversity order is given by $\min(n_d n_s, \min(n_d, n_s) n_r)$ [CV09c].

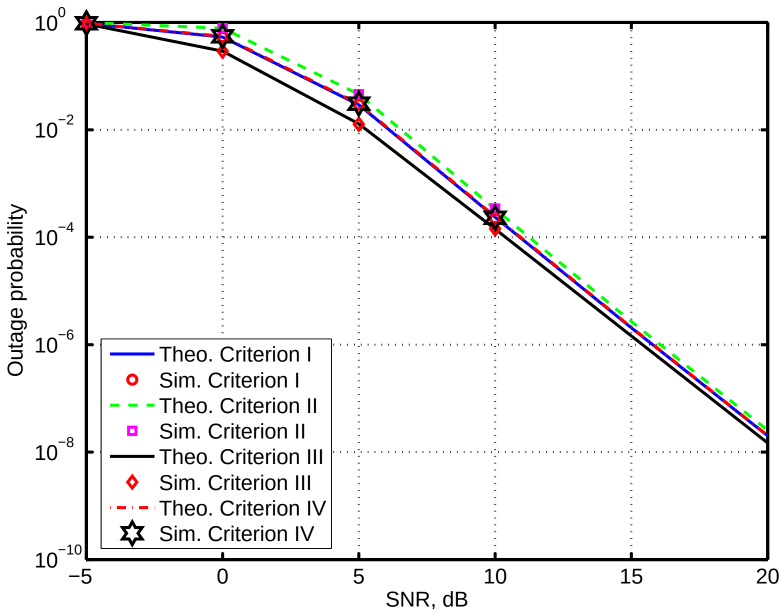

Figure 13: Comparison of analytical and simulated outage probabilities ($n_s = n_r = n_d = 2$).

2.4.3 Numerical results

With the help of Monte-Carlo simulations, the validity of the analytical expression (48) is verified. The analytical and simulated outage probabilities are compared in Figs. 13 and 14 with $n_s = n_r = n_d = 2$ and $n_s = n_r = 2, n_d = 3$. The channel coefficients of $\mathbf{H}_0$, $\mathbf{H}_1$ and $\mathbf{H}_2$ are all zero-mean complex Gaussian random variables with unit variance. It can be observed from these figures that there is fine agreement between the theoretical and simulated results. The fact that the diversity order obtained by the system is given by $\min(n_d n_s, \min(n_d, n_s) n_r)$ is validated by these results. It is interesting to note that the diversity order is same for all the selection criteria which suggests that even by applying the simplest method (Criterion I), which does not need the knowledge of relay-destination channel, the diversity order of the system can be achieved.

2.5 Full-diversity LDPC Codes for slow fading Relay Channels

Cooperative Communications yields significant gains in slow fading environments. Consider a simple relay channel with one source (S), one relay (R) and one destination (D). All channels have a single intrinsic diversity order, because the distribution of the fading gain is different from zero around the origin, the channel is frequency non-selective and devices do not use more than one antenna [TV05]. In the following, we will denote these channels as block-fading (BF) channels [BPS98]. Using cooperative communications, a double diversity order can be obtained, resulting in a high performance gain in terms of

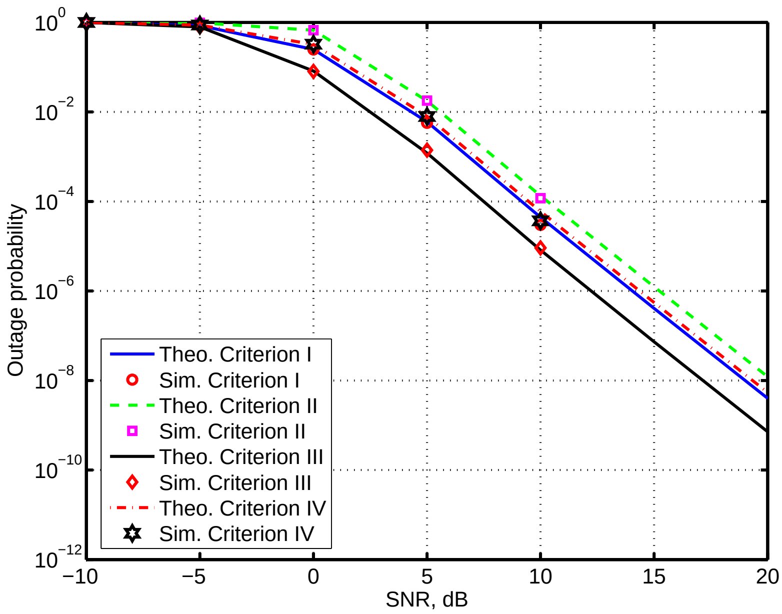

Figure 14: Comparison of analytical and simulated outage probabilities ($n_s = n_r = 2$, $n_d = 3$).

signal-to-noise ratio (SNR) wrt. a given error rate. Furthermore, DF automatically achieves this diversity order, without the need for any special code structure. The reason is that R is repeating the transmission from S (in the case of successful decoding of the message from S at R), and this "repetition code" achieves full-diversity. But there is also its weakness. From a coding point of view, a repetition code is a weak code, in terms of coding gain. Therefore, coded cooperation [Hun04], a variant of DF with a smaller outage probability than the latter, has been introduced. In coded cooperation, R does not simply repeat the transmission from S, but sends additional parity bits, related to the message of S. Unfortunately, a practical implementation of coded cooperation does not automatically achieve full-diversity. Considering algebraic codes and ML-decoding, it is not too difficult to obtain the intrinsic diversity, offered by the relay channel. When using capacity achieving codes, such as low-density parity-check (LDPC) codes [RU08], and their iterative decoding, codes have to be carefully designed to achieve full-diversity [DBM09].

An error correcting code is full-diversity if the diversity order d is equal to L, where L is the number of fading instances per codeword, i.e., the intrinsic channel diversity. In the case of the relay channel, the overall code has $L = 2$ fading instances. According to the blockwise Singleton bound [KH00], the coding rate for a L-order full-diversity code is upper bounded by $R_{cmax} = 1/L$. Hence, for a relay channel we get $R_c \leq 0.5$. An excellent error-correcting code is a code whose performance is close to the outage probability. Therefore, it must fulfill two criteria:

1- full-diversity, i.e., the slope of the word error rate (WER) is the same as the slope of the outage probability at $\gamma \to \infty$;

2- minimizing the gap between the outage probability and the WER performance at high SNR.
The criteria are given in order of importance. The first criterion is independent of the degree distributions of the code [BGiFBZ07], hence serves to construct the skeleton of the code. It guarantees that the gap between the outage probability and the WER performance is not increasing at high SNR. The second criterion serves to select the appropriate degree distributions. In this contribution, the most attention goes to the first criterion. Via techniques from [Bou09], the coding gain can be improved, which is subject of future work.

For the BF relay channel, there was a lack of a near-outage LDPC code. Hu et al. [HD07] obtained interesting results by designing random LDPC codes for cooperation over fading channels, but they did not tune the parity-check matrix in order to achieve full-diversity. Considering diversity, we show the link between a BF relay channel and a BF point-to-point channel. Error-correcting codes designed for BF point-to-point channels can be inserted as a constituent code in the overall code for BF relay channels. The other constituent code provides the rate-compatibility property, necessary for cooperative communications, because R must be able to decode after receiving only the first part of the codeword (hence a higher-rate codeword). WER performance is determined for infinite length codes through density evolution (DE) as well as for finite length codes. We show that our code construction exhibits near-outage limit performance for all block lengths and for a range of coding rates up to 0.5, which is the highest possible coding rate for two cooperating users.

2.5.1 System model and notation

The most elementary example of a cooperative network is the relay channel, introduced by van der Meulen [Meu71]. Channel-State Information (CSI) is assumed at the decoder. We consider half-duplex devices, assuming that simultaneously receiving and transmitting data in the same frequency-band is too complicated due to the limited isolation of directional couplers. In addition, we also restrict the protocol to be orthogonal (S and R transmit in different timeslots) for simplicity.

The transmission of a codeword is organized in two frames which form together one block. Therefore, a codeword C from S is split in two parts: C_1 and C_2. In the first frame of a block, S broadcasts C_1 to R and D. In the second frame, R cooperates and sends C_2 if it is able to decode the transmission of S in the first frame. Hence, two cases are distinguished, as shown in Fig. 15, depending on whether R is able to decode the message from S.

	Frame 1	Frame 2
Source	C_1	
Relay		C_2

	Frame 1	Frame 2
Source	C_1	
Relay		

Figure 15: Both cases encountered in coded cooperation on a relay channel. On the left, the S-R transmission has been decoded successfully. R cooperates in the second frame. On the right, the S-R transmission was not successful.

A codeword will consequently be split over 2 frames. The first part of the codeword has length N_1 and the second part of the codeword has length N_2. We consider codewords to have a total length equal to N binary digits, so that $N = N_1 + N_2$. We define the level of cooperation, β, as the ratio N_2/N.

The S-R, S-D, and R-D channels are modeled as memoryless with additive white Gaussian noise and multiplicative real-valued fading which is Rayleigh distributed. The fading gains (α_{sd} for the S-D channel, α_{rd} for the R-D channel, and α_{sr} for the S-R channel) are constant during 2 frames and independent from block to block due to interleaving. This means that in the left case considered in Fig. 15, C_1 and C_2 are received at D with constant but independent fading gains.

We focus on binary LDPC codes $\mathscr{C}[N,K]_2$ with block length N and dimension K, and coding rate $R_c = K/N$. The code $\mathscr{C}$ is defined by an $(N-K) \times N$ parity-check matrix H, or equivalently, by the corresponding Tanner graph [RU08]. Regular (d_b, d_c) LDPC codes have a parity-check matrix with d_b

number of ones in each column and d_c number of ones in each row. For irregular $(\lambda(x), \rho(x))$ LDPC codes, these numbers are replaced by the so-called degree distributions [RU08]. For simplicity, we only use regular LDPC codes in this contribution.

The average signal-to-noise ratios on the *S-D*, *S-R* and *R-D* channels are the same, and are denoted as $\gamma = \frac{E_b}{N_0} = \frac{1}{2R_c\sigma^2}$.

2.5.2 *Full-diversity on Relay Channels*

When only considering diversity, and making abstraction of coding gain for the moment, designing good codes becomes much easier. In this section we will show that is sufficient to assume an errorless *S-R* channel and furthermore, block Rayleigh fading channels can be reduced to an extremal case, the block Binary Erasure Channel (block BEC), where the fading gains belong to $\{0, +\infty\}$. This is a surprising result, but very useful.

2.5.2.1 Errorless source-relay channels

In a cooperative protocol, where *R* has to decode the transmission from *S* in the first slot, two cases can be distinguished, see Fig. 15. The decoding error probability (i.e. the WER) at *D*, P_e, can be written as follows:

$$P_e = P(\text{case } 1)P(e|\text{case } 1) + P(\text{case } 2)P(e|\text{case } 2). \tag{49}$$

The probability $P(\text{case } 2)$ is equal to the probability of erroneous decoding at *R*; for large γ we have $P(\text{case } 2) \propto \frac{1}{\gamma}$ [TV05]. The probability $P(e|\text{case } 2)$ is equal to the probability of erroneous decoding on the *S-D* channel; hence, for large γ, $P(e|\text{case } 2) \propto \frac{1}{\gamma}$. Now, the error probability P_e is proportional to

$$P_e \propto (1 - \frac{c}{\gamma})P(e|\text{case } 1) + \frac{1}{\gamma^2}, \tag{50}$$

where c is a positive constant. Full-diversity means that $P_e \propto \frac{1}{\gamma^2}$. We see that this only depends on the behavior of $P(e|\text{case } 1)$ at large γ, because the second case where *R* cannot decode the transmission from *S* in the first slot does automatically give rise to a double diversity order without the need for any code structure. This means that as far as the diversity order is concerned, it is sufficient to assume errorless *S-R* channels (yielding $P_e = P(e|\text{case } 1)$) to simplify the analysis.

2.5.2.2 Full-diversity on BF relay channel if and only if full-diversity on BF point-to-point channels

Assuming errorless *S-R* channels, *R* always sends additional parity bits to *D*. It is easy to see the link between a BF relay channel and a BF point-to-point channel with two channel states per codeword. The decoding at *D* boils down to decoding a codeword where the first part C_1 has been faded by α_{sd} and the second part C_2 has been faded by α_{rd}, where α_{sd} and α_{rd} are independent. This is equivalent to a BF point-to-point channel with two independent channel states. Full-diversity LDPC codes on this channel have been studied in [BGiFBZ07].

2.5.2.3 Block BEC channels

In the belief propagation (BP) algorithm, probabilistic messages (log-likelihood ratios) are propagating on the Tanner graph. The behavior of the messages for $\gamma \to \infty$ determines if the diversity order can be achieved [Bou09]. Unfortunately, the BP algorithm is numerical and messages propagating on the graph are analytically intractable. Fortunately, there is another much more simple approach to prove full-diversity. Diversity is defined at $\gamma \to \infty$. In this region the fading can be modeled by a block BEC, an extremal case of block-Rayleigh fading. Full-diversity on the block BEC is a necessary *and sufficient* condition for full-diversity on the block Rayleigh fading channel [BGiFBZ07]. The analysis on a block BEC channel is very simple (bits are erased or perfectly known) but very powerful to check the diversity order of a system.

$$\text{\textit{To have a diversity order } } d=2\text{\textit{, assume that one frame is erased and check whether all information bits can still be recovered.}} \quad (51)$$

This rule will be used in the following of the paper to derive the skeleton of the code.

2.5.3 Coding rate, random codes and full-diversity

In this section, we will derive an upper bound on the coding rate yielding full-diversity. The diversity of a system can be checked by erasing a link and checking whether all information bits can be recovered (see (51)). In this system there exist two links: the *S-D* link which corresponds to N_1 coded bits (linear combinations of the K information bits of S) and the *R-D* link which corresponds to N_2 coded bits (linear combinations of the K information bits of S). The overall coding rate is R_c. let us express N_2 and R_c in function of K, N_1 and β:

$$N_2 = \frac{N_1\beta}{1-\beta} \quad (52)$$

$$R_c = \frac{K(1-\beta)}{N_1} \quad (53)$$

It is clear that error-correcting codes will never be able to decode all the unknowns, i.e., K information bits, if the decoder has less equations than unknowns. Assuming that one link has been erased will give rise to a maximum coding rate such that the number of equations is always bigger than the number of unknowns.

1-Erased R-D link: D must derive K information bits from the codeword of S, hence we get:

$$N_1 \geq K. \quad (54)$$

2-Erased S-D link: D must derive K information bits from the codeword of R, such that $N_2 \geq K$ or via (52)

$$\frac{N_1\beta}{1-\beta} \geq K. \quad (55)$$

Combining (54) and (55) renders $N_1 \geq K\max\left(1, \frac{1-\beta}{\beta}\right)$, such that via (53) we get $R_c \leq \frac{1-\beta}{\max\left(1,\frac{1-\beta}{\beta}\right)}$ or

$$R_c \leq \min(1-\beta, \beta). \quad (56)$$

The maximum achievable coding rate $R_c = 1/2$ can be achieved if $\beta = 1/2$, in which case R and S send K bits each. It can be proven that a random rate-1/2 $(\lambda(x), \rho(x))$ LDPC ensemble cannot achieve full-diversity [Bou09]. Here we propose a semi-random rate-1/2 LDPC ensemble that achieves full-diversity.

2.5.4 Rate-compatible Full-diversity LDPC codes

2.5.4.1 Rootchecks

let us repeat the rule to check if an error-correcting code has full-diversity: assume that one frame is erased and check if all information bits can still be recovered. Applying this to the relay channel leads to two possibilities:

- *S*-*D* channel is erased: $\alpha_{sd} = 0, \alpha_{rd} = \infty$
- *R*-*D* channel is erased: $\alpha_{sd} = \infty, \alpha_{rd} = 0$

Dealing with the last case only is not difficult: let S send its information uncoded and R sends extra parity bits. If D receives the transmission of S without errors, it has all the information bits. So the challenging case is the first one. Let us assume that the bits transmitted by S are filled red and the bits transmitted by R are filled white. Assume all red bits are erased at D. A very simple way to guarantee full-diversity is to connect a red information bit node to a *rootcheck*, which is a special type of check node, where all the leaves have a color, that is different from the color of its root, see Fig. 16. Assigning rootchecks to all the information bits is the key to have full-diversity.

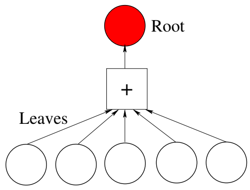

Figure 16: Rootcheck: a guarantee that root can be recovered if the leaves are not erased.

S transmits information bits and parity bits and R transmits information bits and parity bits related to the information of S. The previous description naturally leads to 4 classes of bit nodes. Information bits of S are split into two classes: one class of bits is transmitted on α_{sd} (red) and is denoted as $1i$, the other class is transmitted on α_{rd} (white) and denoted as $2i$; similarly, red and white parity bits are of the classes $1p$ and $2p$ respectively. In the remainder of the contribution, the vectors **1i**, **2i**, **1p**, and **2p** collect the bits of the classes $1i$, $2i$, $1p$, and $2p$ respectively.

Above, we concluded that all information bits should be the root of a rootcheck. The class of rootchecks for **1i** is denoted as $1c$. Translating Fig. 16 to its matrix representation renders:

$$\begin{array}{cccc} 1i & 1p & \{2i,2p\} & \\ \left[\ \mathbf{I} \right. & \mathbf{0} & \left. H_{\text{rest}} \quad\quad \right] & 1c \end{array}$$

The identity[1] concatenated with a matrix of zeros, assures that bits of the class $1i$ are the only red bits connected to check nodes of the class $1c$. Hence, a full-diversity code construction for the relay channel can be formed by assigning this type of rootchecks (introducing the new class $2c$) to all information bits:

$$H = \begin{array}{c} \begin{array}{cccc} 1i & 1p & 2i & 2p \end{array} \\ \left[\begin{array}{cccc} \mathbf{I} & \mathbf{0} & H_{2i} & H_{2p} \\ H_{1i} & H_{1p} & \mathbf{I} & \mathbf{0} \end{array} \right] \end{array} \begin{array}{c} 1c \\ 2c \end{array}.$$

S transmits **1i** and **1p** and R transmits **2i** and **2p**, hence the level of cooperation is $\beta = 0.5$ and the coding rate is $R_c = 0.5$. Because of the rootchecks, this code construction is denoted as a root-LDPC. The reader can easily verify that if only one color is erased, all information bits can be retrieved immediately. This full-diversity code construction has another advantage. If R receives **1i** and **1p** without errors, it can derive **2i** and consequently **2p** (after encoding).

2.5.4.2 Rate-compatibility

The previous paragraph established a code construction which exhibits full-diversity. The analysis assumed an errorless *S*-*R* channel. Now let us go back to a Rayleigh distributed fading gain and additive white Gaussian noise. The first part of a codeword, transmitted during the first frame should protect

[1]Note that the identity matrix can be replaced by a permutation matrix. For the simplicity of the notation, in the rest of this contribution **I** will be used.

information, here **1i** and **1p**, on the noisy *S-R* channel. Consequently, a channel code, compatible with two distinct rates is to be devised. In non-cooperative communications, this property is known as rate-compatibility where parity bits of higher rate codes are embedded in those of lower rate codes [Hag88]. For LDPC codes, two techniques have been used: puncturing and extending [LN02], [HKM04], [YB04]. A fraction of parity bits of a mother code could be punctured to obtain higher rate codes. However, the resulting rate range is limited because the deletion of too many bits has a negative effect on decoding via belief propagation. To obtain a more dynamic range in rates, the technique of extending has been used. The extension is made by adding extra parity bits p_1' as illustrated in eq. (57):

$$H = \begin{array}{c} \\ \\ \end{array}\begin{array}{c} \begin{array}{ccccc} 1i & 1p & p_1' & 2i & 2p \end{array} \\ \left[\begin{array}{ccccc} & H_1 & & \mathbf{0} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} & \mathbf{0} & H_{2i} & H_{2p} \\ H_{1i} & H_{1p} & \mathbf{0} & \mathbf{I} & \mathbf{0} \end{array}\right] \end{array} \begin{array}{c} 1c \\ 2c \\ 3c \end{array}. \tag{57}$$

The overall code is the intersection of two constituent codes defined by H_2 and H_1 padded with zeros on the right. The matrix H_2 is the root-LDPC of the previous paragraph, guaranteeing diversity. H_1 expresses the code of the *S-R* channel. Since R only receives **1i**, **1p**, and $\mathbf{p_1'}$, H_1 can only be defined over these vectors. For simplicity, we only used the technique of extending to acquire rate-compatibility, but this may be further optimized by combining puncturing and extending via known techniques [LN02, HKM04, YB04].

Eq. (57) is a rate-compatible full-diversity LDPC code, but there is a loss in coding gain because **1i** may have have a higher coding gain than **2i** due to the asymmetry of (57) wrt. the information bits. Futhermore, the cooperation level *beta* is now smaller than 0.5, such that the coding rate is limited to $\beta < 0.5$. In a second step, to get a balanced structure, we replace the zero-padded H_1 by the direct sum of two rate R_1 codes defined by H_{1s} and H_{1r} as illustrated by Eq. (58)

$$H = \begin{array}{c} \begin{array}{cccccc} 1i & 1p & p_1' & 2i & 2p & p_2' \end{array} \\ \left[\begin{array}{cccccc} & H_{1s} & & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & & H_{1r} & \\ \mathbf{I} & \mathbf{0} & \mathbf{0} & H_{2i} & H_{2p} & \mathbf{0} \\ H_{1i} & H_{1p} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \end{array}\right] \end{array} \begin{array}{c} 1c \\ 2c \\ 3c \\ 4c \end{array}. \tag{58}$$

Thus, the constituent code H_{1s} protects bits **1i** and **1p** via extra parity bits $\mathbf{p_1'}$. Similarly, in the second frame, extra parity bits $\mathbf{p_2'}$ are generated from **2i** and **2p**. The bottom of the global parity-check matrix simply includes the root-LDPC structure, connecting (**1i**, **1p**) to (**2i**, **2p**). For simplicity we can assume that H_{1s} and H_{1r} belong to the same random rate R_1 LDPC ensemble, defined by the degree distributions $(\lambda_1(x), \rho_1(x))$. Hence, if the degree distribution of the root-LDPC is $(\lambda_2(x), \rho_2(x))$, we refer to the rate-compatible root-LDPC as a $(\lambda_1(x), \rho_1(x), \lambda_2(x), \rho_2(x))$ code. Since we guarantee full-diversity via a root-LDPC with a fixed rate $\frac{1}{2}$, the global coding rate of the rate-compatible root-LDPC code observed at D is $R_c = \frac{R_1}{2}$. As a consequence, the global coding rate R_c can be easily varied through R_1.

2.5.5 Numerical results

We evaluated the asymptotic performance of rate-compatible root-LDPC codes by applying density evolution (DE) [RU08] [BGiFBZ07] on the proposed code construction. To compare with previous work, we simulated the proposed code construction on a cooperative multiple access channel (Cooperative MAC) [LTW04d]. This is a simple superposition of two relay-channels, where two users transmitting data to a single receiver cooperate by alternately being the relay for the other user. The DE trees and DE equations can be found in [DBM09]. In addition, it is interesting to evaluate the finite length performance of the proposed rate-compatible root-LDPC codes. Not only to approve the asymptotic performance, but also to see how to generate an instance of the parity-check matrix, given by Eq. (58). Details of this matrix generation can also be found in [DBM09]. We studied the following scenario:

- The average SNR of the independent interuser channels is 5dB higher than the average SNR on the *S-D* link.
- The average SNR of the *R-D* link is equal to that on the *S-D* link.
- The coding rate is $R_c = \frac{1}{3}$ and the cooperation level is $\beta = 0.5$.

Fig. 17 shows the main results: the word error rate (WER) of a regular $(3,9,3,6)$ rate-compatible root-LDPC code for infinite and finite length.

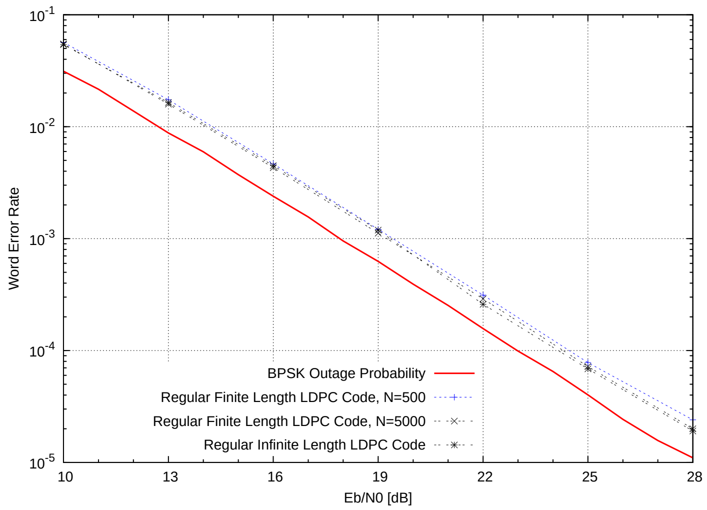

Figure 17: Comparison of rate-compatible root-LDPC codes for different block lengths with iterative decoding on a cooperative MAC for two users, coding rate $R_c = 1/3$. The ratio E_b/N_0 is the average information bit energy-to-noise ratio on the *S-D* link.

As can be seen clearly on the figure, the regular rate-compatible root-LDPC code is always within 1.5dB from the outage probability limit. This distance is respected for many variations of the channel conditions, such as other interuser channel conditions or uplink channel conditions. Note that our code construction can, just as in [HD07], be applied on a full-duplex channel, doubling the overall spectral efficiency. As mentioned before, the coding rate is adjustable by varying the number of parity bits $\mathbf{p_1'}$ and $\mathbf{p_2'}$. In [DBM09], the WER performance of a code with a coding rate $R_c = 0.45$ is illustrated. The coding rate is adjustable as long as $R_{cmax} = 0.5$ is not exceeded.

We will not elaborate on strategies to enhance the coding gain of this code construction by selecting the appropriate degree distributions. Increasing the coding gain boils down to giving full-diversity to the parity bits also [Bou09]. A first step towards good codes is made in [Bou09], but much work still has to be done, and consists of our future research.

2.5.6 Comparison with previous work

Especially rate-compatible punctured convolutional codes (RCPC) have been used in coded cooperation. The main drawback of these codes is that the WER increases with the logarithm of the block length to the power d where d is the diversity order [BCGiF04], [BGiFC05], whereas the WER of near-outage codes should be independent of the block length. This can be seen clearly on Fig. 18, where we show the WER of two rate-compatible non-recursive non-systematic (75,53,47) convolutional codes with block length 500 and 5000 respectively. We used the same channel conditions and coding rate as in the above scenario.

We also compared with another protocol, Decode and Forward (DF), using near-outage LDPC codes for this protocol. Despite the fact that this implementation has near-outage performance, the WER performance is worse than that of our code construction. The reason is that the outage probability limit of DF is higher than that of coded cooperation.

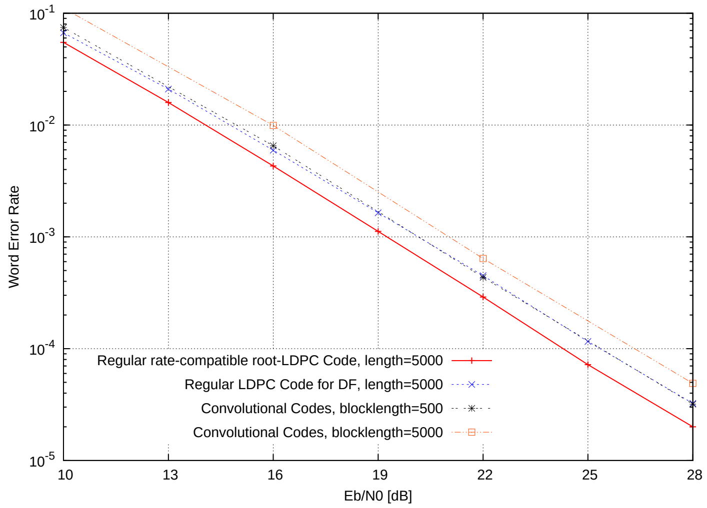

Figure 18: Comparison of rate-compatible root-LDPC codes for coded cooperation with other work on a cooperative MAC for two users. We simulated LDPC codes for Decode and Forward under iterative decoding and an implementation of rate-compatible convolutional codes [Hun04]. The ratio E_b/N_0 is the average information bit energy-to-noise ratio on the *S*-*D* link.

2.5.7 Conclusion

We have studied LDPC codes for relay channels in a slowly varying fading environment under iterative decoding. Random LDPC codes cannot exhibit full-diversity for the maximum achievable coding rate $R_{cmax} = 0.5$ yielding full-diversity. We have introduced the new family of rate-compatible root-LDPC codes, which combines the rate-compatibility property with the full-diversity property for any coding rate $R_c \leq R_{cmax} = \min(\beta, 1-\beta)$, where β is the cooperation level. We have shown that the error rate performance of root-LDPC codes is close to the outage probability limit and this occurs for all block lengths (finite and infinite) and all rates not exceeding R_{cmax}. Its flexibility and high performance makes rate-compatible root-LDPC attractive for wireless cooperative communications scenarios with slowly varying fading.

2.6 A full-diversity joint network-channel code construction for cooperative communications

In the previous section, we constructed a rate-compatible full-diversity LDPC code for the relay channel. This code construction can be extended to as many relays as wanted. However, in some cases two sources share a common relay, which is denoted as a Multiple Access Relay Channel (MARC). If the cooperation level β remains 0.5, than the same code construction as in section 2.5 can be used, but the coding rate will stay limited to $R_c = 0.5$. It is known that network coding [ACLY00] can increase the spectral efficiency in networks. Using network coding, R can send a combination of its incoming bit packets to D (only linear combinations over GF(2) are considered here). From a decoding point of view, this linear combination can be seen as additional parity bits of a linear block code. It can be proven that for a cooperation level $\beta = 1/3$, a double diversity order on the MARC is achievable when the coding rate is smaller than

$R_c = 2/3$. Hence, the problem formulation is how to design a full-diversity joint network-channel code construction for a rate $R_c = 2/3$.

Up till now, no full-diversity capacity-achieving code for the MARC at a coding rate $R_c = 2/3$ has been published. Hausl and Dupraz obtained interesting results on joint network-channel coding for the MARC with turbo codes [HD06] and LDPC codes [HSOB05], but the authors do not have a structure to guarantee full-diversity, which is the most important criterion for a good performance in fading channels. Hence, constructing a full-diversity LDPC code automatically beats previously proposed codes. We present a strategy to produce full-diversity LDPC codes up to a coding rate $R_c = 2/3$. Just as in section 2.5, we will not elaborate on coding gain, which is subject of future research. Simulation of the word error rate (WER) performance of the new proposed family of LDPC codes for the MARC confirms the full-diversity.

Channel-State Information is assumed at the decoder. To simplify the analysis, we consider orthogonal half-duplex devices that transmit in separate time slots. The notation used here is the same as in section 2.5.

2.6.1 System model and notation

We consider a Multiple Access Relay Channel (MARC) with two users S_1 and S_2, a common relay R and a common destination D. Each of the three transmitting devices transmits in a different timeslot: S_1 in timeslot 1, S_2 in timeslot 2 and R in timeslot 3. The three time slots are also called frames. We consider a joint network-channel code over this network: codewords $(c_1, c_2 \ldots c_N)$ to have a total length equal to N binary digits, where S_1 and S_2 transmit N_s bits (note that each user is given an equal time slot because of fairness) and R transmits N_r bits, so that $N = 2N_s + N_r$. We define the level of cooperation, β, as the ratio N_r/N. Because the users do not communicate among each other, the bits $(c_1 \ldots c_{N_s})$, transmitted by S_1, and the bits $(c_{N_s+1} \ldots c_{2N_s})$, transmitted by S_2, are independent.

The S_1-D channel, S_2-D channel, and the R-D channel are modeled as memoryless flat Rayleigh-fading channels with respective (positive) fading gains α_1, α_2 and α_r that are normalized to unit average square. In some parts of the paper, a block binary erasure channel (block BEC) will be assumed. In a block BEC, the fading gains belong to $\{0, \infty\}$. The fading gains are constant during the transmission of one codeword and are independent from codeword to codeword (fading gains are also denoted as channel *states*). Assuming BPSK transmission, the signal received at D can be written as

$$y_l = \begin{cases} \alpha_1 c_l + \eta_l, & \text{if } 1 \leq l \leq N_s \\ \alpha_2 c_l + \eta_l, & \text{if } N_s < l \leq 2N_s \\ \alpha_r c_l + \eta_l, & \text{if } 2N_s < l \leq N \end{cases}, \qquad (59)$$

where $c_l \in \{-1, 1\}$ and η_l denotes real-valued additive white Gaussian noise with variance σ^2.

We assume that no errors occur on the S_1-R and S_2-R channels. This simplifies the analysis and does not change the criteria for the code to attain full-diversity, as shown in section 2.5.2.
We focus on binary LDPC codes $\mathscr{C}[N,K]_2$ with block length N and dimension K, and coding rate $R_c = K/N$, see section 2.5.1.

The average signal-to-noise ratios on the S_1-D, S_2-D and R-D channels are the same, and are denoted as $\gamma = \frac{E_b}{N_0} = \frac{1}{2R_c\sigma^2}$.

2.6.2 Full-diversity on the MARC

An error correcting code is full-diversity if the diversity order d is equal to L, where L is the number of fading instances per codeword, i.e., the intrinsic channel diversity. In the case of the MARC, the overall code has $L = 3$ fading instances, but the information of each user is only sent over two fading instances, because the sources do not listen to each other and transmit only their own information, hence a diversity order $d = 2$ is the highest achievable. Therefore, in the following we will denote a code as a full-diversity code when it achieves $d = 2$.

In section 2.5.2, we derived that full-diversity coding, i.e., a code exhibiting a double diversity order, can be analyzed on a block BEC, namely: assume that one frame is erased and check if all information bits can still be recovered. Furthermore, assuming the S-R-channels to be errorless does not relax the problem.

Rootcheck: Applying the previous rule to the MARC leads to three possibilities:
- the S_1-D channel is erased: $\alpha_1 = 0, \alpha_2 = \infty, \alpha_r = \infty$
- the S_2-D channel is erased: $\alpha_1 = \infty, \alpha_2 = 0, \alpha_r = \infty$
- the R-D channel is erased: $\alpha_1 = \infty, \alpha_2 = \infty, \alpha_r = 0$

Dealing with the last case only is not difficult: let every source send its information uncoded and R sends extra parity bits. If D receives the transmissions of S_1 and S_2 perfectly, it has all the information bits. So the challenging cases are the first two possibilities. Let us assume that the bits transmitted by S_1 are filled red, the bits transmitted by S_2 are filled blue, and the bits transmitted by R are filled white. Assume all red bits are erased at D. Just as in section 2.5.2, full-diversity is guaranteed by connecting red information bit nodes to a *rootcheck*, see Fig. 19.

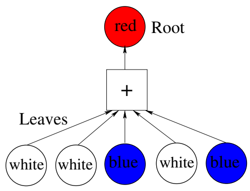

Figure 19: Rootcheck: a guarantee that root can be recovered if leaves are not erased.

Also in the case of the MARC, the key to full-diversity is to assign rootchecks to all the information bits.

Example: The sources S_1 and S_2 transmit information bits and parity bits that are related to their own information, and R transmits information bits and parity bits related to the information of S_1 and S_2. The previous description naturally leads to different classes of bit nodes, 8 to be precise. Information bits of S_1 are split into two classes: one class of bits is transmitted on α_1 (red) and is denoted as $1i_1$, the other class is transmitted on α_r (white) and denoted as $2i_1$; similarly, red and white parity bits derived from the information of S_1 are of the classes $1p_1$ and $2p_1$ respectively. Likewise, bits related to S_2 are split into four classes: blue bits $1i_2$ and $1p_2$, and white bits $2i_2$ and $2p_2$. The subscripts refer to the user. In the remainder of the paper, the vectors $\mathbf{1i_1}$, $\mathbf{2i_1}$, $\mathbf{1p_1}$, and $\mathbf{2p_1}$ collect the bits of the classes $1i_1$, $2i_1$, $1p_1$, and $2p_1$ respectively. A similar notation holds for S_2.

Above, we concluded that all information bits should be the root of a rootcheck. The class of rootchecks for $\mathbf{1i_1}$ is denoted as $1c$. Translating Fig. 16 to its matrix representation renders:

$$\begin{array}{cccc} 1i_1 & 1p_1 & \{1i_2, 1p_2, 2i_1, 2p_1, 2i_2, 2p_2\} & \\ \left[\;\mathbf{I}\right. & \mathbf{0} & \left. H_{\text{rest}} \qquad\qquad\quad \right] & 1c \end{array}$$

The identity[2] concatenated with a matrix of zeros, assures that bits of the class $1i_1$ are the only red bits connected to check nodes of the class $1c$. Because S_1 and S_2 do not communicate between each other, their bits are independent. Therefore, the matrix representation can be more elaborated:

$$\begin{array}{cccccccc} 1i_1 & 1p_1 & 1i_2 & 1p_2 & \{2i_1, 2p_1\} & 2i_2 & 2p_2 & \\ \left[\;\mathbf{I}\right. & \mathbf{0} & \mathbf{0} & \mathbf{0} & H_{\text{rest}} & \mathbf{0} & \left.\mathbf{0}\;\right] & 1c \end{array}$$

Hence, a full-diversity code construction for the MARC[3] can be formed by assigning this type of rootchecks

[2]Note that the identity matrix can be replaced by a permutation matrix. For the simplicity of the notation, in the rest of the paper **I** will be used.

[3]The reader can verify that this is a straightforward extension of full-diversity codes for the block-fading channel [BGiFBZ07].

(introducing new classes $2c$, $3c$, and $4c$) to all information bits:

$$H = \begin{array}{c} \\ \left[\begin{array}{cccccccc} 1i_1 & 1p_1 & 1i_2 & 1p_2 & 2i_1 & 2p_1 & 2i_2 & 2p_2 \\ \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & H_{2i} & H_{2p} & \mathbf{0} & \mathbf{0} \\ H_{1i} & H_{1p} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & H_{2i} & H_{2p} \\ \mathbf{0} & \mathbf{0} & H_{1i} & H_{1p} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \end{array}\right] \end{array} \begin{array}{c} \\ 1c \\ 2c \\ 3c \\ 4c \end{array} \quad (60)$$

S_1 transmits $\mathbf{1i_1}$ and $\mathbf{1p_1}$, S_2 transmits $\mathbf{1i_2}$ and $\mathbf{1p_2}$, and the common relay first transmits $\mathbf{2i_1}$ and $\mathbf{2p_1}$ and then transmits $\mathbf{2i_2}$ and $\mathbf{2p_2}$, hence the level of cooperation is $\beta = 0.5$. The reader can easily verify that if only one color is erased, all information bits can be retrieved after one decoding iteration. This full-diversity code construction has another advantage. If R receives $\mathbf{1i_1}$ and $\mathbf{1p_1}$ perfectly, it can derive $\mathbf{2i_1}$ and consequently $\mathbf{2p_1}$ (after encoding). The same holds for S_2. This code construction is semi-random, because only parts of the parity-check matrix are randomly generated.

2.6.3 Full-diversity codes and Network Coding

Via a similar reasoning as in section 2.5.3, it can be proven that the maximum coding rate yielding full-diversity is $R_c = 2/3$ for a cooperation level $\beta = 1/3$ [DCBM09]. Also, it can be proven that a random rate-2/3 $(\lambda(x), \rho(x))$ LDPC ensemble cannot achieve full-diversity. The proof is the same as in [Bou09], but with three fading instances instead of two. Here we propose a semi-random rate-2/3 LDPC ensemble that achieves full-diversity.

In section 2.6.2, we have seen an example of full-diversity on the MARC. The key idea was to assign a class of rootchecks to each class of information bits. The example attains a double-diversity order, but does not have the highest achievable coding rate exhibiting a double diversity order (because $\beta = 0.5$ instead of $\beta = 1/3$). To increase the coding rate from 0.5 to $2/3$, the relay will have to perform network coding, i.e., transmitting a sequence of bits after linear processing of the incoming streams. A simple example:
- S_1 transmits $[\mathbf{1i_1}\ \mathbf{1p_1}]$ to R and D
- S_2 transmits $[\mathbf{1i_2}\ \mathbf{1p_2}]$ to R and D
- R derives $[\mathbf{2i_1}\ \mathbf{2p_1}]$ and $[\mathbf{2i_2}\ \mathbf{2p_2}]$
- R sends the sum $[\mathbf{2i_1}\ \mathbf{2p_1}] + [\mathbf{2i_2}\ \mathbf{2p_2}] = [\mathbf{3p_1}\ \mathbf{3p_2}]$ to D.

This code has a coding rate $R_c = 2/3$, since $3K$ bits are transmitted for $2K$ information bits. The destination can use a combination of decoders and encoders to recover all information bits via three codes: two channel codes for the transmission from S_1 and S_2 and a network code for the linear processing done at the relay. This network code connects both channel codes. The two channel codes are:

$$H_1 = \left[\begin{array}{cccc} 1i_1 & 1p_1 & 2i_1 & 2p_1 \\ \mathbf{I} & \mathbf{0} & H_{2i} & H_{2p} \\ H_{1i} & H_{1p} & \mathbf{I} & \mathbf{0} \end{array}\right] \begin{array}{c} \\ 1c \\ 2c \end{array} \quad (61)$$

$$H_2 = \left[\begin{array}{cccc} 1i_2 & 1p_2 & 2i_2 & 2p_2 \\ \mathbf{I} & \mathbf{0} & H_{2i} & H_{2p} \\ H_{1i} & H_{1p} & \mathbf{I} & \mathbf{0} \end{array}\right] \begin{array}{c} \\ 1c \\ 2c \end{array} \quad (62)$$

The network code is simply:

$$H_3 = \left[\begin{array}{cccccc} 2i_1 & 2p_1 & 2i_2 & 2p_2 & 3p_1 & 3p_2 \\ \mathbf{I} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{I} \end{array}\right] \begin{array}{c} \\ 1c \\ 2c \end{array} \quad (63)$$

Let us check if this scheme achieves a double diversity order by looking at one of the three possible cases at D: the S_1-D channel has been erased. D knows $[\mathbf{1i_2}\ \mathbf{1p_2}]$ from the S_2-D channel and $[\mathbf{3p_1}\ \mathbf{3p_2}]$ from the R-D channel. It derives $\mathbf{2i_2}$ via the decoder of (62) and $\mathbf{2p_2}$ via the encoder of (62). Via (63) it obtains easily $[\mathbf{2i_1}\ \mathbf{2p_1}]$. From this, D can find $\mathbf{1i_1}$ via the decoder of (61), so that all the information bits have been recovered. The other two cases can be solved in a similar way.

Such a decoding structure with encoders and decoders mixed is not really practical. However, it is possible to determine an overall parity-check matrix such that one decoder is sufficient to obtain all the information bits:

$$\begin{array}{c} \begin{array}{cccccccc} 1i_1 & 1p_1 & 1i_2 & 1p_2 & 2i_1 & 2i_2 & 3p_1 & 3p_2 \end{array} \\ \left[\begin{array}{cccccccc} H_{1i} & H_{1p} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & H_{1i} & H_{1p} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & H_{2i} & H_{2p} \end{array}\right] \end{array} \begin{array}{c} 1c \\ 2c \\ 3c \\ 4c \end{array}, \tag{64}$$

where the checks of the class $4c$ are the sums of the checks of the classes $1c$ of codes (61) and (62). Assume again that the S_1-D channel has been erased. The bits $\mathbf{2i_2}$ are derived via the checks $\mathbf{2c}$, the bits $\mathbf{2i_1}$ are derived via the checks $\mathbf{3c}$, and finally the bits $\mathbf{1i_1}$ are derived via the checks $\mathbf{4c}$.

This code corresponds to a joint network-channel code. The coding rate is $R_c = 2/3$ because the bits $[\mathbf{2i_1}\ \mathbf{2i_2}]$ are not transmitted. This joint channel-network code has full-diversity but note that the bits $[\mathbf{2i_1}\ \mathbf{2i_2}]$ have only two edges in the overall Tanner graph, with a poor coding gain as a consequence! In fact, the set of columns corresponding to the punctured variables $[\mathbf{2i_1}\ \mathbf{2i_2}]$ do not have a random matrix such that these variable nodes cannot conform any degree distribution. The same holds for the set of rows corresponding to the check nodes $\mathbf{3c}$. This problem is tackled in the following section, by considering at the relay different linear combinations of all the information bits than the one presented above.

2.6.4 A full-diversity code construction for the MARC

The parity-check matrix (64) of the overall decoder at the D shows that the bits transmitted by R, $[\mathbf{3p_1}\ \mathbf{3p_2}] = \mathbf{3p}$, are linear combinations of all the information bits $\mathbf{1i_1}$, $\mathbf{2i_1}$, $\mathbf{1i_2}$, and $\mathbf{2i_2}$. Furthermore, the checks $[\mathbf{3c}\ \mathbf{4c}]$ represent rootchecks for all the information bits, guaranteeing full-diversity. The checks $[\mathbf{1c}\ \mathbf{2c}]$ are necessary because the bits $[\mathbf{2i_1}\ \mathbf{2i_2}]$ are not transmitted. Many variations of (64) exist when considering at the relay other linear combinations, while keeping valid rootchecks in $[\mathbf{3c}\ \mathbf{4c}]$. The parity-check matrix (65) is one of the variations having a random matrix in each set of rows and columns, preserving the randomness of the overall code, hence allowing each variable node and check node to conform any degree distribution.

$$H = \begin{array}{c} \begin{array}{ccccccc} 1i_1 & 1p_1 & 1i_2 & 1p_2 & 2i_1 & 2i_2 & 3p \end{array} \\ \left[\begin{array}{ccccccc} H_{1i} & H_{1p} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & H_{1i} & H_{1p} & \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & H_{2i} & \mathbf{I} & H_3 \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{I} & H_{2i} & \end{array}\right] \end{array} \begin{array}{c} 1c \\ 2c \\ 3c \\ 4c \end{array} \tag{65}$$

2.6.5 Numerical Results

We evaluated the finite length performance of the new proposed family of full-diversity LDPC code for the MARC. The parity-check matrix (65) is used by D to decode the information bits. This paper focuses on full-diversity, rather than coding gain. Therefore, a simple $(3,6)$ LDPC code is generated. This means that all the random matrices in (65) are randomly generated satisfying an overall row weight of 6 and an overall column weight of 3. This matrix has a rate 0.5, but because $[\mathbf{2i_1}\ \mathbf{2i_2}]$ are not transmitted, the coding rate is $R_c = 2/3$. At the moment of writing, the authors have no knowledge of another published full-diversity LDPC code for maximum rate, but other interesting results were obtained in [HD06], [HSOB05]. Fig. 20 illustrates that the WER performance of a $(3,6)$ LDPC code with block length $N = 2000$ is close to the outage probability.

2.6.6 Conclusion

We have studied LDPC codes for the multiple access relay channel in a slowly varying fading environment under iterative decoding. LDPC codes must be carefully designed to achieve diversity on this channel and network coding must be applied to increase the achievable coding rate yielding full-diversity at a maximum rate $R_{cmax} = 2/3$. Combining network coding with full-diversity channel coding gave rise to a new family of full-diversity joint network-channel LDPC codes. Full-diversity is achieved for all rates

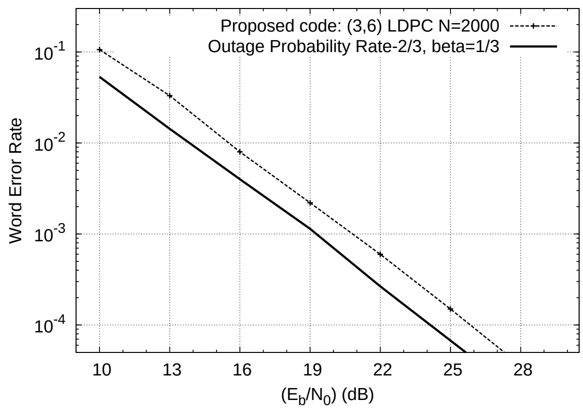

Figure 20: The WER performance of the proposed full-diversity LDPC code for maximum rate $2/3$ is close to the outage probability and performs better than codes from literature, which do not achieve the same slope of the error probability.

not exceeding $R_{cmax} = 2/3$. This code skeleton offers new perspectives to close the gap with the outage probability.

2.7 Ongoing research

In the next months, the issue of power allocation for OFDM transmission with multiple relays will be investigated for the case of individual power constraints at the different relays. In parallel, considering a system with a single relay, a MARC system with two users will be investigated. The question of power allocation, together with the choice of the user(s) to be relayed and of the decoding orders will be analyzed. Concerning LDPC codes for the multiple access relay channel, in a next step, the density evolution simulations will be finalized in order to generalize the previous results for all block lengths. Together with density evolution, the coding gain can be improved of this code construction. This problem is closely related to the optimization of the coding gain in section 2.5.

3 TASK TR6.3: RANDOMIZED DISTRIBUTED SPACE-TIME CODING FOR COOPERATIVE NETWORKS AND ADAPTIVE RELAY SCHEMES

3.1 Relaying schemes

3.1.1 AF-type relaying schemes

While it is quite natural to use feedback information to design DF and EF relaying schemes in order to optimize them by using the corresponding feedback, this becomes much less natural when analyzing the case of the AF protocol. It turns out that simple but useful changes can be made on the AF protocol. Indeed, one of the weaknesses of AF is that it amplifies the reception noise at the relay, which is a critical issue when a cheap relay is used or/and a large amount of interference is received by the relay. This is why we wanted to make the AF protocol more robust to this type of bad scenarios. For this purpose we have studied the following two aspects.

- How to tune the amplification factor in order to maximize the information rate from the source to the destination.
- How to tune the clipping threshold in order to maximize the end-to-end distortion (mean square error on the channel input symbols).

Optimizing the amplification factor. In the vast majority of the papers available in the literature, the amplification factor a_r is chosen in order to saturate the power constraint at the relay. As mentioned in some works [AFM04, KSA04, GJ06, DLK07], this choice can turn out to be sub-optimal in the sense of certain performance criteria. The intuitive reason for this is that the AF protocol not only amplifies the useful signal but also the received noise. While this effect can be negligible in certain scenarios for the standard relay channel, it is generally a dominant effect in interference relay channels [SE07]. Indeed, even if the noise at the relay is negligible, the interference term is generally not. This gives us a particular motivation for choosing the amplification factor adequately that is, to maximize the transmission rate of a given user or the network sum-rate. The proposed derivation differs from [AFM04, GJ06] because we have considered a different system (an IRC instead of a relay channel with no direct link), a specific relaying function (linear relaying functions instead of arbitrary functions) and a different performance metric (individual transmission rate and sum-rate instead of raw bit error rate [AFM04] and mutual information [GJ06]). Our problem is also different from [DLK07] since we do not consider the optimal clipping threshold, for frequency division relay channels, in the sense of the end-to-end distortion. At last, the main difference with [KSA04] is that, for the relay channel, the authors discuss the choice of the optimal amplification gain, in terms of transmission rate, for a vector AF protocol having a delay of at least one symbol duration; here we focus on a scalar AF protocol with no delay and a different system namely the IRC. In this setup, we have found an analytical expression for the best a_r in the sense of the transmission rate achieved by a given user and seen that the a_r maximizing the network sum-rate has to be computed numerically in general. The corresponding analytical result is stated in [DLK07].

Optimizing the clipping threshold. The relay is assumed to implement a linear threshold function which operates independently on the real and imaginary parts of the received signal $x = x^R + jx^I$ is defined as

$$f_\beta^R(x^R) = \left| \begin{array}{cc} x^R & |x^R| \leq \beta \\ \beta \cdot \mathrm{sgn}(x^R) & |x^R| > \beta, \end{array} \right. \tag{66}$$

where we considered the case of the real part and β is the clipping level. We see that the relay acts like a perfect AF relay in the region $[-\beta, \beta]$ and limits values outside this region. Our motivation for using this function is to assess the benefits from clipping the received signal but in some context better relaying functions can be used. For example the authors of [AFM04] derived the best relaying function in the sense of the raw BER when no direct link is assumed and a BPSK modulation is used both at the source and relay. In our context, the goal is different and the extension of [AFM04] to the case of QAM modulations does not seem to be trivial. In the same spirit, [GJ06] proposed an optimized relaying function in the sense of the mutual information when no direct link is assumed. The rationale for the proposed function (66) is that it preserves the important soft information but does not needlessly expend power relaying large noise samples. Furthermore, it only requires the optimization of a single parameter, i.e., the clipping level β. In spite of the seeming simplicity of the relaying function, however, calculating the p.d.f. of saturated Gaussian signals is known to be intractable [Hua83]. After passing the received signal through the saturation function, the signal is scaled by some real parameter α which is chosen to satisfy the average unit power constraint. As already mentioned, one just needs to tune β in order to minimize the end-to-end distortion; the corresponding result is available in [DLK07].

3.1.2 EF-type relaying schemes

Here we focus on information-theoretic relaying schemes but, as already mentioned in the first N++/WP6.2 deliverable, the proposed concepts and coding schemes can be easily exploited to design practical EF protocols. Here, the idea is to know how the EF protocol has to be modified in order to be used in other network topologies than the standard relay channel. More specifically, instead of considering a point-to-point communication helped by a relay we have considered the important case of an interference channel with a relay that is, two interfering point-to-point communications helped by a relay. The corresponding channel is called the interference relay channel (IRC) and has been introduced by [SE07]. It consists of two sources nodes, $\mathscr{S}_i$, two destination nodes $\mathscr{D}_i$ and a relay node $\mathscr{R}$, $i \in \{1,2\}$. The authors of [SE07] focused on the DF protocol. Our contribution has been to analyze the case of the AF and EF protocols [DBL09b]. In [DBL09b] we have proposed two EF relaying schemes: a version based on a single compression level and a second one which is based on two compression levels. The best choice between the two is not ready and depends on the channel gains but a simple reasoning on special cases and extensive simulation analyses leads to the following conclusions.

Special cases. The first special case is the case of asymmetric channels. By asymmetric we mean that the reception noises are ordered as follows $N_2 \gg N_1$ or $N_1 \gg N_2$. Consider w.l.o.g. the case $N_2 \gg N_1$ with $N_2 \to +\infty$ and $N_1 < +\infty$. It is obvious and easy to check that the rate of user two $R_2 \to 0$ for both the single-level and bi-level compression schemes. It is clear that user 1 gets a higher rate with the bi-level compression scheme and as the rate for user 2 tends to zero this protocol is also the better one in terms of network sum-rate. A second case, which is very special but has the advantage of requiring no long derivations is the perfectly symmetric IRC. With this setting, a quick inspection of the compression noise expressions in [DBL09b] shows that the compression noise level of the single-level scheme will always be lower than those obtained with the bi-level scheme. This is because, in symmetric IRCs, the single-level compression scheme does not generate much (exactly zero in the case of the perfectly symmetric IRC) interference whereas in the case of bi-level compression, there is always at least one user undergoing some interference from the other.

Simulation results. In [DBL09b] we have assessed the influence of the relay location on the IRC network sum-rate. This leads to different regions of the (2D) space where the single level scheme performs better/worse than its bi-level counterpart. These regions illustrate a combination of several trade-offs, which we try to explain clearly here. When the qualities of the links $\mathscr{R} \to \mathscr{D}_1$ and $\mathscr{R} \to \mathscr{D}_2$ are quite similar, the single-level scheme is the most efficient because: the compression rate for each receiver is not limited by the other receiver; the relay uses all its transmit power P_r to transmit the cooperative signal; as there is only one cooperation message, the relay generates no additional interference. This scheme is thus the best choice both for the transmission rates R_1 and R_2 and therefore for $R_1 + R_2$. On the other hand, if the qualities of the links $\mathscr{R} \to \mathscr{D}_1$ and $\mathscr{R} \to \mathscr{D}_2$ are markedly different, which happens when the relay is located around the destination $\mathscr{D}_i$ (resp. $\mathscr{D}_j$), it is better for R_i (resp. R_j) to choose the double-level scheme. This is true if the negative effect that the double-level scheme allocates only $\frac{P_r}{2}$ to each cooperation message is compensated by the mentioned positive effect.

3.2 Distributed resource allocation

3.2.1 Distributed power allocation in interference relay channels

In the preceding section we have derived achievable transmission rates for the (single-band) interference relay channel when the relay implements either the amplify-and-forward, decode-and-forward or estimate-and-forward protocol. Here, we have considered wireless networks that can be modeled by a multi-band IRC. We have tackled the existence issue of Nash equilibria (NE) in these networks when each information source is assumed to selfishly allocate its power between the available bands in order to maximize its individual transmission rate. Interestingly, it is possible to show that the three power allocation (PA) games (corresponding to the three protocols assumed) under investigation are concave, which guarantees the existence of a pure NE after Rosen [Ros65]. As the relay can also optimize several parameters e.g., its position and transmit power, it can be considered as the leader of a Stackelberg game where the information sources are the followers. Our theoretical analysis have been illustrated by simulations giving more insights on the addressed issues. These results are available in [DBL09a].

3.2.2 Spectrum sharing in distributed multiple access channels

In the preceding section we have been studying distributed power allocation in IRCs. Another important class of cooperative channels is the class of cooperative multiple access channels. We are currently studying this class of channels. So far we have studied, as in intermediate and more simple case, multiple access channels. More precisely we have studied the network spectral efficiency of distributed vector multiple access channels (MACs) when the number of accessible dimensions per transmitter is strategically limited. Considering each dimension as a frequency band, we call this limiting process bandwidth limiting (BL). Assuming that each transmitter maximizes its own data rate by water-filling over the available frequency bands, we have considered two scenarios. In the first scenario, transmitters use non-intersecting sets of bands (spectral resource partition), and in the second one, they freely exploit all the available frequency bands (spectral resource sharing). In the latter case, successive interference cancelation (SIC) has been used. We have shown the existence of an optimal number of dimensions that a transmitter must use in order to maximize the network performance measured in terms of spectral efficiency. We have provided a closed form expression for the optimal number of accessible bands in the first scenario. Such an optimum point, depends on the number of active transmitters, the number of available frequency bands and the different signal-to-noise ratios. In the second scenario, we have shown that BL does not bring a significant improvement on the network spectral efficiency, when all transmitters use the same BL policy. For both scenarios, we have provided simulation results to validate our conclusions. All these contributions have led to the publication of a joint paper involving CNRS/LSS, Poznan University, and CNRS/Supélec [MDLB09]. This paper received the Best Student Paper Award (Crowncom 2009).

3.3 Ongoing research

Concerning distributed power allocation problems algorithms are currently designed to implement the proposed policies with realistic channel state information assumptions at the terminals. The main issue here is to solve convergence problems in decentralized procedures similar to the Cournot tâtonnement.

As far as spectrum sharing problems are concerned, LSS, PUT, and Supélec are extending their work presented at Crowncom. Technically, the goal is to study the existence of predictable states in distributed wireless networks where spectrum efficiency is what terminals want to selfishly maximize and analyze the equilibrium efficiency. This work should be translated into the submission of a joint journal paper.

4 TASK TR6.4: OBLIVIOUS COOPERATION PROTOCOLS AND SCALING LAWS FOR RELAY NETWORKS

4.1 Analysis of Oblivious Cooperation Protocols

In [GSG+09a, GSG+09b] the problem of multicasting multiple messages with the help of a relay, which may also have an independent message of its own to multicast, is considered. As a first step to address this general model, referred to as the compound multiple access channel with a relay (cMACr), the capacity region of the multiple access channel with a 'cognitive' relay is characterized, including the cases of partial and rate-limited cognition. Achievable rate regions for the cMACr model are then presented based on decode-and-forward (DF) and compress-and-forward (CF) relaying strategies. Moreover, an outer bound is derived for the special case in which each transmitter has a direct link to one of the receivers while the connection to the other receiver is enabled only through the relay terminal. Numerical results for the Gaussian channel are also provided.

In [SSE+09] the problem of communications via a multirelay channel with non ergodic link-failures is analyzed. More specifically, a multi-relay network is considered in which communication from source to relays takes place over a (discrete or Gaussian) broadcast channel, while the relays are connected to the receiver via orthogonal finite-capacity links. Unbeknownst to the source and relays, link failures may take place between any subset of relays and destination in a non-ergodic fashion. Upper and lower bounds are derived on average achievable rates with respect to the prior distribution of the link failures, assuming the relays to be oblivious to the source codebook. The lower bounds are obtained via strategies that combine the broadcast coding approach, previously investigated for quasi-static fading channels, and various robust distributed compression techniques.

Cui et al. in [CHS+09] study the throughput scaling of wireless networks over channels with random connections, in which the channel connections are independent and identically distributed (i.i.d.) according to a common distribution. The channel distribution is quite general, with the only limitations being that the mean and variance are finite. Previous works have shown that, when channel state information (CSI) of the entire network is known a priori to all the nodes, wireless networks are degrees-of-freedom limited rather than interference limited. In this work, we show that this is not the case with a less demanding CSI assumption. Specifically, we quantify the throughput scaling for different communication protocols under the assumption of perfect receiver CSI and partial transmitter CSI (via feedback). It is shown that the throughput of single-hop and two-hop schemes are upper-bounded by respectively, $O(n^{1/3})$ and $O(n^{1/2})$, where n is the total number of source-to-destination pairs. In addition, multi-hop schemes cannot do better than the two-hop relaying scheme. Furthermore, the achievability of the $\Theta(n^{1/2})$ scaling for the two-hop scheme is demonstrated by a constructive example.

4.2 Multiple access relay channel with relay-source feedback

In [GW75, CL81], it has been shown that feedback can sometimes really increase capacity. Inspired by these works, in [HK09] the authors incorporate noiseless feedback from the relay to the source and derive an achievable rate region $\mathscr{R}_{\text{FB}}$ that exceeds the previous rate region on the MARC without feedback in [KvW00]. The presence of the feedback enables the sources to understand each other's message, which in turn allows the sources to cooperate to resolve the residual uncertainty at the receiver in a more efficient way. At the same time, some independent fresh information from the sources can be superimposed upon the resolution information.

In this work, both the discrete and the AWGN case are considered. In the discrete case the MARC with relay-source feedback as shown in Fig. 21.

The channel is assumed to be memoryless and represented by the conditional probability distribution

$$P(y_{rk}, y_{dk}|x_{11}, \cdots, x_{1k}, x_{21}, \cdots, x_{2k}, x_{r1}, \cdots, x_{rk}, y_{r1}, \\ \cdots, y_{rk-1}, y_{d1}, \cdots, y_{dk-1}) = p(y_{rk}, y_{dk}|x_{1k}, x_{2k}, x_{rk})$$

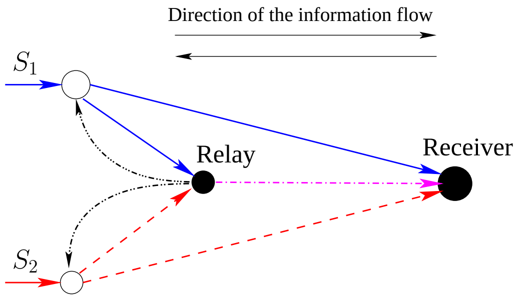

Figure 21: The two-source multiple-access relay channel with feedback from the relay.

where $x_{1j}, x_{2j}, x_{rj}, y_{rj}, y_{dj}$ are the inputs and outputs of the channel at time j, respectively. An $((M_1, M_2), n)$ code for the two-source MARC with feedback from the relay (Fig. 21) can be defined as follows:
i) a set of encoding functions $x_i : 1, 2, \cdots, M_i \times \mathscr{Y}_r^n \rightarrow \mathscr{X}_i^n$, $i = 1, 2$, where the k th component of $\mathbf{x}_i \in \mathscr{X}_i^n$ is determined by the function

$$x_{ik}(w_i, (y_{r1}, y_{r2}, \cdots, y_{rk-1})), \, k = 1, 2, \cdots, n, \, w_i = 1, 2, \cdots, M_i.$$

This means that each transmitter i chooses a sequence of symbols depending on its current message w_i and the past symbols $y_{r1}, y_{r2}, \cdots, y_{rk-1}$ fed back from the relay.
ii) a set of relay functions $\{f_k\}_{k=1}^n$ such that

$$x_{rk} = f_k(y_{r1}, y_{r2}, \cdots, y_{rk-1}), \, k = 1, 2, \cdots, n.$$

iii) a decoding function $g : \mathscr{Y}_d^n \rightarrow \{1, 2, \cdots, M_1\} \times \{1, 2, \cdots, M_2\}$. We write $g(y_{d1}, \cdots, y_{dn}) = (\hat{w}_1, \hat{w}_2)$.

By definition, the rate pair (R_1, R_2) is said to be achievable for the MARC with feedback if there exists an $((M_1, M_2), n)$ code with

$$R_1 \leq \frac{1}{n} \log M_1, \, R_2 \leq \frac{1}{n} \log M_2$$

such that $P_n \rightarrow 0$ as $n \rightarrow \infty$, where P_n is the average error probability. The capacity region is the closure of the set of all achievable rate pairs (R_1, R_2).

Let U be a discrete random variable which takes values in the set $\mathscr{U} = \{1, 2, \cdots m\}$, where $m = \min\{||\mathscr{X}_1|| \, ||\mathscr{X}_2|| \, ||\mathscr{X}_r|| + 1, ||\mathscr{Y}_d|| + 2\}$. $\mathscr{P}$ is the set of all joint distributions of the form

$$\begin{aligned} &P_{UX_1X_2X_rY_rY_d}(u, x_1, x_2, x_r, y_r, y_d) = P_U(u) P_{X_1|U}(x_1|u) \\ &P_{X_2|U}(x_2|u) P_{X_r|U}(x_r|u) P_{Y_rY_d|X_1X_2X_r}(y_r, y_d|x_1, x_2, x_r) \end{aligned}$$

where $P_{Y_rY_d|X_1X_2X_r}(y_r, y_d|x_1, x_2, x_r)$ is fixed by the channel.

In [HK09] the authors prove that the achievable rate region $\mathscr{R}_{\text{FB}}$ on the degraded MARC with relay-source feedback is given by

$$\begin{aligned} \mathscr{R}_{\text{FB}}(R_1, R_2) = \bigcup \{ &(R_1, R_2) : \\ &R_1 \leq \{I(X_1; Y_r | X_2, X_r, U)\} && (67) \\ &R_2 \leq \{I(X_2; Y_r | X_1, X_r, U)\} && (68) \\ &R_1 + R_2 \leq \min\{I(X_1, X_2; Y_r | X_r, U), && (69) \\ &\qquad I(X_1, X_2, X_r; Y_d)\}\} \end{aligned}$$

where the union is taken over $\mathscr{P}$.

The model for the degraded AWGN MARC with feedback from the relay as shown in Fig 22.

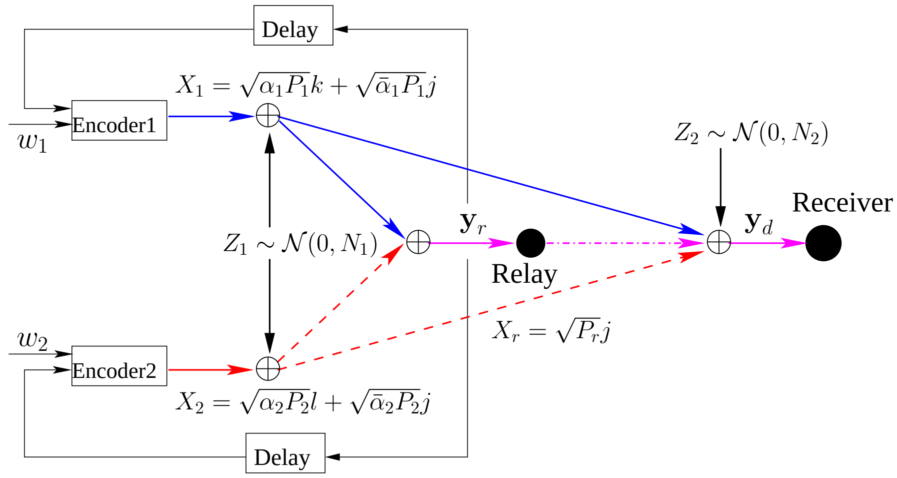

Figure 22: AWGN degraded MARC with feedback from the relay.

Let $C(x) = \frac{1}{2}\log(1+x),\ x \geq 0$. Then, referring to the results presented above, in [HK09] the authors show that the achievable rate region of the degraded Gaussian MARC with feedback from the relay is given by

$$\mathscr{R}_{\text{FB}}(R_1, R_2) = \bigcup \{(R_1, R_2) :$$

$$R_1 \leq \left\{ C\left(\frac{\alpha_1 P_1}{N_1}\right)\right\} \tag{70}$$

$$R_2 \leq \left\{ C\left(\frac{\alpha_2 P_2}{N_1}\right)\right\} \tag{71}$$

$$R_1 + R_2 \leq \min\left\{ C\left(\frac{\alpha_1 P_1 + \alpha_2 P_2}{N_1}\right), C\left(\frac{\alpha_1 P_1 + \alpha_2 P_2 + P_{\text{eff}}}{N}\right)\right\}\Bigg\} \tag{72}$$

where $N = N_1 + N_2$ and the union is over all $0 \leq \alpha_i \leq 1,\ i \in [1, 2, r]$.

4.3 Asymptotic Analysis of Space-Time Codes

4.3.1 Randomized Distributed Space-Time Coding for Multiple-Relay Channel: Asymptotic Analysis of Outage Probability and Diversity

Cooperative transmission has recently emerged as an alternative means of providing spatial diversity without the need of physically collocating multiple antennas in a single terminal. The basic idea behind this type of transmission is the fact that multiple terminals in the system can emulate the operation of a virtual antenna array by conveniently replicating the signal transmitted by a source node [Lan02, Doh03, FK06]. To match with practical requirements, terminals acting as relays are often assumed to only have channel state information associated with the signal that is received (which corresponds to the channel between the source and the relay), and in these circumstances it is convenient to emulate the operation of a distributed space-time code [LW03, LTW04b].

Now, one of the main practical problems associated with the implementation of distributed space time codes is the fact that they are inherently rigid, in the sense that the whole system needs to be designed for each specific number of relays. This means that every time a relay drops in or out of the system, the whole transmission strategy needs to be re-designed, and somehow communicated to the relays. In what follows, we consider a different and much more flexible approach, which will be referred to as *randomized distributed space time block coding* (RD-STBC). This cooperative transmission technique basically implements a distributed version of the well-known linear dispersion codes [Jaf05, Chapter 10] in which the dispersion matrices are randomly and independently chosen for each relay.

More specifically, each relay in the system is associated with an $N \times K$ matrix with independent and identically distributed entries that is known at the destination (these matrices will be referred to as "spreading matrices" for reasons that will become clear in short). The spreading matrices, which are different for each relay, are fixed at the beginning of the transmission and do not change over time. The communication strategy is organized in two phases or time slots, dedicated to source transmission (the first) and relay transmission (the second). After the K samples transmitted by the relays during the first time slot have been received, each relay multiplies the column vector of K samples by the corresponding $N \times K$ matrix, thus generating a column vector containing the N samples to be transmitted during the second time slot. Note that this is much simpler than a conventional distributed space-time code, because relays do not need to change their transmission strategies when new nodes drop in or out of the system, and also because the signal processing at the relays reduces to a mere matrix-vector multiplication.

When adopting the *amplify-and-forward* (AF) relaying strategy (relays forward a noisy copy of the information without trying to decode it), the performance of this cooperative technique is investigated in [GM09] in terms of spectral efficiency. Since the actual performance depends on the choice of spreading matrices, the performance is analyzed in the asymptotic domain assuming that both N and K are large but comparable in magnitude. Using random matrix theory techniques, it is shown that the spectral efficiency converges to a fixed limit, which interestingly is an extraordinarily good approximation of the average behavior of the finite reality. Hence, these asymptotic expressions can be used even when N,K are finite (in practice, no difference between simulated and asymptotic spectral efficiency was observed even when N,K are chosen as low as 4). The analysis carried out in [GM09] assumes that the channel coefficients are frequency non-selective and remain constant during the transmission of a codeword (two time-slots). The effect of introducing quasi-static fading at all the channels (all the channels in the system are fixed during the transmission of a codeword, but vary randomly from one codeword to the next) is investigated in [GMedc]. More specifically, when the *signal-to-noise ratio* (SNR) of the system is large, the effect of fading is analyzed in terms of the outage probability $P_{out}(R)$ (defined as the probability that the system does not achieve a certain spectral efficiency R), which will be approximated as $P_{out}(R) = \kappa SNR^{-d}$. The coefficients κ and d are named *outage gain* and *diversity order*, respectively. The following result is introduced:

Proposition 1 *Consider a system with L AF relays employing the RD-STBC described above. Assume* maximum-likelihood *(ML) reception with full channel state information. Then, the diversity order is a function of the ratio* $\alpha = K/N$*, namely*

$$d = \begin{cases} L+1 & \text{for } \alpha < \dfrac{1}{L-1}, \\ M+1 & \text{for } \dfrac{1}{M} \le \alpha < \dfrac{1}{M-1}, \end{cases}$$

with $1 \le M < L$*. The outage gain* κ *can be hence computed (numerically) according to its definition*

$$\kappa = \lim_{SNR \to +\infty} SNR^d P_{out}(R).$$

The following conclusions can be drawn:

1. the spreading matrices have to be tall ($N > (L-1)K$) to maximize the diversity order. Note, however, that this implies a sensible waste of degrees of freedom, since the source is transmitting K symbols every $K+N$ channel accesses;

2. relaying is always superior to the direct link, which only achieves a unitary diversity order.

The same problem, but with *decode-and-forward* relaying strategy (each relay forwards the source message only if it could decode it), is addressed in [GMeda], where the following result is proved:

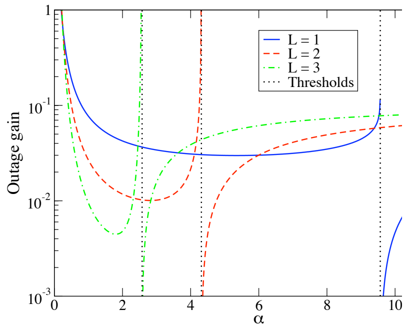

Figure 23: Outage gain as a function of α for unitary channel and noise variances and $R = 0.1$ nat/s/Hz. The diversity order is $L+1$ on the left of the thresholds and 1 on their right.

Proposition 2 *Consider a system with L DF relays employing the RD-STBC described above. Assume maximum-likelihood reception with full channel state information. Then, the diversity order is independent of the ratio $\alpha = K/N$ and equal to the total number of transmitters (source plus relays) $L+1$. The outage gain can be computed numerically.*

In other words, the DF strategy with ML receiver achieves full diversity: each relay offers an independent path from the source to the destination. This was not the case in the AF scenario presented above: as the spreading matrices grow wider (K grows with respect to N), relay contributions become more correlated (and the forwarded noise more colored). This results in a reduction of the equivalent number of independent channels.

In [GMeda], the *linear minimum-mean-square-error* (LMMSE) filter is also considered as a possible alternative to the optimal, but complex, ML receiver. It turns out that the LMMSE filter can exploit diversity only if the spreading sequences are sufficiently long. Otherwise, relays excessively compress information and the LMMSE cannot combine their contributions properly.

Proposition 3 *Consider a system with L DF relays employing the RD-STBC described above. Assume LMMSE reception with full channel state information. Define the function $Q(\alpha) = \exp\left(\frac{1+\alpha}{\alpha}R\right) - 1$ and α_{th} as the unique positive solution to $\alpha = \frac{1}{L}\left(1 + \frac{1}{Q(\alpha)}\right)$. Then the diversity order is given by*

$$d = \begin{cases} L+1 & \text{for } \alpha \leq \alpha_{th}, \\ 1 & \text{for } \alpha > \alpha_{th}. \end{cases}$$

In this case, the outage gain can be computed in closed form and behaves as depicted in Figure 23. Note the importance of identifying the value of α (i.e. allocating time to source and relay transmission phases) that minimizes the outage gain in the full diversity region (on the left-hand side of the threshold). For $\alpha > \alpha_{th}$, the system attains unitary diversity order, as the simple source-destination link. However, the outage gain is always lower than the direct-link one and can be made as small as desired.

Further work is needed to characterize the LMMSE receiver for AF relays and, especially, to analyze more in details the behavior of the outage gain in the different cases. Such a study will give insights on the system performance and allow the comparison between the two relaying strategies. The interest reader can refer to [GMedb, GMedd] for preliminary results in the single-relay case.

4.3.2 Approximately Universal Space-Time Codes for the Parallel, Multi-Block and Cooperative DDF Channels

The work in [EK09] is motivated by the relaying topic of WPR6, in conjunction with the reality that such cooperation takes place in the presence of OFDM capabilities. In this setting, explicit codes are constructed that achieve the diversity-multiplexing gain tradeoff (DMT) of the cooperative relay channel under the dynamic decode-and-forward protocol for any network size and for all numbers of transmit and receive antennas at the relays. Along the way, we prove that codes previously constructed in the literature for the block-fading and parallel channels are approximately universal, i.e., they achieve the DMT for any fading distribution. It is shown how approximate universality of these codes leads to the first DMT-optimum code construction for the general, MIMO-OFDM channel.

4.4 Capacity Analysis of Two-Hop relaying Virtual MIMO Systems in a Poisson Field of Nodes

Virtual (also known as distributed) multiple-input-multiple-output (V-MIMO) systems appear as one of the most interesting paradigms for the deployment of future wireless systems [DDA02, DGA07]. The key aspect of V-MIMO communication systems is the possibility for the devices, which can be equipped with single or multiple antennas, to create clusters of cooperating nodes, which are usually denoted as virtual antenna arrays (VAAs). Cooperation between nodes is exploited to obtain the benefits, in terms of spatial diversity, of conventional MIMO. Although some papers addressing performance analysis of V-MIMO have appeared in the open literature in these past years, to the author's knowledge, few works consider the connectivity problems which usually arise with the formation of the V-MIMO systems.

In this Section, a two-hop V-MIMO system (see Figure 24) is considered, where a source node has to transmit data to a destination node via a relay node, is considered. Several *ancillary* nodes are located in three areas around the *main* nodes, namely source, relay and destination [BZ09a, BZ09b]. Ancillary nodes are supposed to be distributed over the areas according to a Poisson point process (PPP). Three VAAs are formed in this scenario: the source VAA (s-VAA), the relay VAA (r-VAA) and the destination VAA (d-VVA).

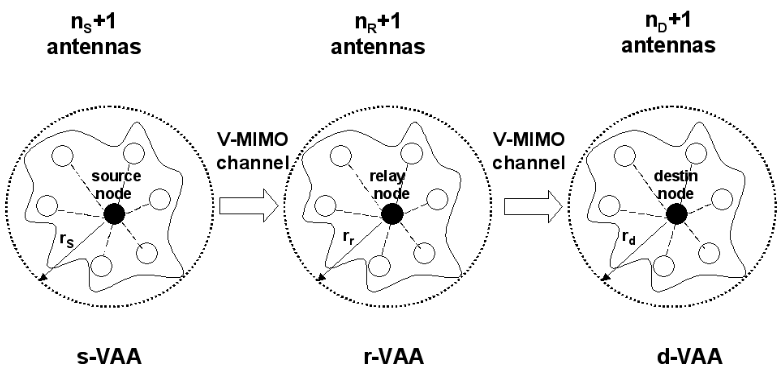

Figure 24: The Virtual MIMO communication system.

Random fluctuations of the wireless channel as well as a distance-dependent deterministic path-loss are accounted for in the radio channel model. Main nodes are supposed to cooperate only with the ancillary nodes which guarantee a feasible quality of the link. Owing to the randomness nature of the channel, the number of transmit and receive antennas is a random variable and a certain outage probability there exists. We define the outage probability as the probability that the source-destination capacity is lower than a given threshold, denoted as C_0.

To form a VAA, the main nodes transmit a query to the ancillary nodes, by using the short-range radio interface. Owing to propagation conditions, only a subset of the ancillary nodes can really cooperate with the main nodes. The number of nodes which actually communicate with source, relay and destination is denoted by n_S, n_R and n_D, respectively, and are called *cooperating* nodes. Being ancillary nodes Poisson distributed, cooperating nodes will be Poisson distributed too (this is a consequence of the Marking Theorem [VDMC08]), with a mean value which depends on the channel model considered.

Once s-VAA, r-VAA and d-VAA are formed, the communication is performed according to the following steps: (i) The source transmits data to the n_S cooperating nodes; (ii) the $n_\mathrm{S}+1$ nodes of the s-VAA transmit data toward the relay through the V-MIMO channel, using the high rate interface (inter-VAA communication); (iii) the $n_\mathrm{R}+1$ nodes of the r-VAA cooperate to decode the received data and forward it toward the destination; (iv) the $n_\mathrm{D}+1$ nodes of the d-VAA receive data from the r-VAA and cooperate to decode it.

Once the channel model is fixed, the distribution of the cooperating nodes is known and the outage probability, denoted as P_out, can be evaluated. In particular, this outage probability is given by: $P_\mathrm{out} = \mathbb{P}\{\bar{C}^{(2)} < C_0\}$, where $\bar{C}^{(2)}$ is the mean (with respect to fast fading fluctuations) capacity in the two-hop communication protocol (see [BZ09a]).

Some example of results achieved with the mathematical framework developed, are shown (see also [BZ09a]). Fig. 25 reports the complementary outage probability, $P_\mathrm{in} = 1 - P_\mathrm{out}$, as a function of C_0, for different values of the signal to noise ratio at the relay and at the destination, ρ, having set the density of ancillary nodes at the three main nodes, denoted as η, equal to 5 10^{-4} [m^{-2}]. As expected, P_in decreases by increasing C_0 and the curves are translated by increasing ρ.

In Fig. 26 the one- and two-hop communication protocols are compared in terms of energy consumption. The total power spent by the network to deliver the data from the source to the destination, depends on the power spent by each node participating in the communication. We denote as $\mathbb{E}\{P_\mathrm{tot}\}$ the averaged (with respect to fast and slow fading, and to the number of cooperating nodes) power spent by all the active nodes in the network.

In Fig. 26, P_in as a function of $\mathbb{E}\{P_\mathrm{tot}\}$, for different values of η, is shown for $C_0 = 5$ [bit/s/Hz] and having fixed a distance source-relay and relay-destination equal to 300 [m]. As expected, the increase in the value of P_in is obtained at the cost of an increasing of the total power spent by the network. For what concerns the comparison between two communication protocols, we can deduce that for low values of η, the one-hop protocol allows to obtain larger P_in. When nodes' density increases, the two-hop protocol can exploit the additional degrees of freedom given by r-VAA and outperforms the single-hop case.

4.5 Asymptotic Capacity of Relay-Assisted Systems

4.5.1 Asymptotic Capacity and Optimal Precoding Strategy for MIMO Relay Systems

In a ideal ad hoc network, where all nodes could perfectly cooperate both at transmission and reception, one could expect the network to behave like a perfect MIMO system. The total capacity would then grow linearly with the number of node pairs as in the point-to-point MIMO channel[Tel95]. However perfect transmit/receive cooperation has a cost, in particular it requires perfect synchronization between cooperating nodes and extensive channel knowledge, which is impossible in practice. A reasonable alternative consists in dividing the network in several cooperative clusters. Each cluster would gather nodes cooperating in a local area to form a virtual antenna array and virtual MIMO transmissions would then occur in multiple stages between these cooperative clusters. This approach, considered in our work in [FZDG08b, FZDG08a], offers a way to leverage the broadcasting nature of the wireless link, by treating simultaneous transmissions as useful signals that are jointly and cooperatively processed at nodes within a cluster.

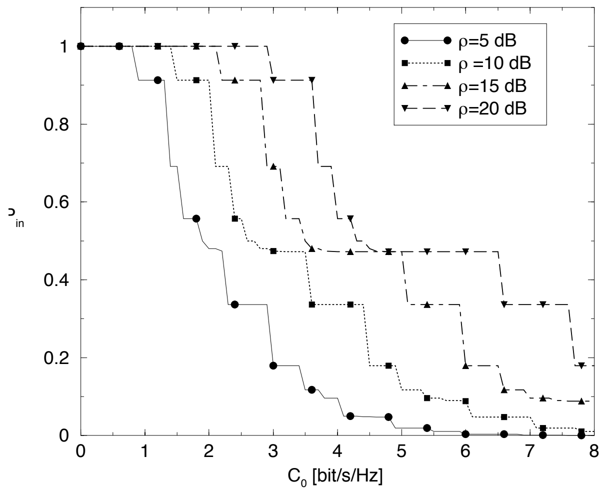

Figure 25: P_{in} as a function of C_0, for different values of ρ.

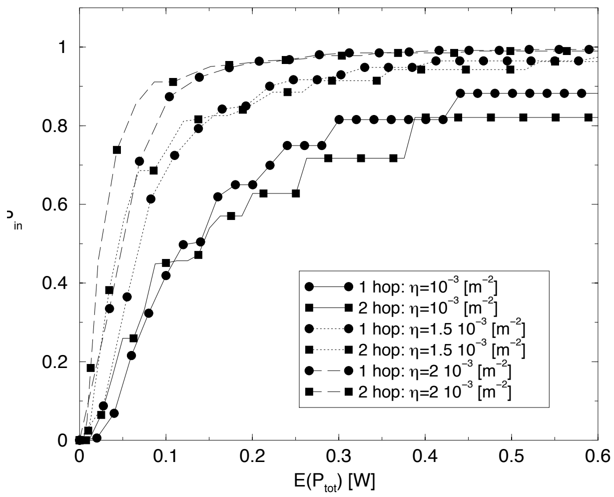

Figure 26: P_{in} as a function of $\mathbb{E}\{P_{\text{tot}}\}$ by varying η, in the one- and two-hop cases.

System model and problem statement

In [FZDG08b, FZDG08a], a cooperative ad hoc network is considered, where nodes are grouped in three types of cooperative clusters:

- a source cluster, consisting of source nodes cooperating to form a virtual transmit antenna array;
- a destination cluster, gathering destination nodes cooperating to form a virtual receive antenna array;
- a series of $N-1$ intermediary relaying clusters, between source and destination clusters, each gathering relay nodes cooperating to form a virtual relay antenna array.

This system forms a virtual MIMO multi-hop relay network. In this transmission chain with $N+1$ levels, it is assumed that each relay receives a faded version of the signal transmitted from the previous level and, after precoding, retransmits it to the next level. We consider the case where all communication links undergo block Rayleigh flat fading and the fading channels at each hop (between two adjacent clusters) may be correlated while the fading channels of any two different hops are independent. Deriving the capacity of such a complex network is intricate. Fortunately, random matrix theory and free probability theory provide useful mathematical results on the eigenvalues and eigenvectors of large random matrices, that can be applied to the field of wireless communications in order to analyze complex communication systems when the system dimensions increase.

As a first step towards a more complete analysis, we start by making some simplifying assumptions. We assume that nodes in a cluster can perfectly exchange information, and that channel knowledge intrinsic to the cluster— channels between nodes within a cluster— is available at all cluster nodes. Thus in each cluster, nodes form a perfect virtual antenna array. We also assume non-noisy communications between relaying clusters, but that noise power is not negligible at the destination cluster. Under those simplifying assumptions, we address the following questions:

- What is the asymptotic capacity of the cooperative-cluster system when the number of nodes in all clusters grow large?
- How should nodes in a cooperative cluster process and transmit cooperatively their wireless signals to maximize the system capacity?

Main results and insights

Using tools from random matrix and free probability theories, we first derive a closed-form expression of the asymptotic instantaneous end-to-end mutual information between the source input and the destination output, as the number of nodes in all clusters grows large. This asymptotic expression is shown to be independent from the channel realizations and to only depend on the channel statistics, with two major consequences. First, in the asymptotic regime the mutual information is not a random variable any more but a deterministic value representing an achievable rate. Second, given the stationarity of channel statistical properties, the asymptotic instantaneous mutual information obtained in the non-ergodic block-fading regime also serves as the asymptotic value of the average end-to-end mutual information between the source and the destination. The expression of the asymptotic average mutual information is the same as the asymptotic ergodic mutual information that would be obtained if the channel was an ergodic process. Intuitively, the time-domain ergodicity is recovered in the spatial domain when the dimensions of the system grow large.

We also obtain the singular vectors of the optimal precoding matrices that maximize the average mutual information of the system with a finite number of antennas at all levels. It is shown that the singular vectors of the optimal precoding matrices are also independent from the channel realizations and can be determined only using statistical knowledge of channel matrices at source and relays. The so-obtained singular vectors turn out to be also optimal in the asymptotic regime of concern.

Finally, we apply the aforementioned results on the asymptotic mutual information and the structure of the optimal precoding matrices to several communications scenarios, with different number of hops, and types of channel correlation. These examples illustrates the gains resulting from the cooperative cluster approach and reveal the few relevant parameters impacting the capacity of cooperative dense ad hoc networks.

4.5.2 Large System Analysis of Protocols for Relay Networks

In [CCF08] a relay assisted CDMA network with a large number of sources and half duplex relays and a unique destination is considered. Two relaying protocols called direct relaying (DR) and full relaying (FR) are proposed. By dividing the relays in groups and adopting different forwarding delays for each group both protocols introduce diversity which depends on the number of groups and on the protocol. In DR mode, the relays forward only signals received directly from the sources. In FR mode, relays forward both signals received by the sources and the other relay groups by applying network coding at the physical layer. This implies a different level of diversity at the destination for the two schemes. Then, an analytical framework for the analysis of the achievable rates in such a network, as the number of nodes and relays becomes asymptotically large, is proposed.

More specifically, in this protocols both source and relay nodes transmit synchronously using code division multiple access (CDMA) scheme with random spreading as multiple access protocol. Each relay decodes a subset of the received information, re-encodes them using the same code adopted by the sources, and retransmits them using CDMA with a delay multiple of a codeword duration T_{cw}. Let us denote with T_t the time interval when the t-th codeword is transmitted. Half of the relays transmit during the time intervals T_t with t even, while the remaining relays transmit in the intervals T_t with t odd in a complementary way. In this framework we consider two different protocols for relay networks based on the decode-and-forward scheme.

Let us introducing first the full relay protocol. Let us consider a relay that in a given time interval transmits the codewords $\mathscr{C}_t$ received directly from the source nodes at time T_t. Then, it also forwards the codewords $\mathscr{C}_{t-1}$ transmitted by the same source nodes in T_{t-1}. This latter codeword is decoded from the signals received by the other relays nodes but not directly from the source. Notice that error propagation in nonzero error capacity channels is avoided thanks to the assumption of a large number of relays in the network. Each relay encodes the subset of received codewords in T_{t-1} and T_t and spread them with distinct spreading sequences. Then, the spread signals are simply combined and transmitted. Furthermore, the relays transmitting in the even intervals are divided into L groups as well as the ones transmitting in the odd interval to obtain $2L$ groups. Each group retransmits the signals with a delay different from the other groups in order to obtain at the receiver multiple copies of the desired information.

In the direct relay protocol, each relay forwards only codewords received directly from the source.

On the one hand, the full relay scheme has the advantage of providing at the destination a number of replicas of the same information equal to $2L+1$ while with the decoupled network coding protocol the number of replicas of the same information is equal to half of the number of the relay groups plus one $L+1$. On the other hand, in the direct relay protocol, the relays observe only once each codewords they intend to forward. In contrast, in the full relay scheme the relay receives $L+1$ replicas for half of codewords and only L replicas for the remaining half of the codewords to be forwarded with obvious cost in achievable rate.

Throughout this work we assume that the channels among all nodes (source-relay nodes, source-destination nodes and relay-destination) are flat fading and the signal at each receiving node (relays or destination) is impaired by white additive Gaussian noise.

By making use of random matrix theory, we analyze the achievable rate per user of this network when the transmitters (relays and sources) do not have complete channel state information and the number of source nodes N_s, relay nodes N_r, and the spreading factor N tend to infinity such that $\frac{N_s}{N} \to \beta < \infty$ and $\frac{N_r}{N_s} \to \gamma < \infty$. The theoretical framework provides the achievable rates for the networks adopting the decoupled networking scheme and the interlaced networking scheme as a function of the noise variance, β, γ, and the limit distributions of the received powers at the relays and at the destination. The system performance at the destination is completely characterized by a positive function of the noise variance, solution of a Tse-Hanly fixed point equation. In the most general case, the performance at the relays are described by two positive functions of the noise variance solution of a system of two fixed point equations. Under easily verified constraints on the limit received power distributions at the relays, the system can be characterized by a single Tse-Hanly equation.

The network performance of the two schemes are assessed under the assumption that the channel gains among all the nodes are Gaussian distributed with zero mean and unit variance. The transmitted power is normalized to keep constant the total transmitted power in the network.

The performance is assessed against the baseline multiple access system where the network consists simply of N_s sources and one destination to get insight on the advantages of cooperation and network coding.

4.6 Ongoing research

An new cooperation between CNRS (Telecom ParisTech) and CTTC is developing in the framework of WPR6.4. Gregoratti and Hachem will jointly work on applications of random matrices to relay networks.

A new cooperation between CNRS (Supelec and Telecom ParisTech), Technion, and NKUA-IASA focuses on oblivious cooperation in downlink multi-cell CDMA systems using random matrix theory. Oblivious cooperation in self-organizing networks might naturally appear inside network capacity formulas. For instance, in a K-cell CDMA-based network, the network-wide maximally achievable throughput is expected to be the sum of K terms, the k^{th} term of which being a function of

- cell k's proper parameters (e.g. the powers allocated to the users inside this cell)
- one or several terms being functionals of the overall network parameters (e.g. all the powers allocated to all users in the network).

These last network-wide parameters can be exchanged at a bandwidth-efficient cost throughout the network, in place of the bandwidth-harvesting exchange of all network parameters. Therefore, one can imagine global network capacity optimization, based on K local capacity optimizations and the signalling of small packets enclosing system-wide parameters.

In the first steps of the investigation of the downlink capacity of CDMA-based networks, it was observed that power optimization (aiming at maximizing network capacity) might consist in some specific cases in a classical water-filling solution inside each cell, depending on the cell users' path-losses and on a single extra system-wide parameter. A sequential power optimization strategy can then be designed which consists, for each cell in turn, in optimizing the power allocation inside its own cell and updating the global parameter to be forwarded to the subsequent cell, until convergence is reached.

When this first study is completed, more advanced models will be considered which are also expected to show separable formulations of their capacity or other performance functionals.

An ongoing cooperation between CNIT (CNR) and CNRS (Eurecom) aims at deepening the analysis in [CCF08] by introducing the effects of the channel attenuation in the analysis. This analysis is performed combining results from random matrix theory and statistical geometry. A network with an large number of relays uniformly distributed over a circumference of radius r from the common destination is considered. The sources are uniformly distributed over a disc of unit radius. Several kinds of protocols will be considered. As an example, we analyzed a direct protocol (see Section 4.5.2) where (i) each relays knows at least the phase of the channel with the destination and (ii) all the receivers (relays and destination) have knowledge of the channel such that coherent communication is possible. The number of the relays scales with the square root of the number of the sources.

Large system analysis of these kinds of networks will enable the design of macroscopic system parameters (e.g. number of relays versus number of users,spreading factor, power allocation, etc.) and answer to several questions such as:

- What is the achievable rate of a relaying-based network with no CSI at the sources.
- How to optimize the relay deployment?
- How much can we gain from opportunistic relaying?

About the MARC channel with feedback, TUM has initiated a cooperation with UCL. TUM visited UCL and joint deepening of the topic is on its way.

5 TASK TR6.5: COOPERATION IN MOBILE AD-HOC NETWORKS (VIRTUAL ANTENNA ARRAYS/VAAS)

Multiple transmission and reception units in wireless devices can improve the quality of communications by means of diversity and multiplexing techniques [BY06], [JKSK07], [HM06], [MCMS07], [LTW04a], [SEA03a], [SEA03b]. However, multiple-input multiple-output (MIMO) units may not be available in certain wireless devices because of cost/power/size constraints; for instance, nodes in a wireless sensor network are usually designed as low-cost, small, and lower-power devices [AMC07]. In the absence of multiple antennas at wireless devices, it is still possible to realize the advantages of MIMO systems, if the nodes cooperate in an appropriate manner. For example, if the nodes share information about the received signals with each other, they can form a virtual multiple-output structure. Similarly, if the nodes transmit cooperatively, they can set up a virtual multiple-input unit [BF07], [GS00], [Say03]. Since cooperative communications provide an opportunity to transmit and receive signals from multiple antennas, diversity gains available in MIMO systems can also be obtained in cooperative systems [NHH04], [Jay06]. In order to exploit the available diversity gains in these virtual MIMO systems, efficient signal processing algorithms should be devised, keeping into account practical power, complexity and cost limitations.

When a number of different transmitting antennas is available, spatial diversity can be exploited. This form of diversity takes advantage from the generation of multiple replicas of modulated signal, each one differently affected by the propagation medium. In addition, multipath diversity can be available in virtual MIMO systems; hence, signals from both different devices and different multipath components should be combined optimally in order to achieve a large diversity order.

In this Chapter, some contributions produced by the Newcom++ partners and regarding novel techniques designed to exploit diversity in wireless ad hoc networks on both the transmit and the receiver side are briefly presented. With regard to the cooperation at the transmit side, the problem of topology creation and distributed node management in order to establish a cooperative transmission in an effective way is tackled. In the field of cooperation at the receive side, linear combining schemes for virtual MIMO systems are studied. Besides this, the concept of cooperation can be also exploited in different scenarios: in the last part of this Chapter, some innovative insights related to the exploitation of user cooperation in cellular networks are also proposed. Finally, the design and analysis of a MAC protocol that allows executing a Cooperative Automatic Retransmission reQuest (C-ARQ) scheme in wireless networks is presented. Whenever a destination station receives a data packet with errors, it requests retransmissions from any of the stations which overheard the original transmission and can act as spontaneous relays or helpers.

At the moment of the publication of this deliverable, the contents surveyed in this Chapter mainly originate from the contributions of various institutions. Within these, a particular emphasis should be put on a tutorial about game theoretical methods for diversity exploitation in wireless network [SPVV], jointly written by the research groups of CNIT/MORE and UCL and currently under review. Game theory is slowly becoming an accepted mathematical tool to devise clever solutions for the resource management problem in various communication scenarios. However, we believe that the literature on the application of game theory to wireless network is still far from reaching a certain maturity. We expect that this collaboration may continue in the next future even in the direction of the exploitation of these concepts for the development of novel fully distributed solutions for ad hoc wireless networks.

5.1 Cooperation at the transmit side: Distributed node management and cooperation

As in a real multiantenna transmitter, in order to exploit the spatial diversity (hence, to achieve a full diversity order) a proper coding is required. Fortunately, the technical literature about distribute coding schemes is huge and some novel insight in this field can be found in the apposite Section.

However, as pointed out in the previous deliverable concerning the state of the art on cooperation, a fundamental problem of cooperative ad hoc networks is represented by devising strategies for the selection of wireless nodes which should assist data communications in order to maximize network performance. Despite its importance, this problem is not usually tackled in most of the available literature about user cooperation, even if some important exceptions can be found (refer to the same deliverable for a deeper insight).

In [SPV09a], the authors adopt a cross layer approach to develop a cooperative link management in wireless ad hoc network. In particular, in this paper the problem of the establishment of a multi-hop cooperative link based on a double string topology (in which a couple of nodes for each relay stage are exploited) is tackled. The link is established by an utility-based distributed routing algorithm that aims at guaranteeing a balanced use of the available radio resources (in terms of energy and bandwidth) of network nodes, jointly improving the network lifetime and its ability to satisfy connectivity requests. The considered topology lends itself to the use of cooperative transmission techniques in a natural fashion, so that large energy saving (or, equivalently, more reliable links) with respect to traditional solutions can be achieved.

Another fundamental point raised in the previous deliverable is the feasibility of cooperation techniques taking into consideration the presence of selfish nodes. In fact, cooperation among multiple terminals in wireless networks can certainly improve performance or extend lifetime in the long run, but requires each relay to sacrifice a fraction of its own resources (e.g., its energy) in the short run for the benefit of other nodes. These considerations evidence the importance of developing communication strategies able to incentive the cooperation among multiple nodes with the aim of maximizing their own degree of satisfaction. This complicated problem can be tackled pragmatically resorting to *game theory*.

Within this field, two research works [SPV09c], [SPV09b] show how this tool can allow to tackle complicated problems related to the node management in a cooperative link from an innovative point of view. In particular the proposed solutions evidence that cooperative communication techniques can be exploited without the drawbacks represented by the large management complexity and the transmission overhead needed in wireless ad hoc networks. These results show how to bring cooperative techniques from theory to implementable real life applications.

The scenario considered in both the above mentioned works is that of a wireless ad hoc network consisting of N energy-constrained selfish nodes, each endowed with a single antenna. In this network data packets are delivered from an arbitrary source node to an arbitrary destination node thanks to a number of relays establishing a cooperative wireless link. This link, in the most general case, will be established by a multi-hop connection in which each relay consist of a cluster of cooperative nodes.

The study proposed in [SPV09c] considers a network consisting of selfish nodes, each exploiting the network to carry its own information flow, but unwilling to spend its limited resources to relay data packets originating from other nodes. Instead, the analysis carried out in [SPV09b] refers to both the case of a network populated by selfish nodes and to that of a network consisting of nodes prone to cooperation.

In [SPV09c] the authors develop a novel strategy for a cooperative transmission accomplished by a relay cluster towards a destination, functionally equivalent to a transmission selection scheme but managed in a fully distributed fashion. This strategy consists of an autonomous choice, made by each potential relay in an autonomous fashion, between two simple alternatives: transmitting (hence, forwarding the information data packet) or remaining silent. For each node, this choice is based only on the status of the node itself (hence, on its available resources and its experienced channel conditions other than its personal needs of future cooperation expected from other nodes) and on the behavior of the other neighboring nodes. This allows to coordinate the transmissions among the potential relays without any explicit information exchange between them, so avoiding the transmission overhead that usually characterizes cooperative transmission protocols. Thanks to this feature, this solution offers a larger throughput and higher energy efficiency than other traditional communication techniques exploiting transmission selection.

Moreover, it has to be pointed out that the effectiveness of this transmission strategy is not affected by the presence of selfish nodes and offer a viable way to exploit cooperative transmission techniques even in these circumstances.

In the work mentioned above, the authors consider unsynchronized and uncoded transmissions from a relay cluster so that a single node is allowed to transmit at a time in order to not make its transmission collide. This transmission scheme is generalized in [SPV09b] where a coded transmission based on distributed randomized – orthogonal space time coding (DR-OSTC) is considered. In the new scenario, multiple nodes can join a transmission cluster in order to actively contribute to the same transmission. In particular, this work proposes a novel forwarding strategy, able to manage the node participation in the cooperative coded transmission introduced before. Even in this case, the proposed scheme works in a fully distributed fashion since it consists of autonomous choices made by each node and operates without any need of explicit information exchange, neither for the selection of the nodes that has to join an active relay set nor for the codeword assignment.

The advantage of having a control on the number of nodes joining a cooperative transmission cluster achieved without the problems related with a traditional node management protocol allow, on the one hand, to eliminate the overhead associated with this control and, on the other hand, to preserve the high energy efficiency of the cooperative link. In fact, the solution proposed in [SPV09b] makes a cooperative link able to exploit only the needed number of potential relay nodes for its transmission, avoiding an energy waste; at the same time, this solution make the link performance weakly affected by an imperfect estimation of the characteristics of the link itself.

5.2 Cooperation at the receive side: Linear Combining Schemes for Virtual MIMO Systems

In order to make the best possible use of diversity in virtual MIMO systems, it is very important to combine various sources of diversity in an optimal manner. For this reason, is also convenient to exploit the different multipath components of a received signal in order to further enhance the diversity order achieved.

In [GPKM06], linear *minimum mean-squared error* (MMSE) receivers are studied for optimal combination of signal from multipath and repetition diversity domains. Similarly, [YV04] considers MMSE multiuser detection for cooperative diversity in *code-division multiple-access* (CDMA) systems. In addition to coherent multiuser detection [VWL06], non-coherent processing is studied in [CL04] from a cooperative diversity perspective.

Optimal combination of signals from various diversity sources can require high computational complexity. In other words, when the optimal symbol detector is designed based on all the signals from various diversity sources, such as spatial diversity and multipath diversity, the resulting complexity can be prohibitive [Ver98]. In those cases, it is practical to consider linear receiver structures, and to try to find the optimal "linear" receiver. In [ZL05], optimal linear receivers are derived for impulse radio ultra-wideband systems. Similarly, [GKPM04] considers the optimal linear combination of signal components from repetition and multipath diversity domains. In some cases, even the computational complexity of optimal linear combining can be quite high. In those cases, suboptimal combination in multiple stages can reduce the complexity by sacrificing a certain degree of optimality [WMK04], [GMKP06].

Consider a wireless system in which K nodes form a virtual transmitter, and send their signals to a receiver. Assume that each signal propagates through a tapped-delay line channel with L multipath components. Then, the lth multipath component (MPC) of the received signal from the kth node can be expressed as

$$r_{k,l} = \sqrt{E_k}\,\alpha_{k,l} b + n_{k,l}\ , \tag{73}$$

where $\alpha_{k,l}$ represents lth channel coefficient (normalized) of node k, $\sqrt{E_k}$ is the signal energy for the kth node, b is the information symbol, and $n_{k,l}$ is the zero-mean noise component.

Let $\mathbf{r}$ contain all the received samples in (73); i.e.,

$$\mathbf{r} = b\beta + \mathbf{w}\ , \tag{74}$$

where β represents the signal part, and $\mathbf{w}$ includes the noise samples. Note that the noise $\mathbf{w}$ can include the effect of multiple access and narrowband interference; hence, it is not white in general.

A linear receiver combines the elements of $\mathbf{r}$ and obtains a decision statistic as follows:

$$y = \theta^T \mathbf{r}\ , \tag{75}$$

where θ is the weighting vector. The MMSE weights that maximize the *signal to interference plus noise ratio* (SINR) of the received signal in (74) can be obtained as [Poo94]

$$\theta_{\text{MMSE}} = \left(\beta\beta^T + \mathbf{R_w}\right)^{-1}\beta = c\mathbf{R_w}^{-1}\beta \tag{76}$$

where $\mathbf{R_w} = E\{\mathbf{ww}^T\}$ is the correlation matrix, and $c = (1+\text{SINR})^{-1}$ with $\text{SINR} = \beta^T\mathbf{R_w}^{-1}\beta$.

Note that the linear MMSE receiver requires inversion of an $K \times L$ matrix to calculate the decision variable. Hence, it can have high complexity when the number of nodes or multipath components is large.

The complexity can be reduced if the combination of received samples in (73) is performed in two steps (suboptimally) [GPKM06]. First, the samples are divided into N_1 groups:

$$\mathbf{r}_j = b\beta_j + \mathbf{w}_j\ , \tag{77}$$

for $j = 1,\ldots,N_1$. Then, the samples in each group are combined by the MMSE criterion using the following weighting vectors:

$$\theta_j = \left(\beta_j\beta_j^T + \mathbf{R}_{\mathbf{w}_j}\right)^{-1}\beta_j = c_j\mathbf{R}_{\mathbf{w}_j}^{-1}\beta_j\ , \tag{78}$$

where $c_j = (1+\beta_j^T\mathbf{R}_{\mathbf{w}_j}^{-1}\beta_j)^{-1}$ and $\mathbf{R}_{\mathbf{w}_j} = E\{\mathbf{w}_j\mathbf{w}_j^T\}$. At this point, N_1 combined samples are obtained: $\{\theta_j^T\mathbf{r}_j\}_{j=1}^{N_1}$. Note that the MMSE combining in each group ignores the information about the other groups, which causes a loss in optimality. However, this is the main source of complexity reduction.

Let $\tilde{\mathbf{r}}$ represent the set of combined samples after the first step; i.e.,

$$\tilde{\mathbf{r}} = \left[\theta_1^T\mathbf{r}_1 \cdots \theta_{N_1}^T\mathbf{r}_{N_1}\right]^T \tag{79}$$

which can be expressed as

$$\tilde{\mathbf{r}} = b\tilde{\beta} + \tilde{\mathbf{w}}\ , \tag{80}$$

where $\tilde{\beta} = [\theta_1^T\beta_1 \cdots \theta_{N_1}^T\beta_{N_1}]^T$ and $\tilde{\mathbf{w}} = [\theta_1^T\mathbf{w}_1 \cdots \theta_{N_1}^T\mathbf{w}_{N_1}]^T$. Then, the MMSE weighting vector can be obtained as

$$\gamma = \left(\tilde{\beta}\tilde{\beta}^T + \mathbf{R}_{\tilde{\mathbf{w}}}\right)^{-1}\tilde{\beta} = \tilde{c}\mathbf{R}_{\tilde{\mathbf{w}}}^{-1}\tilde{\beta} \tag{81}$$

with $\mathbf{R}_{\tilde{\mathbf{w}}} = E\{\tilde{\mathbf{w}}\tilde{\mathbf{w}}^T\}$.

It can be shown that the complexity of the two-step MMSE algorithm is $\mathscr{O}(N^{1.8})$ compared to $\mathscr{O}(N^3)$ of the optimal MMSE scheme [GPKM06]. Note that this complexity reduction is obtained by sacrificing certain degrees of optimality from the spatial and/or multipath diversity domains.

It is observed that the two-step MMSE algorithm becomes the optimal linear scheme for any given channel gains, when the noise samples in $\mathbf{w}_1 \ldots, \mathbf{w}_{N_1}$ in (77) are mutually uncorrelated.

In conclusion, optimal combination of signal samples from multipath components of a number of nodes has been considered in the framework of linear estimation. The optimality criterion has been selected as the MMSE, and both the optimal and a two-step suboptimal technique have been described. As two special cases, the proposed two-step approach covers the scenarios in which the receiver combines only spatial components in an MMSE-optimal manner, and it combines only the multipath components in an MMSE-optimal manner.

5.3 Cooperation in cellular networks

This paragraph focus on a variety of aspects of mobile networks adhering to a meticulous information theoretic analysis. Specifically, in the research surveyed, a simple cellular model of the *Wyner* family is studied and different fundamental aspects are analyzed. In these work, advanced multi-cell processing techniques are addressed, and further on issues fundamental to mobile ad-hoc network, as user cooperation, as well as finite backhaul capabilities of decoding node points are examined. Thus this gives the interpretation of a central-processing decoder connected to virtual sensing antennas via backhaul limited links. The impact of fading is closely studied, and in the multi-cell decoding regime, it is demonstrated that fading may have a beneficial impact providing in fact (nature based) user activity.

Further structures that account for DS-CDMA technique of a changing user environment are also studied, where the advantages of multiuser detection (including successive interference cancellation) of strongest users is in focus.

In [SSPS], multicell processing in the form of joint encoding for the downlink of a cellular system is studied under the assumption that the base stations (BSs) are connected to a *central processor* (CP) via finite-capacity links (finite-capacity backhaul). To obtain analytical insight into the impact of a finite capacity backhaul on the downlink throughput, the investigation focuses on a simple linear cellular system (as for a highway or a long avenue) based on the Wyner model.

Different transmission schemes are proposed that require varying degrees of knowledge regarding the system codebooks at the BSs. Achievable rates are derived in closed-form and compared with an upper bound. Performance is also evaluated in asymptotic regimes of interest (high backhaul capacity and high SNR) and further corroborated by numerical results. The major finding of this work is that even in the presence of oblivious BSs (that is, BSs with no information about the codebooks) multicell processing is able to provide ideal performance with relatively small backhaul capacities, unless the application of interest requires high data rate (i.e., high SNR) and the backhaul capacity is not allowed to increase with the SNR. In these latter cases, some form of codebook information at the BSs becomes necessary.

Multi-user communication channels characterized by short-range intra-cell broadcasting (similar to the Wyner model) have been studied through the theory of products of random matrices in [LZS09]. Here, the authors study the fluctuations of the per-cell sum-rate capacity in the non-ergodic regime and provide results of the type of *central limit theorem* (CLT) and *large deviations* (LD).

In [LSSZ09] the authors study the spectrum of certain large random Hermitian Jacobi matrices. These matrices are known to describe certain communication setups. In particular they are interested in an uplink cellular channel which models mobile users experiencing a soft-handoff situation under joint multicell decoding. Considering rather general fading statistics , this research provide a closed form expression for the per-cell sum-rate of this channel in high-SNR, when an intra-cell TDMA protocol is employed. Since the matrices of interest are tridiagonal, their eigenvectors can be considered as sequences with second order linear recurrence. Therefore, the problem is reduced to the study of the exponential growth of products of two by two matrices.

For the case where K users are simultaneously active in each cell, in [LSSZ09] a series of lower and upper bound on the high-SNR power offset of the per-cell sum-rate, which are considerably tighter than previously known bounds, are obtained.

The problem of multi-user detection for randomly spread *direct-sequence* (DS) *code-division multiple access* (CDMA) over flat fading channels is considered in [SSKL09]. The analysis focuses on the case of many users, and large spreading sequences such that their ratio, which is the system load, is kept fixed. Spectral efficiency of practical linear detectors such as matched filter and decorrelator employing *successive interference cancellation* (SIC) at the receiver is derived. This is used to extend the notion of strongest users detectors for SIC receivers. The strongest users detectors system design relies on an outage approach where each user transmits in a single layer (fixed rate). This constraint is relaxed in [SSKL09], and the concept is combined with a broadcast approach, providing higher achievable rates.

5.4 Persistent Relay CSMA: A MAC Protocol for a Cooperative ARQ Scheme

The focus of this section is on the design and analysis of a MAC protocol that allows executing a Cooperative Automatic Retransmission reQuest (C-ARQ) scheme in wireless networks. These schemes exploit the broadcast nature of the wireless channel in the following manner; whenever a destination station receives a data packet with errors, it requests retransmissions from any of the stations which overheard the original transmission and can act as spontaneous relays or helpers. Retransmissions from these relays may allow for the exploitation of either space or time diversity and lead to higher throughput, extended coverage, or improved energy-efficiency. However, efficient Medium Access Control (MAC) protocols are necessary to coordinate the access of the relays to the wireless channel. This is the main motivation for the design of the Persistent Relay Carrier Sensing Multiple Access (PRCSMA) protocol detailed in the following.

5.4.1 Protocol Description

The main goal of PRCSMA is to enable IEEE 802.11-based stations to execute a C-ARQ scheme. A key constraint in the design of PRCSMA was to modify the legacy MAC rules as less as possible in order to facilitate backwards compatibility. When using PRCSMA, all the stations must listen to every ongoing transmission and keep a copy of any received data packet regardless of its destination address until it is acknowledged by the intended destination. Whenever a data packet is received with errors at the destination, a cooperation phase is initiated by broadcasting a Call for Cooperation (CFC) control packet after sensing the channel idle for a SIFS period. Regular data transmissions in IEEE 802.11 are done after a longer silence period (DIFS), and thus cooperation phases have priority over regular data traffic. The CFC packet invites all the stations which overheard the original transmission to become active relays. Those stations which become active relays execute the rules of the IEEE 802.11 MAC protocol with the two following modifications: i) all the relays backoff before attempting to transmit for the first time in order to avoid a certain collision, and ii) there is no expected ACK associated to each retransmission.

5.4.2 PRCSMA Analytical Model

In a C-ARQ scheme, any station should be able to assess the suitability of initiating a cooperative phase before actually sending the CFC packet and involving more users (than the source and the destination in a communication link. Therefore, it is necessary to develop models that allow estimating the average delay associated to a C-ARQ scheme, defined as the expected duration of a whole packet transmission time including the cooperation phase. Accordingly, an analytical model to evaluate the average packet transmission delay when using PRCSMA is presented in this section. The model is valid for a saturated IEEE 802.11 network.

The backoff counter of a PRCSMA station can be modeled using the embedded Markov Chain presented and analyzed by Wu et al. in [WPL$^+$02]. A time-slotted system is considered where a total of n stations are within the transmission range of each other. A slot is defined as the unit of time between consecutive backoff counter decrements and it has a different duration depending on whether a slot is idle or busy. The main assumption of the model is that the probability of having a collision when attempting to transmit in a given time slot, p, is considered to be constant along time. $W_0 = CW_{min}$ is the size of the initial contention window (CW), m is the maximum backoff stage, and m' is the maximum number of retransmission attempts before discarding a packet. It is worth noting that if $m' > m$, then the backoff window will remain at the maximum stage (m) for the last $m' - m$ transmission attempts. Therefore, the probability that one station attempts to transmit in a given slot, denoted by τ, is derived in [WPL$^+$02] as

$$\tau = \frac{1-p^{m+1}}{1-p} b_{0,0}, \tag{82}$$

where,

$$b_{0,0} = \begin{cases} \frac{2(1-2p)(1-p)}{W_0(1-(2p)^{(m+1)})(1-p)+A} & , \text{ m} \leq \text{m}' \\ \frac{2(1-2p)(1-p)}{W_0(1-(2p))^{(m'+1)}(1-p)+A+B} & , \text{ m} > \text{m}' \end{cases} , \tag{83}$$

and $A = (1-2p)(1-p^{m+1})$ and B $=W_0 2^{m'} p^{m'+1}(1-2p)(1-p^{m-m'})$. Therefore, the probability of collision p in a given slot is equal to

$$p = 1-(1-\tau)^{n-1}. \tag{84}$$

The probability that at least one of the n stations attempts to transmit in a given slot, P_{tr}, can be expressed as

$$P_{tr} = 1 - (1-\tau)^n, \tag{85}$$

and the probability of having a successful slot given that a station transmits, p_s, is given by

$$p_s = \frac{n\tau(1-\tau)^{n-1}}{P_{tr}}. \tag{86}$$

Finally, the probabilities of having an idle (P_i), successful (P_s) or collided (P_c) slot can be then written as

$$\begin{aligned} P_i &= 1 - P_{tr}, \\ P_s &= P_{tr} \cdot p_s = n\tau(1-\tau)^{n-1}, \\ P_c &= P_{tr}(1-p_s). \end{aligned} \tag{87}$$

Using these expressions, the average packet transmission delay is analyzed as follows.

The average packet transmission delay in the context of PRCSMA is defined as the average duration of the first failed transmission plus the average time required to completing a successful cooperation phase given that a number of K retransmissions are required to properly decode a packet received with errors at destination. This average delay is denoted by $\mathrm{E}[T_{COOP}]$ and can be calculated as

$$\mathrm{E}[T_{COOP}] = T_{min} + \mathrm{E}[T_{cont}]. \tag{88}$$

T_{min} is the minimum delay achievable in the case of attaining a perfect scheduling among all the active relays, i.e. avoiding contention. However, the perfect scheduling among the relays is not feasible unless perfect *a priori* knowledge of the relays is available and some deterministic scheduling can be run. Otherwise, the unavoidable contention among the relays may lead to silence periods and collisions that will increase the average packet transmission delay. The term $\mathrm{E}[T_{cont}]$ denotes the expected delay caused by the contention among the relays when accessing to the channel.

T_{min} can be computed as

$$T_{min} = T_0 + T_{CFC} + KT_{DR} + T_{ACK} + 4T_{SIFS}. \tag{89}$$

T_0 is the duration of the first transmission from the source station to the intended destination station. T_{CFC} and T_{ACK} are, respectively, the transmission time of the CFC and the ACK packets. T_{DR} is the time required to retransmit a single packet considering that all the relay stations transmit their cooperative packets at a same common transmission rate. This value depends on whether the basic access mechanism or the collision avoidance handshake RTS/CTS is executed by the relays, and it is equal to $T_{DR|BASIC}$ or $T_{DR|COLAV}$ respectively, and calculated as

$$\begin{aligned} T_{DR|BASIC} &= T_{DIFS} + T_{DATA} + T_{SIFS}, \\ T_{DR|COLAV} &= T_{DIFS} + T_{RTS} + T_{SIFS} + T_{CTS} + T_{SIFS} + T_{DATA} + T_{SIFS}. \end{aligned} \tag{90}$$

T_{DIFS} and T_{SIFS} are the duration of DIFS and SIFS silence periods, respectively, and T_{RTS} and T_{CTS} are the transmission times of a RTS and CTS packets. T_{DATA} is the duration of the transmission of a data packet (using the maximum available transmission rate between the relays and the destination).

On the other hand, and within the context of IEEE 802.11 (and thus of PRCSMA), the contention time of a packet is independent of the contention time of any other packet, and thus $\mathrm{E}[T_{cont}]$ can be calculated as

$$\mathrm{E}[T_{cont}] = K\mathrm{E}[T_c], \tag{91}$$

with $\mathrm{E}[T_c]$ the average contention time required to successfully retransmit a single packet. The average time elapsed between successful transmissions can be decomposed in a number of idle and collided slots with different durations. This term can be derived as follows. Recall that a successful transmission is carried out in a given slot with a probability P_s. Therefore, the average number of slots before having a successful transmission is denoted by $\mathrm{E}[X]$ and it can be calculated as

$$\mathrm{E}[X] = \sum_{k=0}^{\infty} (k+1)(1-P_s)^k P_s = P_s \left[-\frac{\partial}{\partial P_s} \sum_{k=0}^{\infty} (1-P_s)^{k+1} \right] = \frac{1}{P_s}. \tag{92}$$

According to this, the average number of non-successful slots before having a successful transmission is equal to $\mathrm{E}[X]-1$. Therefore, the total contention time will be equal to

$$\mathrm{E}[T_c] = (\mathrm{E}[X]-1)\,\mathrm{E}[T_{nss}], \tag{93}$$

where $\mathrm{E}[T_{nss}]$ is the average duration of a slot given that the slot is not successful. A slot is not successful if it is idle or collided. As previously discussed, a given slot will be idle with probability P_i, and its duration will be equal to the basic slot time, denoted by σ. On the other hand, a given slot will suffer a collision among stations with probability P_c. As for the case of the duration of a successful transmission expressed in (90), the duration of a collision depends on whether collision avoidance is used or not, and is given in (94) as

$$\begin{aligned} T_{col|BASIC} &= T_{DIFS} + T_{DATA} + T_{SIFS}, \\ T_{col|COLAV} &= T_{DIFS} + T_{RTS} + T_{SIFS} + T_{CTS\_TIMEOUT}. \end{aligned} \tag{94}$$

The term $T_{CTS\_TIMEOUT}$ is the duration of the CTS time-out period after with a collision is considered to have occurred if no CTS packet is received by the station transmitting the corresponding RTS.

Applying Bayes' theorem, the average duration of any slot given that the slot is either idle or collided can be expressed as

$$\mathrm{E}[T_{nss}] = \left(\frac{P_i}{1-P_s}\right)\sigma + \left(\frac{P_c}{1-P_s}\right) T_{collision}. \tag{95}$$

Finally, the average total contention time can be expressed as

$$\mathrm{E}[T_{cont}] = K\left(\frac{1}{P_s}-1\right)\left[\left(\frac{P_i}{1-P_s}\right)\sigma + \left(\frac{P_c}{1-P_s}\right) T_{collision}\right]. \tag{96}$$

It is worth recalling that probabilities P_s, P_c, and P_i, calculated with (87), depend on the number of active relays n, the size of the initial backoff window W_0, the maximum backoff stage m, and the maximum number of transmission attempts before discarding a packet m'.

5.4.3 Model Validation

The aim of this section is to validate the accuracy of the model presented in the previous section through computer simulations. To this end, a C++ simulator has been implemented to simulate a network formed by a total of n stations, all of them within the transmission range of each other. We assume that all the stations always have a data packet ready to be transmitted. Note that under these saturated conditions all the stations will always have a non-zero value of the backoff counter unless they are actually transmitting. The configuration parameters of the stations in the network are summarized in Table I and they have been set in accordance to the PHY layer of the Standard IEEE 802.11g [IEE].

Table 1: System Parameters

Parameter	**Value**	**Parameter**	**Value**
MAC header	34 bytes	DATA packets	1500 bytes
PHY header	96 μs	SlotTime, SIFS	10 μs
ACK, CFC	14 bytes	DIFS	50 μs
RTS	20 bytes	CTS	14 bytes

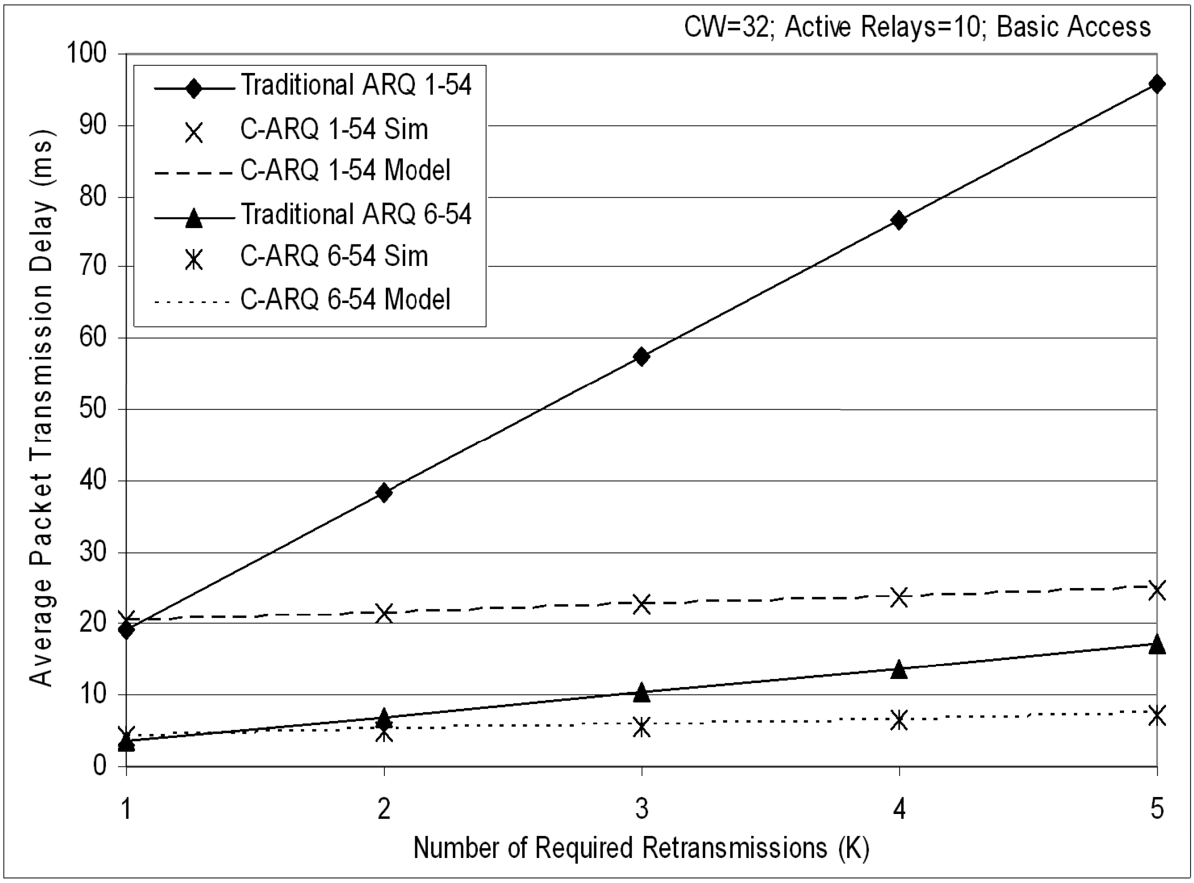

Figure 27: Average Packet Transmission Delay

In order to evaluate the impact of the transmission rates on the performance of the PRCSMA, the size of the initial CW has been set to 32 and the number of active relays (stations contending for the channel) in each cooperation phase has been set to 10. All the relay stations use the basic access method of PRCSMA (IEEE 802.11) to get access to the channel. The average packet transmission delay is illustrated in Figure 27 as a function of K and for different sets of transmission rates (the numbers separated by a dash in the legend of the plot indicate the data transmission rate from the source and the relays, respectively). First, it should be emphasized the almost perfect match between the analytical model and the simulations. As it could be expected, the ratio between the transmission rate from the source and from the relays determines how efficient the C-ARQ is in comparison to the traditional non-cooperative ARQ approach, where the retransmissions are only requested from the source at the best available transmission rate between the source and the intended destination station and without contention between consecutive retransmissions.

5.5 Ongoing research

Future work in this area will concern various topics; among these we mention the problem of node organization in multi source-destination and multihop ad hoc networks. In these scenarios research efforts will mainly devoted to the development of protocols able to set up efficiently, opportunistically and in a fully distributed way proper topologies for exploiting specific cooperative communication schemes.

REFERENCES

[ACLY00] R. Ahlswede, N. Cai, S-Y. R. Li, and R. W. Yeung. Network Information Flow. *IEEE Trans. on Inf. Theory*, 46(4):1204–1216, July 2000.

[AFM04] I. Abou-Faycal and M. Médard. Optimal uncoded regeneration for binary antipodal signaling. In *Proc. IEEE Int. Conf. on Communications (ICC'04)*, Paris, France, pp. 742–746, Jun. 2004.

[AMC07] I. F. Akyildiz, T. Melodia, and K. R. Chowdury. Wireless multimedia sensor networks: A survey. *IEEE Wireless Commun.*, 14(6):32–39, Dec. 2007.

[BCGiF04] J.J. Boutros, E. Calvanese, and A. Guillén i Fàbregas. Turbo code design for block fading channels. In *Proc. Allerton Conf. on Communication and Control, Illinois*, pages 943–953, 2004.

[BF07] A. Boukerche and X. Fei. Energy-efficient multi-hop virtual mimo wireless sensor network. In *Proc. IEEE Wireless Communications and Networking Conference*, pages 4301–4306, Mar. 2007.

[BGiFBZ07] J.J. Boutros, A. Guillén i Fàbregas, E. Biglieri, and G. Zémor. Low-Density Parity-Check Codes for Nonergodic Block-Fading Channels. *IEEE Trans. on Inf. Theory*, submitted Oct. 2007.

[BGiFC05] J.J. Boutros, A. Guillén i Fàbregas, and E. Calvanese. Analysis of coding on non-ergodic block fading channels. In *Proc. Allerton Conf. on Communication and Control, Illinois*, pages 1096–1106, 2005.

[Bou09] J.J. Boutros. Diversity and coding gain evolution in graph codes. In *Proc. ITA'2009 San Diego*, pages 34–43, 2009.

[BPS98] E. Biglieri, J. Proakis, and S. Shamai. Fading channels: information-theoretic and communications aspects. *IEEE Trans. on Inf. Theory*, 44(6):2619–2692, Oct. 1998.

[BY06] A. Bilgin and A. O. Yilmaz. Cooperative diversity techniques for wireless networks. *IEEE Signal Processing and Communications Applications*, pages 1–4, Apr. 2006.

[BZ09a] C. Buratti and Alberto Zanella. Capacity analysis of two-hop virtual MIMO systems in a Poisson field of nodes. In *Proc. of IEEE Vehicular Technologies, VTC fall 2009*, pages 1–6, Barcelona, Spain, April 2009.

[BZ09b] C. Buratti and Alberto Zanella. Performance of distributed multi-stage virtual MIMO systems with random position of ancillary nodes. In *NEWCOM++ special session at the 12th URSI national symp.*, Warsaw, Poland, June 2009.

[CCF08] Laura Cottatellucci, Terence Chan, , and Nadia Fawaz. Large system design and analysis of protocols for decode-forward relay networks. In *Proc. of 6th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks and Workshops, 2008. WiOPT 2008.*, pages 591 – 596, Berlin, Germany, April 2008.

[CHS+09] Shengshan Cui, Alezander M. Haimovich, Oren Somekh, H. Vincent Poor, and Shlomo Shamai (Shitz). Throughput scaling of wireless networks over fading channels. In *Proc. IEEE International Conference on Communications (ICC 2009)*, June 2009.

[CL81] T. Cover and C. Leung. An achievable rate region for the multiple-access channel with feedback. *IEEE Trans. Inform. Theory*, 27(3):292–298, May 1981.

[CL04] D. Chen and J. N. Laneman. Noncoherent demodulation for cooperative diversity in wireless systems. In *Proc. IEEE Global Telecommunications Conference*, pages 31–35, Dec. 2004.

[CV09a] B. K. Chalise and L. Vandendorpe. Joint optimization of multiple MIMO relays for multipoint to multi-point communications. In *Proceedings of the Fifth IEEE Workshop on Signal Processing for Advances on Wireless Communications*, Perugia, Italy, 2009.

[CV09b] B. K. Chalise and L. Vandendorpe. Performance analysis of linear receivers in a MIMO relaying system. *IEEE Commun. Lett.*, 13(5), May 2009.

[CV09c] B. K. Chalise and L. Vandendorpe. Performance analysis of selective source beamforming for AF-MIMO relay channel. *in preparation*, 2009.

[DBL09a] B. Djeumou, E. V. Belmega, , and S. Lasaulce. Interference relay channels - part ii: Power allocation games. *IEEE Trans. on Communications*, March 2009. submitted.

[DBL09b] B. Djeumou, E. V. Belmega, , and S. Lasaulce. Interference relay channels - parti: Transmission rates. *IEEE Trans. on Communications*, March 2009. submitted.

[DBM09] D. Duyck, J.J. Boutros, and M. Moeneclaey. Low-Density Graph Codes for Slow Fading Relay Channels. *IEEE Trans. on Inf. Theory*, submitted Feb. 2009.

[DCBM09] D. Duyck, D. Capirone, J.J. Boutros, and M. Moeneclaey. A full-diversity joint network-channel code construction for cooperative communications. In *PIMRC 2009*, Submitted June 2009.

[DDA02] M. Dohler, J. Dominguez, and H. Aghvami. Link capacity analysis for virtual antenna arrays. In *Proc. of IEEE Vehicular Technology Conference, VTC 2002-Fall.*, September 2002.

[DGA07] M. Dohler, A. Gkelias, and H. Aghvami. Two-hop distributed mimo communication system. *IEEE Electronics Letters*, 39:1350–1351, September 2007.

[DLK07] B. Djeumou, S. Lasaulce, and A. G. Klein. Practical quantize-and-forward schemes for the frequency division relay channel. *EURASIP Journal on Wireless Communications and Networking*, 2007, Article ID 20258, 11 pages, doi:10.1155/2007/20258, 2007.

[Doh03] M. Dohler. Virtual antenna arrays. Technical Report PhD Thesis, King's College London, University of London, London, 2003.

[EK09] Petros Elia and P. Vijay Kumar. Space-time codes that are approximately universal for the parallel, multi-block and cooperative ddf channels. In *Proc. of IEEE International Symposium on Information Theory*, Seoul, Korea, July 2009.

[FK06] Frank H. P. Fitzek and Marcos D. Katz, editors. *Cooperation in Wireless Networks: Principles and Applications – Real Egoistic Behavior is to Cooperate!* Springer, Dordrecht, The Netherlands, 2006.

[FZDG08a] N. Fawaz, K. Zarifi, M. Debbah, and D. Gesbert. Asymptotic capacity and optimal precoding in mimo multi-hop relay networks. December 2008. submitted to IEEE Trans. on Information Theory.

[FZDG08b] N. Fawaz, K. Zarifi, M. Debbah, and D. Gesbert. Asymptotic capacity and optimal precoding strategy of multi-level precode & forward in correlated channels. In *Proc. IEEE ITW'08, Information Theory Workshop*, May 2008.

[GC07] Bo Gui and Leonard J. Cimini. Bit loading algorithms for cooperative ofdm systems. In *Military Communications Conference, 2007. MILCOM 2007. IEEE*, pages 1–7, Oct. 2007.

[GJ06] K. S. Gomadam and S. A. Jafar. On the capacity of memoryless relay networks. In *Proc. IEEE Int. Conf. on Communications (ICC'06)*, Istanbul, Turkey, pp. 1580–1585, Jun. 2006.

[GKPM04] S. Gezici, H. Kobayashi, H. V. Poor, and A. F. Molisch. Optimal and suboptimal linear receivers for time-hopping impulse radio systems. In *Proc. IEEE Conference on Ultra Wideband Systems and Technologies*, pages 11–15, May 2004.

[GM09] D. Gregoratti and X. Mestre. Random DS/CDMA for the amplify and forward relay channel. *IEEE Trans. Wireless Commun.*, 8(2):1017–1027, February 2009.

[GMKP06] S. Gezici, A. F. Molisch, H. Kobayashi, and H. V. Poor. Low-complexity mmse combining for linear impulse radio uwb receivers. In *Proc. IEEE International Conference on Communications*, pages 11–15, Jun. 2006.

[GMeda] D. Gregoratti and X. Mestre. Decode and forward relays: Full diversity with randomized distributed space-time coding. In *Proc. IEEE ISIT 2009*, Seoul, Korea, June 28–July 3 2009, to be published.

[GMedb] D. Gregoratti and X. Mestre. Diversity analysis of a randomized distributed space-time coding in an amplify and forward relay channel. In *Proc. IEEE ICC 2009*, Dresden, Germany, June 14–18 2009, to be published.

[GMedc] D. Gregoratti and X. Mestre. Diversity order for the amplify-and-forward multiple-relay channel with randomized distributed space-time coding. In *Proc. EUSIPCO 2009*, Glasgow, Scotland, August 24–28 2009, to be published.

[GMedd] D. Gregoratti and X. Mestre. The single relay channel: Does randomized coding increase diversity? In *Proc. ICT-MobileSummit 2009*, Santander, Spain, June 10–12 2009, to be published.

[GPKM06] S. Gezici, H. V. Poor, H. Kobayashi, and A. F. Molisch. Optimal and suboptimal linear receivers for impulse radio uwb systems. In *Proc. IEEE International Conference on Ultra-Wideband (ICUWB 2006)*, pages 161–166, Waltham, MA, Sept. 2006.

[GS00] A. Ganesan and A. M. Sayeed. A virtual mimo framework for multipath fading channels. In *Proc. Thirty-Fourth IEEE Conference on Signals, Systems and Computers*, pages 537–541, Oct. 2000.

[GSG+09a] Deniz Gunduz, Osvaldo Simeone, Andrea Goldsmith, H. Vincent Poor, and Shlomo Shamai (Shitz). Relaying simultaneous multicast messages. In *Information Theory Workshop on Networking and Information Theory (ITW2009)*, June 2009.

[GSG+09b] Deniz Gunduz, Osvaldo Simeone, Andrea Goldsmith, H. Vincent Poor, and Shlomo Shamai (Shitz). Relaying simultaneous multicasts' via structured codes. In *IEEE Symposium on Information Theory (ISIT2009)*, June 2009.

[GW75] N. Gaarder and J. Wolf. The capacity region of a multiple-access discrete memoryless channel can increase with feedback (corresp.). *IEEE Trans. Inform. Theory*, 21(1):100–102, Jan 1975.

[Hag88] J. Hagenauer. Rate-compatible punctured convolutional codes (RCPC codes) and their applications. *IEEE Trans. on Commun.*, 36(4):389–400, Apr. 1988.

[HD06] C. Hausl and P. Dupraz. Joint network-channel coding for the multiple-access relay channel. In *Proc. IEEE Comm. Soc. on Sensor and Ad Hoc Comm. and Networks, SECON '06*, pages 817–822, 2006.

[HD07] J. Hu and T.M. Duman. Low Density Parity Check Codes over Wireless Relay Channels. *IEEE Trans. on Wireless Commun.*, 6(9):3384–3394, Sep. 2007.

[HK09] J. Hou and R. Koetter. On the multiple access relay channel with relay-source feedback. In *Proc. IEEE Information Theory Workshop*, 2009.

[HKM04] J. Ha, J. Kim, and S.W. McLaughlin. Rate-Compatible Puncturing of Low-Density Parity-Check Codes. *IEEE Trans. on Inf. Theory*, 50(11):2824–2836, Nov. 2004.

[HM06] A. Host-Madsen. Capacity bounds for cooperative diversity. *IEEE Trans. Inf. Theory*, 52(4):1522–1544, Apr. 2006.

[HSOB05] C. Hausl, F. Schreckenbach, I. Oikonomidis, and G. Bauch. Iterative network and channel decoding on a tanner graph. 2005.

[Hua83] T. C. Huang. Signaling performance over a piecewise linear limited channel in the presence of interference and gaussian noise. *IEEE Trans. Commun.*, 31:861–870, Jul. 1983.

[Hun04] T.E. Hunter. *Coded cooperation: a new framework for user cooperation in wireless systems*. PhD thesis, University of Texas at Dallas, 2004.

[IEE] IEEE. Ieee std. 802.11g, supplement to part 11: Wireless lan medium access control (mac) and physical layer (phy) specifications; further high-speed physical layer extension in the 2.4ghz band.

[Jaf05] H. Jafarkhani. *Space-Time Coding: Theory and Practice*. Cambridge University Press, New York, NY, USA, 2005.

[Jay06] S. K. Jayaweera. Virtual mimo-based cooperative communication for energy-constrained wireless sensor networks. *IEEE Trans. Wireless Commun.*, 5(5):984–989, May 2006.

[JKSK07] G. Jakllari, S. V. Krishnamurthy, M. Faloutsos S.V., and P. V. Krishnamurthy. On broadcasting with cooperative diversity in multi-hop wireless networks. *IEEE J. Sel. Areas Commun.*, 25(2):484–496, Feb. 2007.

[KH00] R. Knopp and P.A. Humblet. On coding for block fading channels. *IEEE Trans. on Inf. Theory*, 46(1):189–205, Jan. 2000.

[KSA04] M. A. Khojastepour, A. Sabharwal, and B. Aazhang. Lower bounds on the capacity of gaussian relay channel. In *Proc. IEEE Conf. on Information Sciences and Systems (CISS'04)*, Princeton, NJ, USA, pp. 597–602, Mar. 2004.

[KvW00] G. Kramer and A. van Wijngaarden. On the white gaussian multiple-access relay channel. In *Proc. of IEEE Int. Symp. Information Theory*, page 40, Sorrento, Italy, June 2000.

[Lan02] J. Nicholas Laneman. *Cooperative diversity in wireless networks: Algorithms and architectures*. Ph.d. dissertation, Massachusetts Institute of Technology, Cambride, MA, September 2002.

[LL06] G. Li and H. Liu. Resource allocation for ofdma relay networks with fairness constraints. *Selected Areas in Communications, IEEE Journal on*, 24:2061–2069, Nov. 2006.

[LN02] J. Li and K. Narayanan. Rate-compatible low density parity check codes for capacity-approaching arq schemes in packet data communications. In *Proc. IAESTED Int. Conf. on Comm., Internet, and Info. Tech.(CIIT '02), St. Thomas, Virgin Islands, USA*, pages 201–206, 2002.

[LSSZ09] N. Levy, O. Somekh, S. Shamai (Shitz), and O. Zeitouni. On certain large random hermitian jacobi matrices with application to wireless communication. *IEEE Trans. Inf. Theory*, 55(4):1534–1554, Apr. 2009.

[LTV08] Jerome Louveaux, Rodolfo Torrea, and Luc Vandendorpe. Efficient algorithm for optimal power allocation in ofdm transmission with relaying. In *IEEE ICASSP08*, pages 3257–3260, Las Vegas, CA, May 2008.

[LTW04a] J. N. Laneman, D. Tse, and G. W. Wornell. Cooperative diversity in wireless networks: Efficient protocols and outage behavior. *IEEE Trans. Inf. Theory*, 50(12):3062–3080, Dec. 2004.

[LTW04b] J. N. Laneman, D. N. C. Tse, and G. W. Wornell. Cooperative diversity in wireless networks: Efficient protocols and outage behavior. *IEEE Trans. Inf. Theory*, 50(12):3062–3080, Dec. 2004.

[LTW04c] J. N. Laneman, D. N. C. Tse, and G. W. Wornell. Cooperative diversity in wireless networks: Efficient protocols and outage behaviour. *IEEE Transactions on Information Theory*, 50:3062–3080, Dec. 2004.

[LTW04d] J.N. Laneman, D. Tse, and G.W. Wornell. Cooperative diversity in wireless networks: Efficient protocols and outage behavior. *IEEE Trans. on Inf. Theory*, 50(12):3062–3080, Dec. 2004.

[LW03] J. N. Laneman and G. W. Wornell. Distributed space-time coded protocols for exploring cooperative diversity in wireless networks. *IEEE Transactions on Information Theory*, 49:2415–2425, Oct. 2003.

[LZS09] N. Levy, O. Zeitouni, and S. Shamai (Shitz). Central limit theorem and large deviations of the fading wyner cellular model via product of random matrices theory. *Problems of Information Transmission*, 45(1):5–21, Mar. 2009.

[MCMS07] V. Mahinthan, L. Cai, J. W. Mark, and X. Shen. Maximizing cooperative diversity energy gain for wireless networks. *IEEE Trans. Wireless Commun.*, 6(7):2530–2539, July 2007.

[MDLB09] S. Medina, M. Debbah, S. Lasaulce, and H. Bogucka. On the benefits of bandwidth limiting in decentralized vector multiple access channels. In *4th ICST/IEEE Intl. Conf. on Cognitive Radio Oriented Wireless Networks and Communications (CROWNCOM)*, June 2009.

[Meu71] E.C. van der Meulen. Three-Terminal Communication Channels. *Advances in Applied Probability*, 3(1):120–154, 1971.

[NHH04] A. Nosratinia, T. E. Hunter, and A. Hedayat. Cooperative communication in wireless networks. *IEEE Commun. Mag.*, 42(10):74–80, Oct. 2004.

[Poo94] H. V. Poor. *An Introduction to Signal Detection and Estimation*. Springer-Verlag, 1994.

[Ros65] J. Rosen. Existence and uniqueness of equilibrium points for concave n-person games. *Econometrica*, 33:520–534, 1965.

[RU08] T.J. Richardson and R.L. Urbanke. *Modern Coding Theory*. Cambridge University Press, 2008.

[Say03] A. M. Sayeed. A virtual mimo channel representation and applications. In *Proc. IEEE Military Communications Conference*, pages 615–620, Oct. 2003.

[SE07] O. Sahin and E. Erkip. Achievable rates for the gaussian interference relay channel. In *Proc. IEEE Global Communications Conference (GLOBECOM'07)*, Washington D.C., USA, pp. 786–787, Nov. 2007.

[SEA03a] A. Sendonaris, E. Erkip, and B. Aazhang. User cooperation diversity, part i: System description. *IEEE Tran. Commun.*, 51(11):1927–1938, 2003.

[SEA03b] A. Sendonaris, E. Erkip, and B. Aazhang. User cooperation diversity, part ii: Implementation aspects and performance analysis. *IEEE Trans. Commun.*, 51(11):1939–1948, 2003.

[SPV09a] S. Sergi, F. Pancaldi, and G.M. Vitetta. Cooperative communication techniques for wireless ofdma-based ad-hoc networks. In *Proc. IEEE International Conference on Communications*, Jun. 2009.

[SPV09b] S. Sergi, F. Pancaldi, and G.M. Vitetta. A game theoretical approach to cluster management in a dr-ostc based cooperative miso link. Technical report, University of Modena and Reggio Emilia, 2009.

[SPV09c] S. Sergi, F. Pancaldi, and G.M. Vitetta. A game theory approach to selection diversity in wireless ad-hoc networks. In *Proc. IEEE International Conference on Communications*, Jun. 2009.

[SPVV] S. Sergi, F. Pancaldi, G.M. Vitetta, and L. Vandendorpe. Game theory and diversity exploitation in wireless communication networks. *Submitted for publication at IEEE Signal Process. Mag., Special Issue on Game Theory in Signal Processing and Communications*.

[SSE+09] Osvaldo Simeone, Oren Somekh, Elza Erkip, H. Vincent Poor, and Shlomo Shamai (Shitz). Multirelay channel with non-ergodic link failures. In *Information Theory Workshop on Networking and Information Theory (ITW2009)*, July 2009.

[SSKL09] A. Steiner, S. Shamai (Shitz), U. Katz, and V. Lupu. Successive interference cancellation and strongest users for randomly spread ds-cdma. In *Proc. 4th International Conference on Cognitive Radio Oriented Wireless Networks and Communications*, 2009.

[SSPS] O. Simeone, O. Somekh, H.V. Poor, and S. Shamai (Shitz). Downlink multicell processing with limited backhaul capacity. *to appear in EURASIP Journal on Advances in Signal Processing, special issue on Multiuser MIMO Transmission with Limited Feedback, Cooperation, and Coordination*.

[Tel95] Emre Telatar. Capacity of multi-antenna Gaussian channels. *Technical Memorandum, Bell Laboratories (Published in European Transactions on Telecommunications, Vol. 10, No.6, pp. 585-595, Nov/Dec 1999)*, 1995.

[TV05] D.N.C. Tse and P. Viswanath. *Fundamentals of Wireless Communication*. Cambridge University Press, 2005.

[VDLZ08] Luc Vandendorpe, Rodolfo Torrea Durán, Jerome Louveaux, and Abdellatif Zaidi. Power allocation for ofdm transmission with df relaying. In IEEE, editor, *ICC08*, pages 3795–3800, Bejing, China, May 2008.

[VDMC08] R. Verdone, D. Dardari, G. Mazzini, and A. Conti. *Wireless sensor and actuator networks*. Elsevier, 2008.

[Ver98] S. Verdu. *Multiuser Detection*. Cambridge University Press, 1998.

[VLOZ08] Luc Vandendorpe, Jérôme Louveaux, Onur Oguz, and Abdellatif Zaidi. Improved ofdm transmission with df relaying and power allocation for a sum power constraint. In IEEE, editor, *ISPWC08*, pages 665–669, Santorini, Greece, May 2008.

[VWL06] L. Venturino, X. Wang, and M. Lops. Multiuser detection for cooperative networks and performance analysis. *IEEE Trans. Signal Process.*, 54(9):3315–3329, Sept. 2006.

[WMK04] S.-H. Wu, U. Mitra, and C.-C. J. Kuo. Multistage mmse receivers for ultra-wide bandwidth impulse radio communications. In *Proc. IEEE Conference on Ultra Wideband Systems and Technologies*, pages 16–20, May 2004.

[WPL+02] H. Wu, Y. Peng, K. Long, S. Cheng, and J. Ma. Performance of reliable transport protocol over ieee 802.11 wireless lan: Analysis and enhancement. In *Proc. of the INFOCOM*, pages 559–608, June 2002.

[YB04] M. Yazdani and A.H. Banihashemi. Irregular rate-compatible ldpc codes for capacity-approaching hybrid-arq schemes. In *Proc. Canadian Conf. on Electrical and Computer Engineering, Saskatoon, Saskatchewan Canada*, pages 303–306, 2004.

[YV04] C. Yang and B. Vojcic. Mmse multiuser detection for cooperative diversity cdma systems. In *Proc. IEEE Wireless Communications and Networking Conference*, pages 42–47, Mar. 2004.

[ZL05] S. Zhao and H. Liu. On the optimum linear receiver for impulse radio systems in the presence of pulse overlapping. *IEEE Commun. Lett.*, 9(4):340–342, Apr. 2005.