A LOW COMPLEXITY SPACE-FREQUENCY MULTIUSER SCHEDULING ALGORITHM

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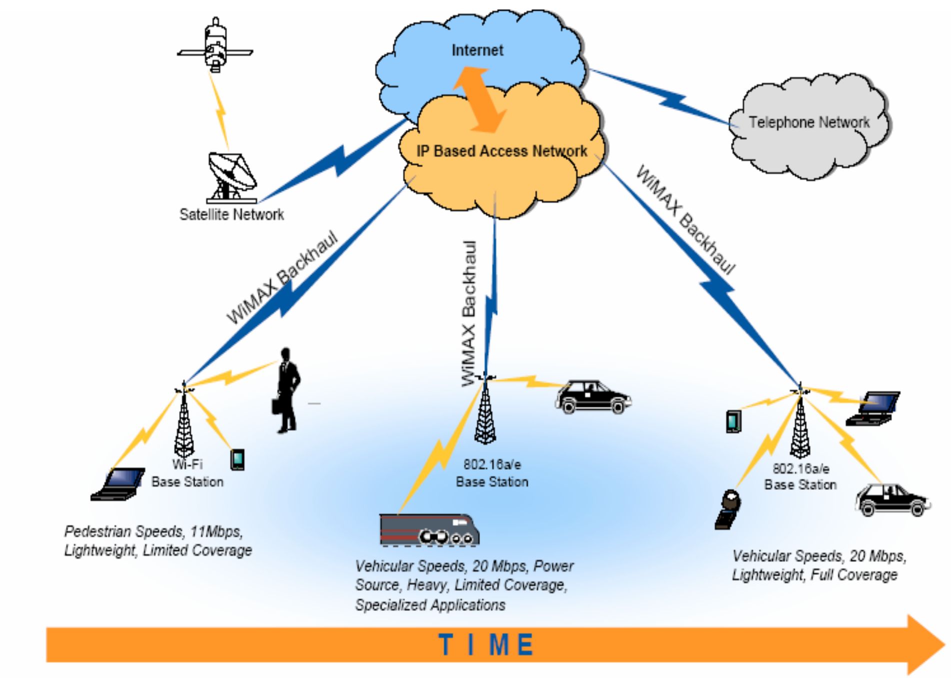
Goal

There is a need of gaining in efficiency by incorporating spatial schedulers in OFDMA (e.g. IEEE802.16, 3GPP/LTE).

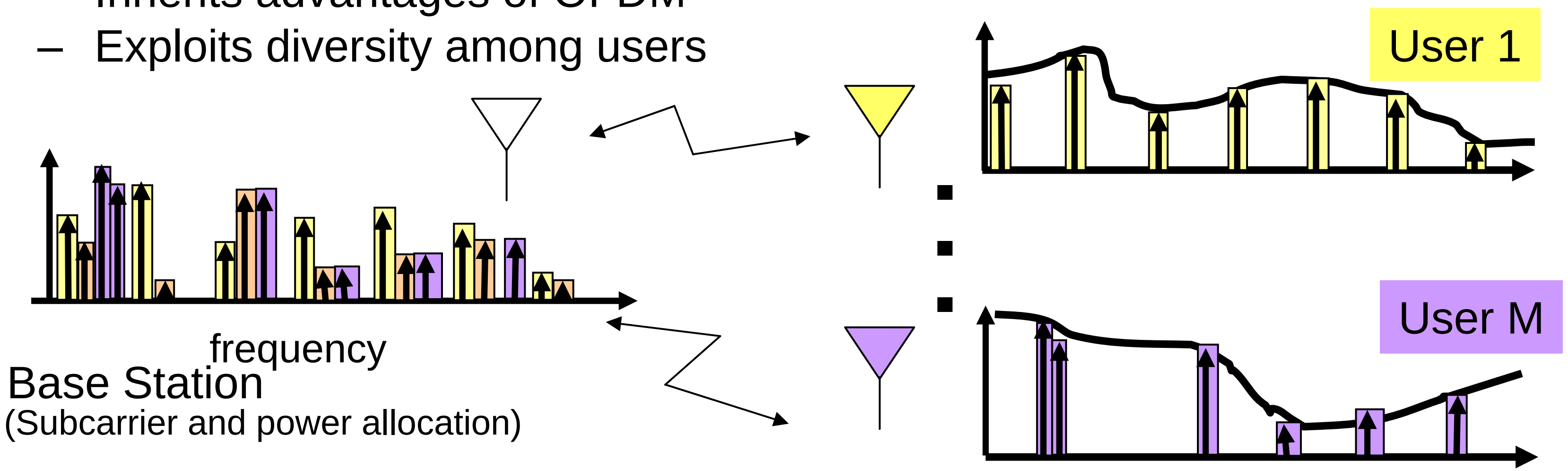
The present work proposes a Space-Frequency-Time Scheduler that differentiates services among user applications and presents low complexity by on-line solutions.

The performance is evaluated under real system conditions.



Introduction

- OFDMA is adopted by IEEE 802.16a/d/e and 3GPP-LTE
- Allows multiple users to transmit simultaneously on different subcarriers
  - Inherits advantages of OFDM
  - Exploits diversity among users



•How do we allocate M data subcarriers and total power P to K users to optimize some performance metric? (voice and data applications)

•Difficult discrete problem

•Brute force optimal solution: search through K^M subcarrier allocations and determine power allocation for each

•The complexity increases if spatial diversity is incorporated at each subcarrier

Ex: The optimal MIMO BC with OFDMA

$$\mathbf{y}_k = \mathbf{H}_k \sum_{i=1}^{K} \mathbf{x}_i + \mathbf{w}_k \qquad with \quad \mathbf{H}_k = diag\left\{\mathbf{H}_{k1} \cdots \mathbf{H}_{kM}\right\}$$

$$\Theta = \left\{ \mathbf{Q} = \left[\mathbf{Q}_1 \cdots \mathbf{Q}_K\right], tr\left(\sum_{k=1}^{K} \mathbf{Q}_k\right) \le P_T \right\} \quad with \quad \mathbf{Q}_k = diag\left\{\mathbf{Q}_{k1} \cdots \mathbf{Q}_{kM}\right\}$$

$$\Pi : set\ of\ all\ possible\ permutations\ on \left\{1, \ldots, K\right\}$$

The achievable rate vector is

$$\mathbf{r}\left(\mathbf{Q}, \pi\right) = \left(r_1\left(\mathbf{Q}, \pi\right), \ldots, r_K\left(\mathbf{Q}, \pi\right)\right)$$

$$r_{\pi(i)} = \log \frac{\det\left(1 + \mathbf{H}_{\pi(i)}\left(\sum_{j \ge i} \mathbf{Q}_{\pi(j)}\right)\mathbf{H}_{\pi(i)}^H\right)}{\det\left(1 + \mathbf{H}_{\pi(i)}\left(\sum_{j > i} \mathbf{Q}_{\pi(j)}\right)\mathbf{H}_{\pi(i)}^H\right)}$$

The capacity region is

Ex: The optimal MIMO BC with OFDMA

$$C = co\left(r\left(\mathbf{Q},\pi\right):\mathbf{Q}\in\Theta,\pi\in\Pi\right)$$

Accordingly, each element of C can be written as a time sharing

$$\mathbf{r} = \sum_{w=1}^{W} \alpha_w \mathbf{r}\left(\mathbf{Q}^{(w)},\pi^{(w)}\right) \qquad W \le K$$

$$If\ \mathbf{x}_p = \left(\alpha_w,\mathbf{Q}^{(w)},\pi^{(w)}\right)_{w=1}^{K}$$

then

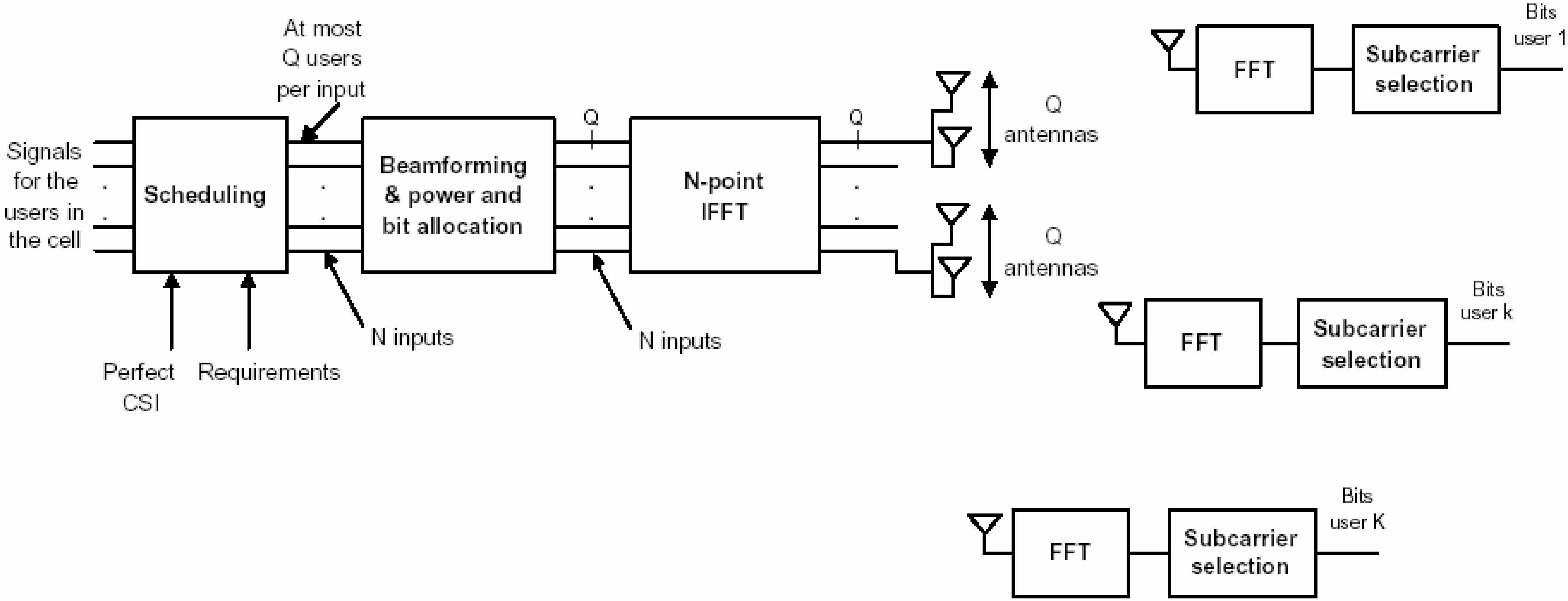
General problem

$$\max_{\mathbf{x}_p} \mathbf{u}\left(\mathbf{r}\left(x_p\right)\right)$$

$$s.t. \quad \mathbf{x}_p \in \mathbf{X}_p$$

u: utility, monotonically increasing

Problem Statement



There are several variables involved to sum rate maximization.

- Best frequencies have to be taken.
- Find and optimal space strategy.
- Allocate power subject to constraints along frequency and space.
- Best users have to be taken.

The problem of low complexity solution is open in the literature.

Contents

- Problem formulation
- Low Complexity Scheduler: ergodic formulation
- Dual Optimization: parallel implementation
- Spatial scheduler
- Waterfilling
- Parameter update
- Simulations
- Results

Problem Statement

$$y_{k,m} = a_{k,m}\sqrt{p_{k,m}}\,\alpha_{k,m,k_q} s_{k,m} + \sum_{s \neq k} a_{s,m}\sqrt{p_{s,m}}\,\alpha_{k,m,s_q} s_{s,m} + w_{k,m}$$

In BC

$$\left|\alpha_{k,m,k_q}\right|^2 = c_{k,m,k_q} = \left|\mathbf{h}_{k,m}^T \mathbf{b}_{m,k_q}\right|^2$$

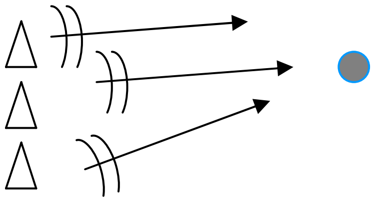
The proposed utility will be based on $\mathrm{SINR}_{\text{user, freq}}$. Assuming perfect isolation between frequencies, interferences are only present spatially

$$\gamma_{k,m,k_q}(\mathbf{a},\mathbf{p}) = \frac{a_{k,m} p_{k,m} c_{k,m,k_q}}{\sigma^2 + \sum_{s \neq q}^{N_T} a_{s,m} p_{s,m} c_{k,m,k_s}}$$

user (→ k) frequency (→ m)

$$a_{k,m} \in \{0,1\} \qquad k = 1 \ldots K \quad m = 1 \ldots M$$

Scheduling variable (→ $a_{k,m}$)



Problem Statement

If we define for each user *k*

$$r_k \triangleq \sum_{m=1}^{M} \mathrm{E}_{\gamma}\left\{\log_2\left(1+\gamma_{k,m,k_q}\right)\right\}$$

$$\hat{p}_k \triangleq \sum_{m=1}^{M} \mathrm{E}_{\gamma}\left\{a_{k,m} p_{k,m}\right\}$$

The main goal is to maximize sum rate

M carriers, K users

MISO, no precoder design

$$\left\{\begin{aligned} &\max_{\mathbf{a},\mathbf{p}} R(\mathbf{a},\mathbf{p}) = \sum_{k=1}^{K} r_k(\mathbf{a},\mathbf{p}) \\ &s.t. \quad r_k \geq \phi_k R, \quad k=1,\ldots,K \\ &\qquad \sum_{k=1}^{K} \phi_k = 1 \\ &\qquad \sum_{k=1}^{K} \hat{p}_k \leq \overline{P} \\ &\qquad p_k \geq 0 \qquad k=1,\ldots,K \end{aligned}\right.$$

Proportional rate (service class)

Low complexity scheduler

- We propose a new scheduler that is based on the following strategy:
  - Ergodic costs and constraints: to incorporate the Time diversity and to reduce complexity
    - Stochastic approximation: to obtain an on-line adaptive algorithm
  - Dual optimization techniques: $O(M_{carriers}, K_{users})$: canonical distributed algorithm
  - Opportunistic user selection

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Dual optimization

Lagrangian becomes $$L = R\left(1-\boldsymbol{\mu}^T\phi\right)+\lambda\overline{P}-\lambda\sum_{k=1}^{K}\hat{p}_k+\sum_{k=1}^{K}\mu_k r_k$$

as $1-\boldsymbol{\mu}^{\mathrm{T}}\phi=0$

then $$L=\lambda\overline{P}-\lambda\sum_{k=1}^{K}\hat{p}_k+\sum_{k=1}^{K}\mu_k r_k$$

Defining $$L_k=\mu_k r_k-\lambda_k\hat{p}_k \qquad \lambda_k=\lambda$$

$$\Downarrow$$

$$\boxed{L=\sum_k L_k+\lambda\overline{P}}$$

Dual optimization

The dual objective is defined as

1

$$g(\lambda,\boldsymbol{\mu}) = \max_{\mathbf{a},\mathbf{p}} \left(\sum_k L_k(\mathbf{a},\mathbf{p},\lambda_k,\boldsymbol{\mu}) + \lambda \overline{P} \right) =$$

$$= \sum_k L_k^*(\mathbf{a}^*,\mathbf{p}^*,\lambda_k,\boldsymbol{\mu}) + \lambda \overline{P}$$

The dual problem is

2

$$\min g(\lambda,\boldsymbol{\mu})$$

$$s.t. \quad \lambda \ge 0, \boldsymbol{\mu} \in D \quad D = \left\{ \boldsymbol{\mu} \ge \mathbf{0}, \boldsymbol{\mu}^T \phi = 1 \right\}$$

IN PARALLEL
[Chiang07]

$$\begin{cases} \max \ \mu_k r_k(\mathbf{a},\mathbf{p}) + \lambda \overline{P} \\ subgradient \ \ for \ \mu_k \\ \\ subgradient \ \ for \ \lambda \end{cases}$$

always convex and involves only (K+1) variables

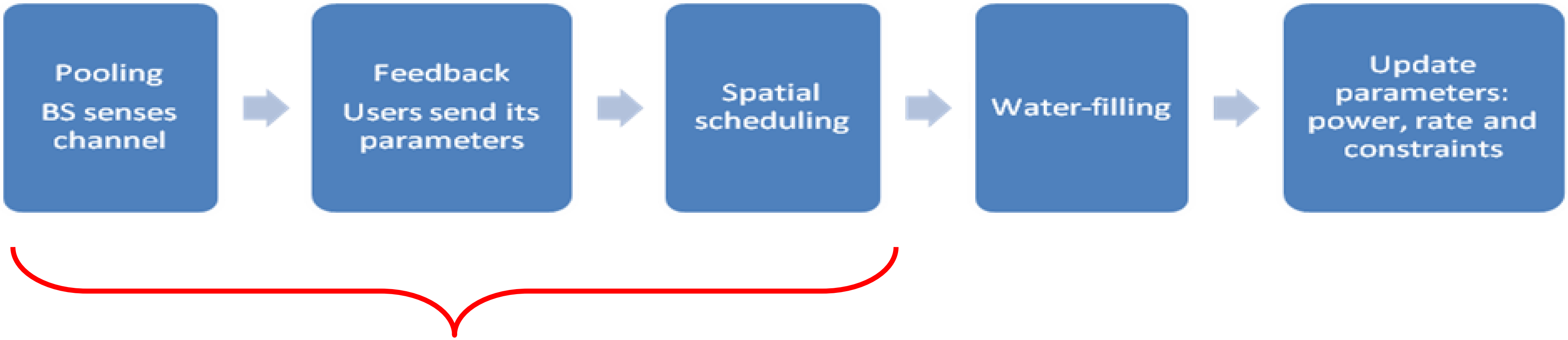
λ, μ are obtained by subgradient

Complexity

User-freq. Power Allocation: O(MK), once user selection is done

λ, μ: (K+1)

Algorithm structure



Spatial Scheduler (BC)

$$\gamma_{k,m,k_q}^{(Q_m)} = \frac{\left| \mathbf{h}_{k,m}^{T} \mathbf{b}_{m,q} \right|^2}{\sigma_n^2 + \sum\limits_{\substack{q^{'} \neq q \\ q^{'} \in S_m^{(Q_m)}}} \left| \mathbf{h}_{k,m}^{T} \mathbf{b}_{m,q^{'}} \right|^2} \qquad m \in S_m^{(Q_m)} \qquad k \in U_m^{(Q_m)}$$

Multibeam opportunistic scheme is used

+ optimal beam and user search (if low #users)

$$k_{m,j,q}^{*} = \arg\max_{k} \gamma_{k,m,j,q}^{(Q_m)}$$

$$j^{*} = \arg \max_{1 \le j \le \binom{N_T}{Q_m}} \sum_{q=1}^{Q_m} \log_2 \left(1 + \gamma_{k_{m,j,q}^{*},m,j,q}^{(Q_m)} \right)$$

$$\binom{Q_m^{*}}{k_{m,j,q}^{*}, m, j^{*}, q} \square\; k, m, k_q$$

$$Q_m^{*} = \arg \max_{1 \le Q_m \le NT} \sum_{k \in U_m^{*(Q_m)}} \sum_{q \in S_m^{*(Q_m)}} \log_2 \left(1 + \gamma_{k,m,j^{*},q}^{(Q_m)} \right)$$

Power Waterfilling

$$\frac{\partial L_k}{\partial p_{k,m}} = 0 \qquad \forall k$$

$$p^{*}_{k,m} = \left[\frac{\mu_k}{\lambda \ln(2)} - \frac{1}{\gamma^{1}_{k,m,k_q}} \right]^{+}$$

$$\gamma^{1}_{k,m,k_q} = \frac{c_{k,m,k_q}}{\sigma^2 + \sum_{s \neq q}^{N_T} a_{s,m} P c_{k,m,k_s}}$$

Parameter update

Per user rate and power are estimated as

$$R_k[n] = \sum_{m \in M} R_{k,m}\left(\gamma_{k,m,k_q}(p^*_{k,m}[n])\right)$$

$$P[n] = \sum_{m \in M}\sum_{k \in K} p^*_{k,m}[n]$$

Stochastic Subgradient method, across time

$$\lambda[n+1] = \left[\lambda[n] - \delta\left(\overline{P} - P[n]\right)\right]^+$$

$$\boldsymbol{\mu}[n+1] = \Pi_{\mathcal{D}}\left(\boldsymbol{\mu}[n] - \delta\left(\mathbf{R}[n] - \phi R[n]\right)\right)$$

$$\mathcal{D} = \left\{\boldsymbol{\mu} \geq \mathbf{0} \middle| \boldsymbol{\mu}^T \phi = 1\right\}$$

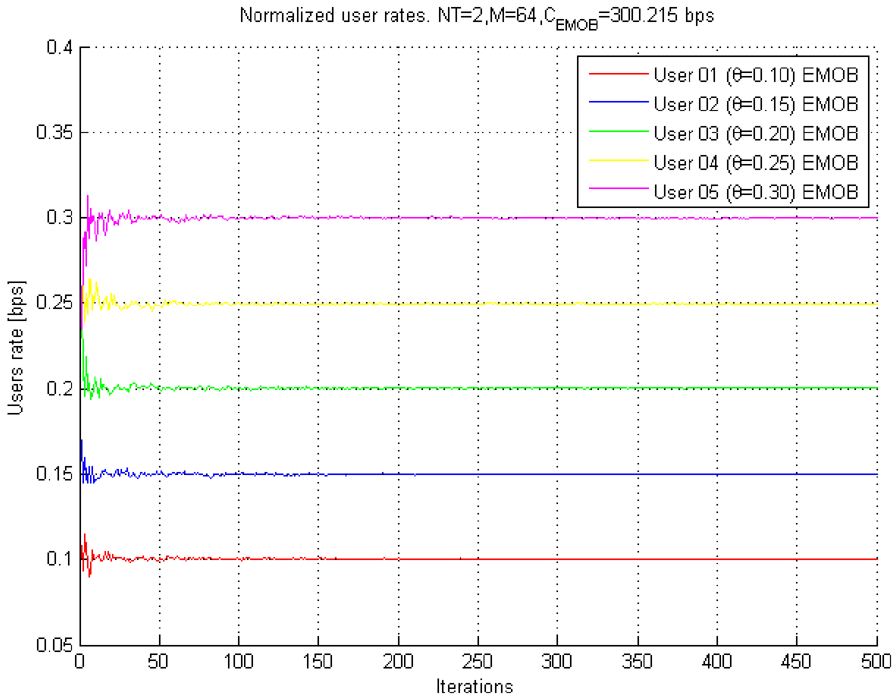
$$\mathbf{R}[n] = \left[R_1[n] \quad \cdots \quad R_K[n]\right]^T$$

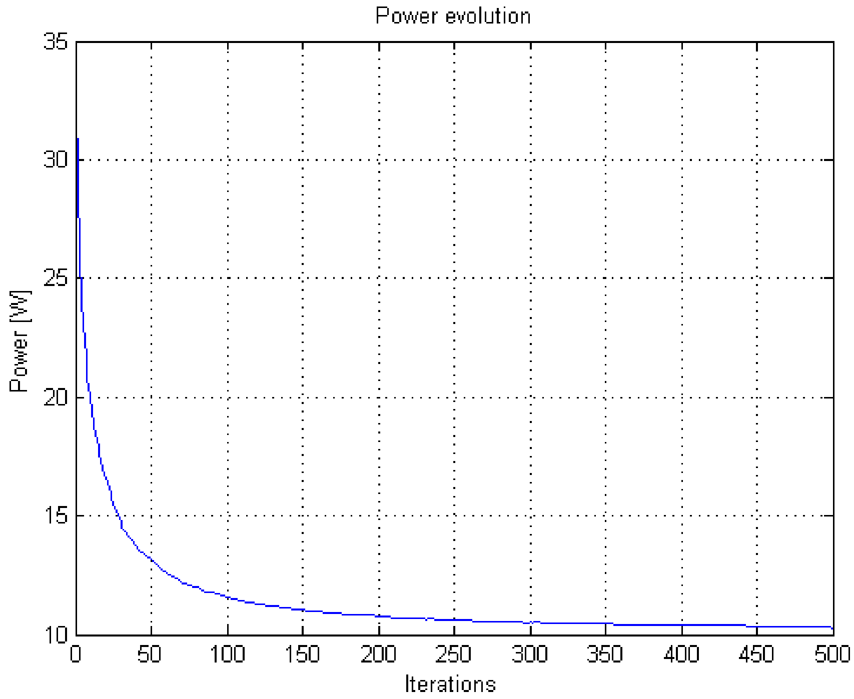
$$R[n] = \sum_{k \in K} R_k[n]$$

Results: Scenario 1

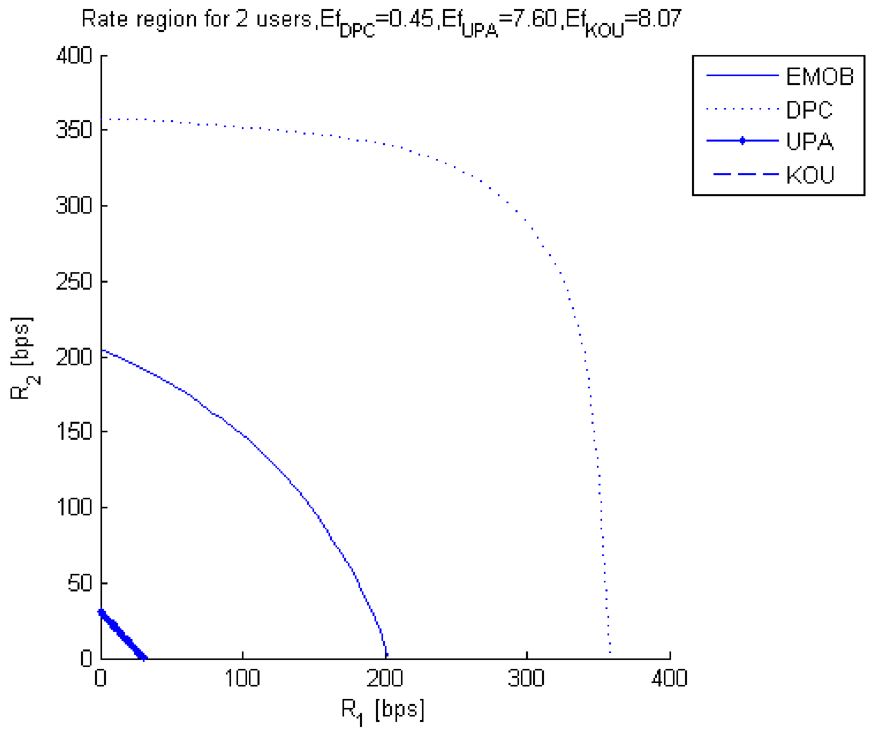
5 users, 64 subcarriers, same distance to BS, Rayleigh channel

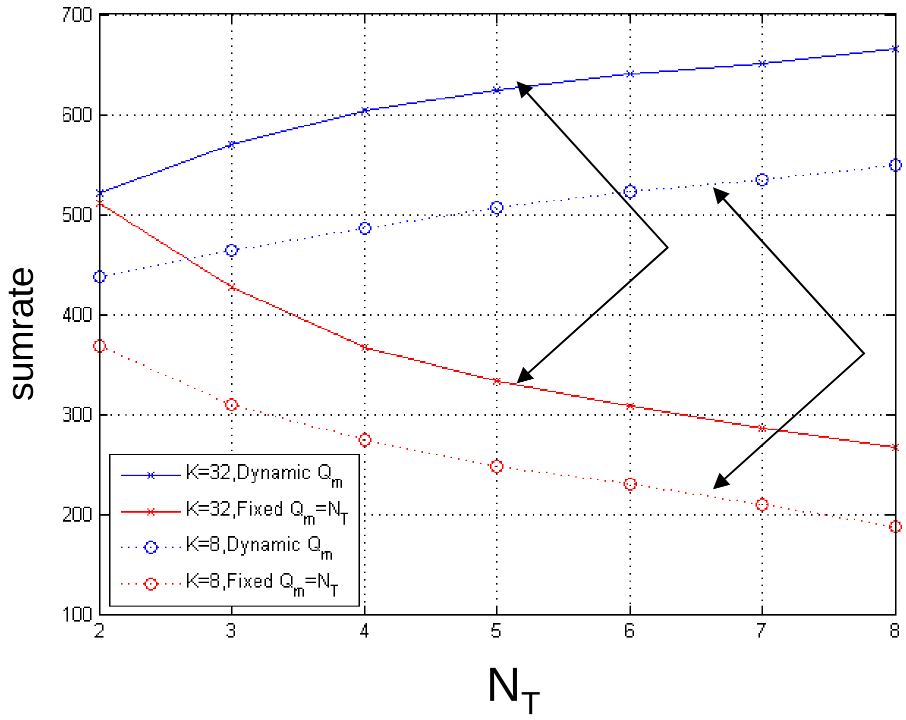
$\bar{P} = 10\ dB$





Results: Scenario 1



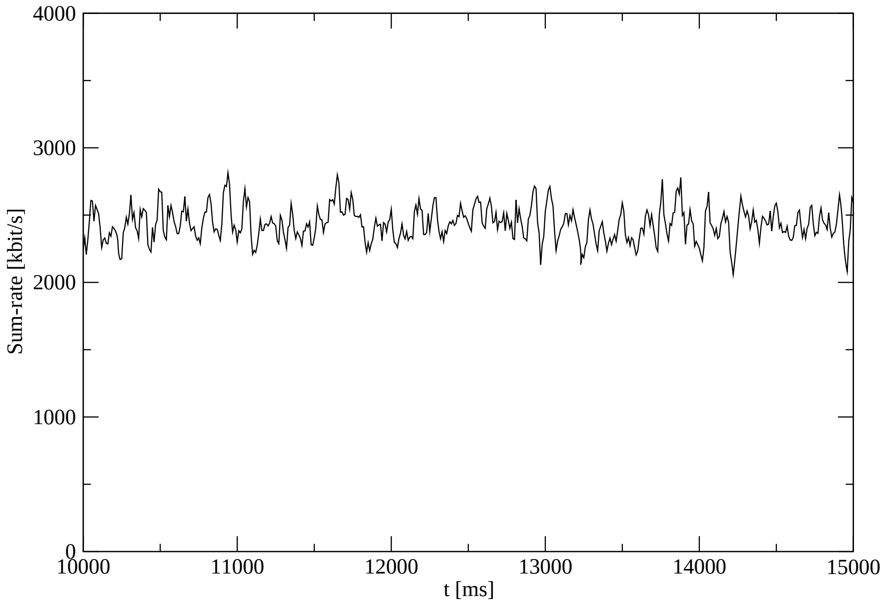


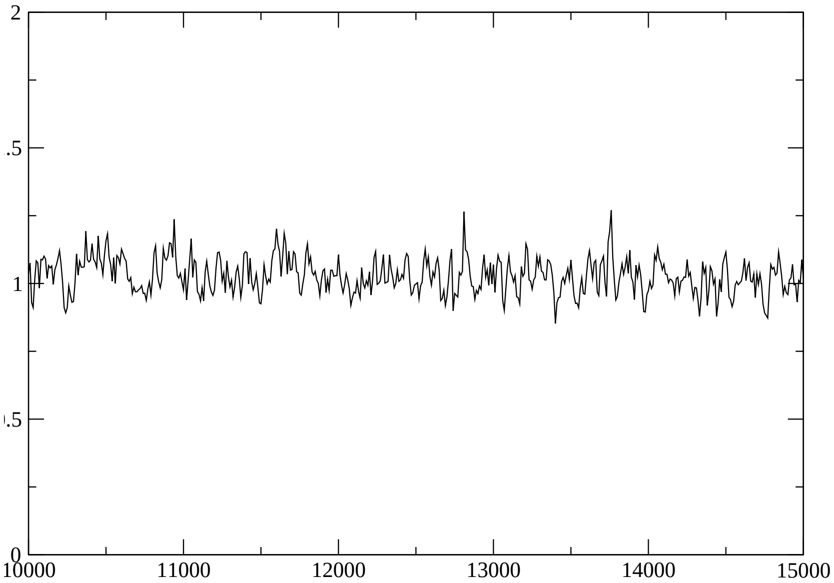
To counteract low K in MOB

Results: Realistic Scenario 2

M= 128 sub-carriers, Bw= 125 Mhz, CSI update every frame (10 ms)
- Uniformly distributed (500 m)
-With the same distance (250 m)

Path loss, shadowing and fast fading is included, spatial model as in 3GPP pedestrian
Delay spread: 2-3 microseconds
Doppler bandwidth 6 Hz
P= 1W
ϕ

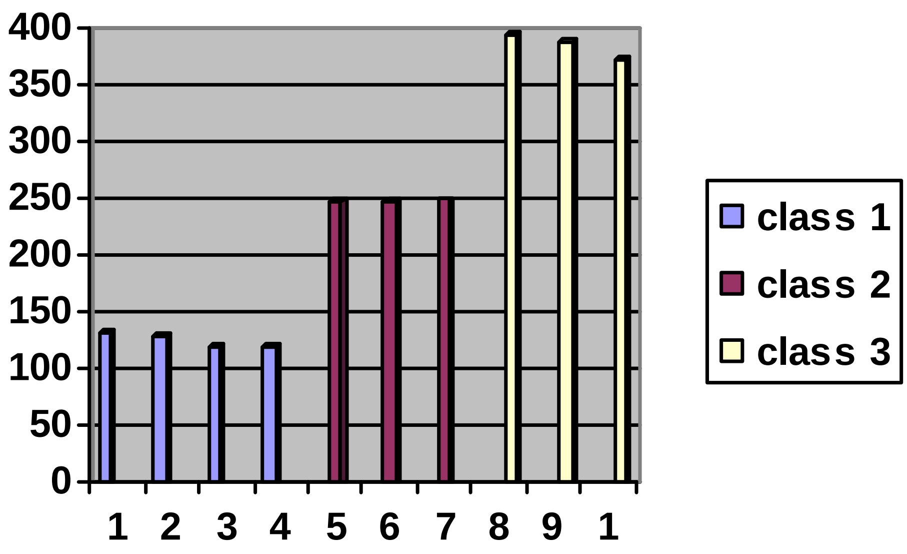




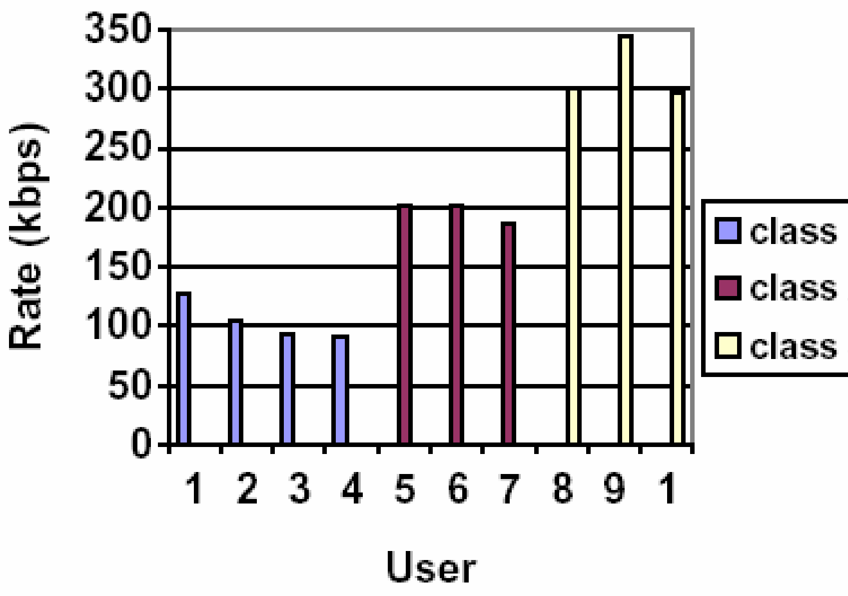
Results: Realistic Scenario 2

<u>Average user rates</u>

10 users, 3 antennas and 3 different user classes (class 1: weight 0.5/K - class 2: weight 1/K - class 3: weight 1.5/K). Fig. 1 refers to equal distance users at 250 m, whereas fig. 2 refers to uniformly distributed users in a circular area of radius 500 m.



250 m



500 m

Conclusions

- Low complexity algorithm (dual decomp., estochastic, opportunisitic)
- This algorithm guarantees weighted rate to all users.

$$\frac{R_i}{\theta_i} = \cdots = \frac{R_j}{\theta_j}$$

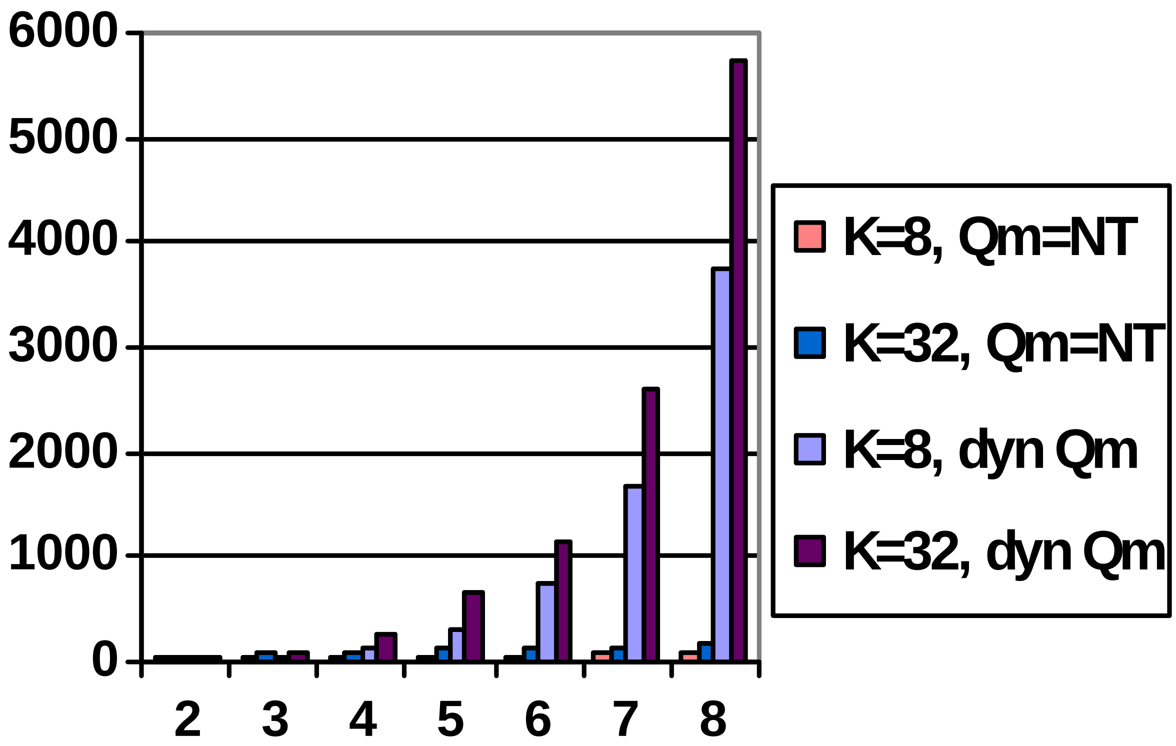
- It takes care of maximization through all frequencies range.
- It can be arranged to many scenarios.
- Different Space division access strategies can be used on it.
- Only partial CSIT (alternatives are opportunistic splitting, persistent scheduling,…).
- Evaluation in a realistic scenario
- Opens different lines for further work

Thank you

Questions ?

Results: Scenario 1

Time Complexity

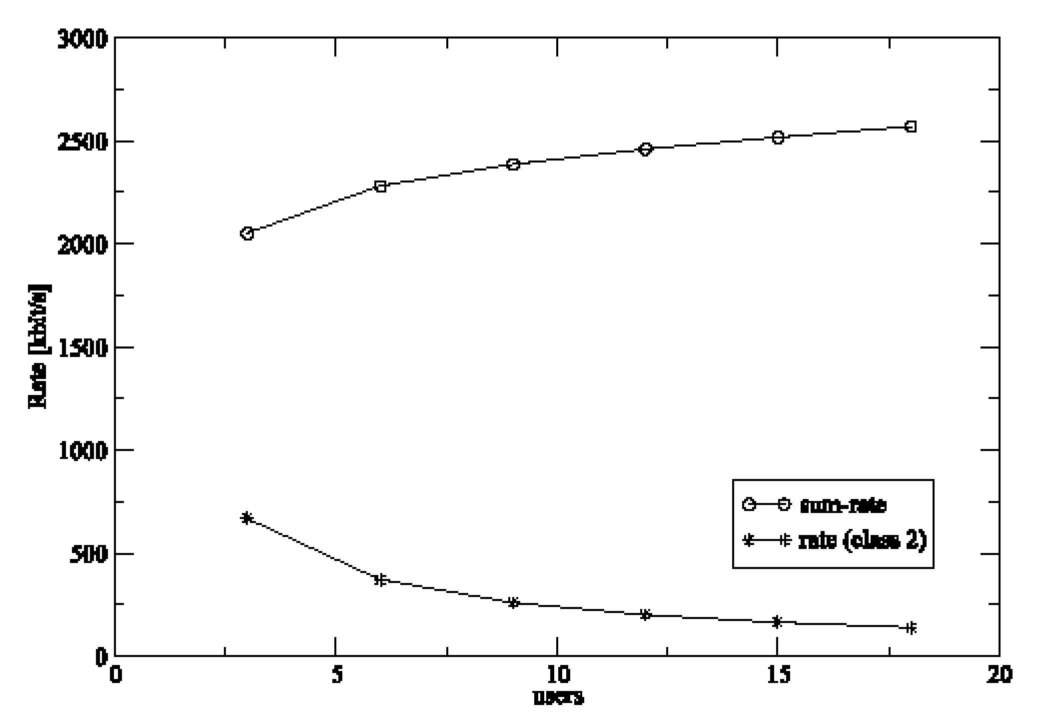


	K=8	K=32
[13]	283	3349
Our algorithm ($Q_m = N_T$ / $Dyn\ Q_m$)	28 / 31	50 / 62

Comparison with DPC $O(K^2 M \log N)$

Results: Realistic Scenario 2

N= 2, 250 m $\phi = \left[0.5/K \quad 1/K \quad 1.5/K \right]$



Fairness and multiuser diversity !!!